Pricing Stock Market Volatility: Does it Matter whether the Volatility is Related to the Business Cycle?

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ABSTRACT
This paper investigates the impact of business cycle-related market volatility on expected returns. We develop a model that enables us to decompose the market volatility into two components: business cycle-related volatility and unrelated volatility. Then, the risk-return relation is assessed based on these two components. Our empirical results demonstrate that business cycle-related market volatility is priced in the stock market, whereas the unrelated component is not. Furthermore, our procedure identifies a few periods of high volatility that are not related to recessions, including the 1987 crash and the 1998 Russian default. (JEL: G12, C32, C51)

KEYWORDS: business cycles, expected returns, market volatility, Markov-switching, risk-return trade-off, long-run risk (LRR) asset pricing model

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Many researchers have posited that there may be a link among business cycles, stock market volatility, and expected returns. For example, this proposed link is frequently used to explain why certain financial/macroeconomic variables, such as the term spread or the short-term interest rate, have an explanatory power on stock return movement (Campbell and Shiller, 1989; Fama and French, 1988, 1989; Campbell and Diebold, 2009; Rapach et al., 2010; and Henkel et al., 2011). In other words, these different variables somehow act as a proxy for future business conditions/cycles, likely capturing time-varying market risk/volatility, which in turn drives the time variation in expected excess returns. Hereafter, we refer to the sequential link among business cycles, market risk, and market return as the “underlying mechanism” of the literature on return predictability. This underlying mechanism gives rise to the need to test directly the link between business cycles and expected returns via stock market volatility.

Schwert (1989), Hamilton and Lin (1996), and Campbell and Diebold (2009) provide evidence of the first link of the business cycle-market volatility-expected return mechanism. Their findings suggest that stock market volatility is counter-cyclical. On the other hand, there has also been a great deal of research into the second link of the mechanism (literature on the risk-return trade-off). Bollerslev and Zhou (2006) describe what they refer to as the “volatility puzzle.” According to their research, some studies (e.g., French et al., 1987; Chou, 1988; Campbell and Hentschel, 1992; Bali and Peng, 2006; and Guo and Whitelaw, 2006) have reported consistently positive and significant estimates of the risk premium in the market. In contrast, others (e.g., Campbell, 1987; Turner et al., 1989; Breen et al., 1989; Chou et al., 1992; and Glosten et al., 1993) have documented negative values, unstable signs, or otherwise insignificant estimates.

Given the evidence of the link between business cycles and market volatility (the first link of the underlying mechanism), we reinvestigate the risk-return relationship conditioning on business cycles in this paper. Existing research on the risk-return trade-off ignores the possibility that different sources of market volatility might exert different effects on market returns. As an example of different sources of market volatility, Figure 1 represents the time series plot of the risk-neutral implied volatility for the S&P500 (the VIX), which can be viewed as a measure of market volatility.\footnote{For reference, the NBER recession dates are shown in the shaded areas in Figure 1. The plot of the VIX is drawn only after January 1990 as a result of data availability constraints at the Chicago Board Options Exchange (CBOE).} We observe an increase in market volatility during recessions (shown in the shaded areas), as pointed out by Schwert (1989) and Hamilton and Lin (1996). However, there also exist volatility spikes in 1997, 1998, 2002, 2010, and 2011, which do not coincide with economic recessions. Thus, if one looks at the risk-return relationship...
conditioning appropriately on the business cycles, the volatility puzzle may be resolved.

In this paper, we consider the possibility that market volatility can be decomposed into two components: (i) business cycle-related volatility and (ii) unrelated volatility. Then, we investigate the risk-return relationship based on these two components. Towards this end, we develop a bivariate regime-shift model of stock returns and output growth within Campbell and Shiller’s (1989) log-linear present value framework.

Our empirical results demonstrate that business cycle-related market volatility is priced in the stock market, whereas the non-cyclical counterpart of market volatility is not. Thus, the well-known volatility puzzle may derive from the failure to account for the business cycle-related component of market volatility. In other words, the previous findings may have reflected the average effect of both cyclical volatility and non-cyclical volatility. Depending on which component predominates in a particular dataset, sample period, or model specification, the result can be positive, negative, or mixed. In addition, we identify a few periods of high volatility that are not related to recessions, most notably the 1987 crash and the 1998 Russian default.

Our empirical findings are consistent with the recently developed long-run risk (LRR) literature, pioneered by Bansal and Yaron (2004). The LRR asset pricing model demonstrates that long-run volatility movements, mainly measured by consumption growth volatility, have strong explanatory power for asset prices (e.g., Bansal and Yaron, 2004; Bansal et al., 2005; Bansal et al., 2007; and Bollerslev, Tauchen, and Zhou, 2009). In our bivariate model, the cyclical volatility component of stock returns plays a similar role as the long-run stochastic volatility in the LRR literature, because our cyclical volatility component is not related to the transitory shock either in output growth or in stock returns. This point will be discussed further in a later section (Section 2.2).

Bansal et al. (2005) employ various measures of economic uncertainty in investigating the risk-return relationship. They find significant results with consumption growth volatility (and GDP growth volatility as well), but insignificant results with stock market volatility. They claim that consumption volatility (rather than stock market volatility) is an effective barometer of fundamental economic uncertainty. Given that stock market volatility has been used as a standard measure for uncertainty in the previous literature, their puzzling findings have provided us further motivation to investigate market volatility, especially linking it to economic fundamentals, as specified in our bivariate model.

The rest of this paper is organized as follows. Section 1 describes the model proposed in this paper. The key features of the proposed model and the estimation procedure are discussed in this section. Section 2 describes the data and presents estimation results. Further discussion on the LRR literature is
also presented. Concluding remarks are presented in Section 3. All technical derivations are provided in the Appendix.

1 THE MODEL

To investigate the market risk-return relationship conditioning on business cycles, we consider two different sources of market volatility: a business-cycle component and a non-cyclical component. As mentioned previously, the stock returns may themselves constitute a very noisy signal while identifying the cyclical volatility movement. As a result, using stock returns alone to identify regime changes can prove inferior to applying a joint approach that can employ some additional information to clear up the noisy signal. Therefore, we develop a bivariate regime-shift model by applying both the regime-shift stock return model of Kim et al. (2004) and the regime-shift output growth model of Hamilton (1989) to differentiate between cyclical and non-cyclical volatility movements. As will be demonstrated graphically in a later section, our joint approach can identify all NBER recessions more effectively than a single equation approach.

1.1 Key features of the proposed model

Several key building blocks of the proposed model are explained in this section. First, we consider stock prices in natural logarithm (denoted as \( p_t \)) consisting of two components: the fundamental component (denoted as \( p_t^* \)), which is derived from the future dividend flow as per Campbell and Shiller (1989), and the transitory component (denoted as \( z_t \)) which is simply the deviation of the stock price from its fundamental component. Therefore, we have the following:

\[
p_t = p_t^* + z_t.
\] (1)

If we define the stock return as \( r_t = \Delta p_t = p_t - p_{t-1} \), it is obvious that the stock return itself consists of two components: the fundamental component (\( \Delta p_t^* \)) and the transitory component (\( \Delta z_t \)), as:

\[
r_t = \Delta p_t^* + \Delta z_t.
\] (2)
We assume that unexpected shocks to these two components (denoted as $e_t$ and $v_t$, respectively) are subject to Markov-switching variances as follows:

$$e_t \sim N(0, \sigma_{e,t}^2), \quad \sigma_{e,t}^2 = \sigma_{e,0}^2(1 - S_{it}) + \sigma_{e,1}^2 S_{it}, \quad \sigma_{e,0}^2 < \sigma_{e,1}^2,$$

$$v_t \sim N(0, \sigma_{v,t}^2), \quad \sigma_{v,t}^2 = \sigma_{v,0}^2(1 - S_{it}) + \sigma_{v,1}^2 S_{it}, \quad \sigma_{v,0}^2 < \sigma_{v,1}^2,$$

$$\Pr[S_{it} = 1 | S_{i,t-1} = 1] = p_i \quad \text{and} \quad \Pr[S_{it} = 0 | S_{i,t-1} = 0] = q_i, \quad i = 1, 2,$$

where $S_{it}, (i = 1, 2)$ are unobservable binary state variables ($S_{it} = 1$ for the high volatility regime and $S_{it} = 0$ otherwise); $q_i$ and $p_i$ are transition probabilities governing the evolution of $S_{it}$. We note that two different state variables (each having two different volatility regimes) allow us four different volatility regimes for stock returns. Hence, our specification for volatility regimes can be considered as more general than that of Kim et al. (1998), where three different volatility regimes are used to capture the heteroskedastic characteristics in monthly stock returns.

Secondly, we devise a mechanism to connect the regime-switching process in the fundamental stock return to the recession-related regime variable in Hamilton’s (1989) business cycle model, as follows:

$$y_t = \mu_i + \psi_{y,1} y_{t-1} + \psi_{y,2} y_{t-2} + u_t, \quad u_t \sim N(0, \sigma_u^2),$$

$$\mu_i = \mu_0 (1 - S_{i,t-1}) + \mu_1 S_{i,t-1}, \quad \mu_0 > \mu_1,$$

where $y_t$ denotes the monthly output growth rate; $S_{it}$ identifies recession periods with low mean growth, $\mu_i$. We also use the recession-identifying state variable, $S_{it}$, to identify the high-volatility regime in the fundamental stock return process in (3), reflecting the well-documented counter-cyclicality of stock market volatility. Therefore, the high-volatility regime in the fundamental stock return can be interpreted as “recession-related,” whereas the high-volatility regime in the transitory stock return is “recession-unrelated.” We also note that, in equation (6), recession lags one period behind a high-volatility regime of the stock market to take into account that stock market participants are usually forward-looking.\(^3\) The

\(^3\) The forward-looking feature of market participants (or the lag feature of the output equation) is
lagged version of Hamilton’s (1989) model is first introduced in Hamilton and Lin (1996), using a single underlying state for a bivariate system of stock returns and output. One advantage of our two state variable approach over that of Hamilton and Lin (1996) is that it offers a better description of the time series properties of stock return volatility, since not all high-volatility regimes are related to recessions (e.g., the heightened volatility following the 1987 stock market crash and the 1998 Russian default).

This second feature summarized in equation (6) can be modified in many interesting ways to link one part of the volatility process to fundamental macroeconomic variables. It is possible to use interest rate dynamics or the consumption growth dynamics if one believes that one of these alternative macroeconomic variables better identifies the cyclical movement in the risk-return relationship than does the output variable. An important point of this modification is to impose reasonable dynamics for each candidate variable. As an example, interest rates can be modeled as having regime-shifts in both mean and variance (see Garcia and Perron, 1996; Ang and Bekaert, 2002; Bansal and Zhou, 2002; and Dai, Singleton, and Yang, 2007), while consumption growth has a regime shift in mean alone (see Cecchetti et al., 1990; and Kandel and Stambaugh, 1990). For the output growth rate (i.e. our measure of the fundamental macroeconomic variable), Hamilton (1989) shows that an autoregressive model (AR(2)) with mean-shift well identifies business cycles for his 1952-1984 sample period. However, it has been documented in the business cycle literature that there is no empirical evidence for a volatility shift in output growth. Hamilton and Lin (1996) confirm this feature using a slightly modified model of output growth. Therefore, following the business cycle literature, we use the AR(2) model with a mean shift only as specified in (6).

As the high-volatility regime associated with \( \epsilon_t \) (the unexpected shock to the fundamental stock return) is identified as a recession-related regime by sharing the same state variable, \( S_{t-1} \), in equations (3) and (6), another high-volatility regime identified by \( S_{2,t} \) (which is not functionally related to recessions/economic conditions) becomes the non-cyclical regime of the stock market. This non-cyclical regime is expected to capture several historical events, such as the 1987 stock market crash and the 1998 Russian default period. Consistent with this notion, we specify a model for the transitory return as follows:

\[
z_t = \tau S_{2,t} + \phi_1 z_{t-1} + \phi_2 z_{t-2} + v_t.
\]

conceptually consistent with our modeling framework that uses Campbell and Shiller’s (1989) log-linear present value framework, presented in Section 1.2. Campbell and Shiller’s (1989) model allows both expected returns and expected future cash flows to affect asset prices, imposing a forward-looking feature of stock prices.

6
Kim and Kim (1996) introduced the transitory component of a stock price, which captures historical (temporary) liquidity crisis periods. Following Kim and Kim (1996), we use the AR(2) dynamics for the transitory component of stock prices, as specified in (7).4

Lastly, we consider a linear function for the relationship between stock market volatility and expected returns, as follows:

$$E[\Delta \rho_{t+j} | I_t] = \beta_1 E[\sigma_{t+j}^2 | I_t] + \beta_2 E[\sigma_{t+j}^z | I_t],$$

where $I_t$ is the conditioning information set available at time $t$. The $\beta_1$ parameter reflects the marginal effect of market volatility arising from the business cycle-related component on the expected return, and $\beta_2$ is the effect of the non-cyclical counterpart. The specification of the risk-return relationship in equation (8) can be elaborated further to allow for four different risk premiums (two types of volatility coupled with two separate regimes, i.e., high-cyclical, low-cyclical, high-transitory, and low-transitory volatilities), but we do not pursue this elaborated extension in this paper.

1.2 The proposed empirical model

Incorporating the features described in Section 1.1, we derive an empirical model of stock returns and output growth within Campbell and Shiller’s (1989) log-linear present value framework. It is shown in the Appendix that the stock return equation in (2), together with equations (3)-(5) can be rewritten as follows:

$$r_t = \beta_1 E[\sigma_{t+j}^2 | I_{t-1}] + \beta_2 E[\sigma_{t+j}^z | I_{t-1}]$$

$$+ \delta \left( E[\sigma_{t+j}^2 | I_{t-1}'] - E[\sigma_{t+j}^2 | I_{t-1}] \right) + \delta_z \left( E[\sigma_{t+j}^z | I_t'] - E[\sigma_{t+j}^z | I_{t-1}] \right) + \Delta z_t + \epsilon_t,$$

where $I_t$ is the conditioning information set available at time $t$. The $\beta_1$ parameter reflects the marginal effect of market volatility arising from the business cycle-related component on the expected return, and $\beta_2$ is the effect of the non-cyclical counterpart. The specification of the risk-return relationship in equation (8) can be elaborated further to allow for four different risk premiums (two types of volatility coupled with two separate regimes, i.e., high-cyclical, low-cyclical, high-transitory, and low-transitory volatilities), but we do not pursue this elaborated extension in this paper.

4 A later section (Section 2.1) shows that the estimates of the parameters in equation (7) ($\tau$, $\phi_1$, and $\phi_2$) are significant, with the AR coefficients being transitory. As expected, the estimates of the unobserved non-cyclical component of stock prices ($z_t$) will be shown later to capture the historical (temporary) liquidity crisis periods.
where \( \delta_i = -\beta_i / (1 - \rho \lambda_i) \), \( \lambda_i = p_i + q_i - 1 \), \( i = 1, 2 \), \( \rho = 0.997 \).\(^5\) We note that the specification for \( z_i \) in (9) has already been given in (7). The information set \( I'_t \) denotes the information available to investors at the end of trading period \( t \), including observations on trades that continuously occur during the period.

Then, equation (9), along with the output equation in equation (6), completes the bivariate model of stock returns and output. We define \( \rho_{e,u} \) as the correlation between the two shocks (\( e_t \) and \( u_t \)) in the bivariate system and assume that the two shocks are jointly normal, as follows:

\[
\begin{bmatrix}
    e_t \\
    u_t
\end{bmatrix}
\sim N\left(0, \begin{bmatrix}
    \sigma^2_{e_t} & \rho_{e,u} \sigma_{e_t} \sigma_{u_t} \\
    \rho_{e,u} \sigma_{e_t} \sigma_{u_t} & \sigma^2_{u_t}
\end{bmatrix}\right).
\]

(10)

Here \( e_t \) denotes “news” about future dividends that arrives during trading period \( t \), not the dividends growth itself (see Appendix for explanation). We treat the \( e_t \) component as a residual of the return equation, as in Campbell (1991). However, our approach is different from that of Campbell (1991) in that we allow regime dependence in the volatility. Thus, there are two possible routes to connect dividend growth and output growth; one is the correlation coefficient \( \rho_{e,u} \) between their innovations, and the other is the shared regime switching variable \( S_{i,t} \).

To estimate the stock return equation in (9), we need to consider a discrepancy between the two information sets available to investors and econometricians. In particular, the investors’ information set is regarded as larger than that available to econometricians or researchers. This difference is because market participants continuously observe trade that occurs during the period, while researchers collect data discretely at either the beginning or the end of each period. Note that the information set \( I'_t \) in equation (9) contains information on trades continuously occurring during the period \( t \), which is unavailable to econometricians. To handle this difficulty, we use the actual values, \( \sigma^2_{e_t} \) and \( \sigma^2_{u_t} \), as a proxy for \( E[\sigma^2_{e_t} | I'_t] \) and \( E[\sigma^2_{u_t} | I'_t] \).\(^6\) Using such proxy variables has been justified by the results of Turner et al. (1989), Kim et al. (2004), and Henry and Scruggs (2007). Therefore, equation (9) is replaced by the

\(^5\) The particular value 0.997 for \( \rho \) is explained in the Appendix.

\(^6\) A slightly stronger assumption is that investors observe the variances perfectly, which is equivalent to assuming that \( I'_t = \{S_{i,t}\} \).
following estimable equation:

\[
 r_i = \beta_1 E[I_{i,1}^2] + \beta_2 E[I_{i,2}^2] + \delta_1 (\sigma_{i,1}^2 - E[I_{i,1}^2]) + \delta_2 (\sigma_{i,2}^2 - E[I_{i,2}^2]) + \Delta z_i + e_i, 
\]

\[ (9') \]

where \( \delta_i = -\beta_i / (1 - \rho \lambda), \lambda = p_i + q_i - 1, i = 1, 2, \) and \( \rho = 0.997. \)

Finally, note that, if we ignore the alternative source of stock market volatility identified by the output growth equation in (6), our bivariate model simplifies to that of Kim et al.(2004).

1.3 Estimation procedure

The proposed bivariate model for stock returns and output growth represented by equations (9') and (6), along with equations (3)-(5), (7), and (10), includes two different sets of parameters that we need to make inference for: (i) the hyper-parameter set \( \theta = (\beta_1, \beta_2, \tau, \phi_1, \phi_2, \sigma_{i,0}, \sigma_{i,1}, \sigma_{i,2}, p_i, q_i, q_2, \mu, \psi, \psi', \sigma_a, \rho_{\sigma}, \varphi, \psi, \lambda)' \); and (ii) the set of unobserved variables, \( \tilde{S}_i = (S_{i,1}, S_{i,2}, \cdots, S_{i,T})', i = 1, 2, \) and \( \tilde{z} = (z_1, z_2, \cdots, z_T)' \). The model is estimated employing Kim’s (1993) filter for unobserved component models with Markov-switching heteroskedasticity.

Kim’s (1993) filter is simply a combined version of the conventional Kalman filter and the Hamilton filter. Thus, just like the conventional Kalman filter or the Hamilton filter, Kim’s (1993) filter provides estimates for the second set of parameters of the model (unobserved variables) at any point of time \( t \), based on information up to time \( t-1 \) (i.e., \( E[S_{i,t} | I_{t-1}], E[S_{i,t}^2 | I_{t-1}], E[z_i | I_{t-1}], \) and \( Var[z_i | I_{t-1}] \)). These filtered estimates are derived conditional on some values of the hyper-parameters. Since we can obtain \( E[S_{i,t} | I_{t-1}] \) and \( E[S_{i,t}^2 | I_{t-1}] \), we can compute \( E[\sigma_{i,t}^2 | I_{t-1}] \) and \( E[\sigma_{i,t}^2 | I_{t-1}] \) using equations (3) and (4). Therefore, we have all the variables to construct the joint density of \( r_i \) and \( y_i \) conditional on past information, \( f(r_i, y_i | I_{t-1}), t = 1, \cdots, T \) (and the log-likelihood function as well). Thus, similar to the conventional state-space models or Markov-switching models, the inference problem for our proposed model consists of (i) estimating the hyper-parameters of the model by maximizing the log-likelihood function over the full sample and (ii) making inferences about the unobserved variables conditional on the
hyper-parameter estimates of the model.\footnote{Readers are referred to Kim (1993) and Kim and Kim (1996) for detailed derivation of the Kim’s (1993) filter.}

There is an issue that merits some discussion. Just like the other two filters mentioned above, Kim’s (1993) filter provides “real-time” or “forward-looking” estimates of the decomposed volatility components (i.e., the cyclical and non-cyclical volatilities, $E[\sigma_{c,t}^2 | I_{t-1}]$ and $E[\sigma_{v,t}^2 | I_{t-1}]$) at any point of time $t$ if the true values of the hyper-parameters are known and can be used in estimation. We refer to these estimates as either “real-time” or “forward-looking” because these estimates at time $t$ are obtained using information available up to time $t-1$ only. However, the true values of the hyper-parameters are unknown in reality; therefore, certain estimates must be used. Usually, such estimation is carried out using the maximum likelihood method over the full sample period. Therefore, some “future” information indirectly enters the volatility estimates, which makes them neither purely real-time, nor fully forward-looking. Nevertheless, we conjecture that: (i) the volatility estimates can still provide some partial “real-time” classification or decomposition to investors; and (ii) using the full sample is not likely to cause a biased inference if the true values of the hyper-parameters are constant over time, because the maximum likelihood (ML) estimator for the hyper-parameters is consistent.

In principle, one can completely eliminate such a problem by estimating the hyper-parameters using observations only up to time $t-1$. Although such estimates over a sub-sample are less efficient than the full sample estimates, both must be consistent for the true values. In addition, the difference between them should be small in large samples, which can at least partially justify using the full sample estimates. Alternatively, it is plausible to assume that investors might have accumulated a lot of information and observations (beyond the full sample period used by econometricians). This information may be sufficient to allow them to have already obtained consistent estimates for the hyper-parameters. In such a case, the full sample estimates can be considered as proxies for such consistent estimates available to investors, but unavailable to econometricians.

2 EMPIRICAL FINDINGS

In this section, we describe the data used in our analysis and report the estimation results based on the proposed bivariate model developed in the previous section. After explaining the estimation results, we will discuss how our findings are related to important contemporary research strands, such as literature on
return predictability, liquidity-based asset pricing, and long-run risk.

2.1 Data and estimation results

In our empirical analysis, we use excess stock returns on a market portfolio, constructed using the CRSP value-weighted portfolio and the 30-day US Treasury bill rate over the sample period, January 1959 to June 2012.\textsuperscript{8} To assess the prevailing business conditions, we utilize the Conference Board Coincident Economic Index.

Figure 2 plots the excess returns and output growth rate over the sample period. The shaded areas in the figure are the NBER recession dates. We can see that there is a lot of noise added to the cyclical movements in stock returns.\textsuperscript{9} The plot of output growth shows a noticeable reduction in volatility since 1984 (known as the “Great Moderation” in literature) and the deep and long recession in the late-2000s (known as the “Great Recession”).\textsuperscript{10} In our estimations, we consider the possible effects of the “Great Moderation” and “Great Recession” to better identify the output dynamics. We incorporate the following dummy variables in output equation (6): (i) a dummy variable for the variance $\sigma_u^2$, which accounts for the impact of the “Great Moderation” since 1984; and (ii) a dummy variable for the mean growth ($\mu_c$ and $\mu_t$), which accounts for the impact of the “Great Recession” since 2008.\textsuperscript{11}

To compare our results with earlier literature, we report the maximum likelihood estimation result of Kim et al.’s (2004) model in Table 1. Their model is equivalent to our proposed model if a single source of market volatility is considered. Standard errors shown in parentheses in Table 1 are typically calculated from the inverse of the Fisher information matrix (i.e., the negative of the second derivative of

\textsuperscript{8} The excess return data can be also found in the following webpage of the Fama-French Data Library: (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The starting point of the sample period is determined according to the availability of our output measure (the Conference Board Coincident Economic Index).

\textsuperscript{9} A similar phenomenon has been illustrated in the time series plot of VIX in Figure 1, although the time span is short as a result of the data availability.

\textsuperscript{10} Readers are referred to Stock and Watson (2003) (and references therein) for a detailed explanation on the effects of the “Great Moderation” in the business cycle literature. The “Great Recession” (or the 2008-2012 Global Recession, the Lesser Depression, or the Long Recession) is a recently stylized name, which considers the late-2000s recession to be different to previous postwar recessions as it has a lower mean growth rate and a slower recovery (Stock and Watson, 2012).

\textsuperscript{11} As we will show later, both effects turn out to be significant and these modifications provide reasonable parameter estimates (and thus reasonable inferences on the probabilities of a recession or a boom) that are consistent with business cycle literature.
the log-likelihood function evaluated at the hyper-parameter estimates). The market premium parameter $\beta_1$ is significant, showing a positive relation between market volatility and the expected returns.$^{12}$ We also reproduce the time series of $\Pr[S_{t+1} = 1|I_t]$, i.e., the conditional probabilities of being in a high volatility state, in Figure 3. The estimated probabilities of the high volatility regime capture historical liquidity crises, as well as recession periods.

Table 2 reports the parameter estimates of the proposed bivariate model of stock returns and output growth (given by equations (9') and (6) along with equations (3)-(5), (7), and (10)). Standard errors shown in the table are computed again from the Fisher information matrix, as before. The first market premium parameter, $\beta_1$, shows the impact of business cycle-related volatility on the expected returns, whereas $\beta_2$ is the impact of the non-cyclical component. Most interestingly, the business cycle-related volatility has significant explanatory power with respect to movements in the expected return ($\beta_1$ is 0.045 and significant), unlike the non-cyclical volatility ($\beta_2$ is insignificant). In other words, the risk-return trade-off holds only for shocks that are related to business cycles.

The low and high volatilities (standard deviations) that are related to the business cycle are estimated to be 2.688 and 4.544, while the corresponding non-cyclical counterparts are 1.353 and 4.957, respectively. The estimated value (0.92) of the transition probability, $p_1$, implies that the duration of the high-cyclical volatility state (or recession) is about one year.$^{13}$ The estimates of the transition probabilities ($p_2$ and $q_2$) indicate that the low-volatility state dominates the high-volatility state for the non-cyclical component. Specifically, the duration of the high-volatility state for the non-cyclical component implied by $p_2$ is only three months, whereas the business cycle-unrelated low-volatility state lasts longer than five years. All the parameter estimates of non-cyclical components of stock prices ($\tau$, $\phi_1$ and $\phi_2$) are significant. We note that the AR coefficients turn out to be transitory, which is consistent with Kim and Kim (1996).

The mean growth rates during booms ($\mu_b$) is estimated to be 0.355, and -0.096 during recessions

$^{12}$ Although the risk premium parameter $\beta_1$ of Kim et al.'s (2004) model is significant indicating the positive relation between market volatility and expected return, most conventional estimates of such a risk-return relationship without conditioning on regime-shifts have produced mixed, insignificant, and/or conflicting findings (Bollerslev and Zhou, 2006).

$^{13}$ As well-known, the expected duration of recessions implied by the transition probability $p_1$ is given by $1/(1-p_1)$.
The “Great Recession” dummy variable gives significantly lower output growth during a recession \( (\mu_{1,\text{dummy, post-2008}}) \) is significantly estimated to be -0.712, although no such effect during a boom \( (\mu_{0,\text{dummy, post-2008}}) \) is statistically insignificant. The coefficients of the autoregressive terms \( (\psi_1, \psi_2) \) are transitory, as in the Hamilton-type (1989) business cycle literature. The post-1984 volatility estimate \( (\sigma_{\text{post-1984}}) \) is significantly smaller than the pre-1984 volatility estimate \( (\sigma_{\text{pre-1984}}) \). All these parameter estimates (including the regime transition probabilities \( p_i \) and \( q_i \)) on the output equation (and thus the inferences on the probabilities of a recession or a boom in Figure 4) are consistent with the business cycle literature.

The correlation estimate \( (\rho_{ew}) \) is insignificant. This insignificant estimate is consistent with the assumption of independence between innovation to dividend growth and that to consumption growth in the literature (e.g., Cecchetti et al., 1993, and Bansal and Yaron, 2004), if we expect output growth to play a role similar to that of consumption growth. Therefore, as explained previously, the remaining possible connection between output growth and dividend growth would be achieved through the shared state variable, \( S_{1,t} \).

Figure 4 presents the time series of \( \Pr[S_{1,t} = 1 | I_i] \), \( i = 1,2 \), i.e., the conditional probabilities of the high-volatility state for business cycle-related and unrelated shocks. The regime probabilities of the cyclical shocks clearly identify the NBER recession periods shown in the shaded area. The non-cyclical counterpart shown at the bottom in Figure 4 identifies historical liquidity crises and is not persistent, i.e., they are of relatively short duration. The point at which the non-cyclical volatility spikes are not persistent may be the reason why the transitory volatility shock is not priced in the expected returns. It is worth noting here that the 1973-4 and the 2008-9 high-volatility periods (unlike other high-volatility periods) are characterized by both cyclical and non-cyclical volatility components.

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14 The “Great Moderation” and “Great Recession” dummy variables play an important role in identifying business cycles in the output equation, as in the business cycle literature. Thus, they affect the identification of cyclical market volatility and market premium parameter estimates (and all other hyper-parameter estimates in the system).

15 Comparing Figure 3 and the cyclical regime probabilities (the solid line) in Figure 4, we note that our joint model identifies the NBER recessions more accurately than the single equation approach adopted by Kim et al. (2004).

16 The periods 1973-4 and 2008-9 periods are unique, when compared to other stock market crashes, in that they have both cyclical (and persistent) and non-cyclical (and transitory) volatility components at the same time. This may be due to a series of distinctly different events occurring around the world, for example, the collapse of the Bretton Woods system just before 1973-4, the US dollar devaluation under
Figure 5 depicts the estimates of non-cyclical components of stock prices, $z_t$, in equation (9'), along with their one-standard-error confidence bands. As explained in Section 1.3, Kim’s (1993) filter provides the estimates for $z_t$ and its variance (or standard error).\textsuperscript{17} During the sample period, a few episodes of non-cyclical components were readily identified, most notably the 1987 stock market crash and the 1998 Russian default.

Overall, the estimated bivariate model passes the usual diagnostic tests, such as the Q-test for autocorrelation at the usual 5% significance level, and shows a reasonable model fit; for example, the $R^2$ measure suggested by McElroy (1977) is 31.7%\textsuperscript{18}

2.2 Further discussion on related literature

There are a few issues that relate our results to earlier literature that merit discussion. First, our main findings that (i) cyclical volatility is priced by higher expected return and (ii) non-cyclical volatility is not provide one possible explanation as to why earlier literature regarding the risk-return relationship has generated mixed, insignificant, and/or conflicting findings, as cited in the introduction and in the previous work of Bollerslev and Zhou (2006). The marginal effect of expected market volatility on expected returns in earlier literature appears understated as a result of its failure to sort out the impact of transitory shocks (outliers). In other words, the previous findings may have reflected the average effect of both cyclical volatility and non-cyclical volatility. Depending on which component predominates in a particular dataset, sample period, or model specification, the results can be positive, negative, or mixed. Moreover, our findings regarding the market premium parameter ($\beta$) for cyclical volatility also corroborate the conventional explanation used in the return predictability literature, because we can explain the link between business cycles and excess returns through business cycle-related volatility. Therefore, if some variables constitute a proxy for business cycles, our results indicate that those variables should have some predictive power on excess returns.

Second, our empirical findings on the non-cyclical component of market volatility might have

\textsuperscript{17} Readers are referred to Kim (1993) and Kim and Kim (1996) for a detailed derivation of the expectation and the standard errors of unobserved variables.

\textsuperscript{18} The $R^2$ measure suggested by McElroy (1977) is a multivariate extension of the conventional $R^2$, which can be used in multivariate models.
implications in terms of the liquidity-based asset pricing literature. Although we do not employ a specific measure for liquidity (or illiquidity), the non-cyclical market volatility spikes identified in our model coincide with the historical liquidity crisis periods (illiquid periods) identified in liquidity-based asset pricing literature. Thus, our non-cyclical market volatility may be interpreted as a temporary market-wide liquidity risk measure used in literature. If so, our result that non-cyclical market risk is not priced in the market may indicate that the temporary market-wide liquidity risk is also not priced in the market. Pastor and Stambaugh (2003) and Henry and Scruggs (2007) introduced measures of market-wide liquidity risk in assessing the impact of liquidity risk on expected returns. Illiquid periods identified by those measures are closely associated with our high market volatility periods (including both cyclical and non-cyclical components). This observation may generate a potentially interesting research topic in which one can determine the impact of liquidity risk on expected returns by taking into account the following two factors: (i) identifying the impact of liquidity risk from the impact of business cycle-related volatility risk and (ii) decomposing the impact of liquidity risk into a persistent component and a temporary component.

Last but not least, our empirical findings appear to be consistent with recently developed long run risk (LRR) literature (pioneered by Bansal and Yaron (2004)). In this literature, it is demonstrated that long-run volatility movements (mainly measured by consumption growth volatility) have large explanatory power for asset prices (see Bansal and Yaron, 2004; Bansal et al., 2005; Bansal et al., 2007; and Bollerslev, Tauchen, and Zhou, 2009). In our bivariate model, the cyclical volatility component of stock returns plays a similar role to the long-run stochastic volatility in the LRR literature. To be precise, our cyclical volatility component is not related to the transitory shocks either in output growth or in stock returns, whereas the non-cyclical volatility component is related to the transitory shock in stock returns. Moreover, the cyclical volatility is driven by the same regime-shift process as the output growth. Thus, such a cyclical volatility process can be viewed as a “stochastic” volatility because its risk cannot be “hedged.” Such a stochastic (or cyclical) volatility component may require separate risk premium compensation, if it moves together with the unpredictable fundamental in the economy (this paper uses output growth as a proxy for this pricing kernel). However, the transitory movement in stock return volatility is not related to the fundamental (or pricing kernel): therefore, it may not need to be separately compensated, as confirmed by the empirical results shown in Section 2.1.19

For the purpose of comparison, we conduct an additional empirical analysis using consumption  

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19 We note that volatility risk (one of priced components in the LRR model) is generally ignored in the risk-return literature. We acknowledge that volatility risk does not have any role in our model either.
growth instead of output growth. The results are reported in Table 3. In this new analysis, the first parameter, $\beta_1$, shows the impact of increased economic uncertainty along with bad long-run growth prospects (low mean growth of consumption) on the expected returns, whereas $\beta_2$ shows the impact of the economic growth-unrelated counterpart. All the empirical results are qualitatively the same, as explained in the previous sub-section. Specifically, consumption growth-related volatility has significant explanatory power on the movements of expected returns, whereas the unrelated counterpart does not. If we believe that consumption growth accurately reflects business cycles, $\beta_1$ can again be interpreted as an impact of the business cycle component of market volatility, whereas $\beta_2$ is the business cycle-unrelated counterpart (see Bansal et al., 2007, Cecchetti et al., 1990, and Kandel and Stambaugh (1990) for business cycle behavior in consumption dynamics).

3 CONCLUDING REMARKS

In this paper, we investigate the risk-return relationship conditioning on business cycles. For this purpose, we decompose the market volatilities into two components (business cycle-related volatility and unrelated volatility), and investigate the effects of these volatilities on expected returns.

Our empirical results show that cyclical risks are priced in the stock market. This result confirms the “underlying mechanism” of return predictability literature, in other words, the business cycle-risk-return relationship. In contrast, the temporary high volatilities observed during liquidity crises are not compensated for by higher expected returns. Furthermore, a few high volatility period episodes that are not related to recessions were also identified, most notably the 1987 crash and the 1998 Russian default. These results provide new evidence for risk-return literature, as well as support for the business cycle-based risk-return relationship elucidated in return predictability literature.

20 We use the monthly consumption of nondurables and services in real terms, as obtained from FRED (which is collected by the Federal Reserve Bank of St. Louis), for the same sample period employed in the previous section.

21 Bansal et al. (2005) employ various measures of economic uncertainty in investigating the risk-return relationship. They find significant results with consumption growth volatility (and GDP growth volatility), but insignificant results with stock market volatility. They show that the consumption volatility (rather than stock market volatility) is an effective barometer of fundamental economic uncertainty.
APPENDIX

Campbell and Shiller (1989) use a first-order Taylor series approximation to derive the following log-linear present value relationship for the fundamental component of stock price:

\[
p_j^* = \frac{k}{1 - \rho} + E \left[ \sum_{j=0}^\infty \rho^j (1 - \rho) d_{t+j} - \Delta p^*_{t+j} \right] I_t,
\]

where \( d_{t+j} \) is the log dividend at time \( t+1+j \) claimed at the beginning of the period, and \( \rho \) and \( k \) are linearization parameters defined by \( \rho = \exp \left( \frac{1}{1 + \exp (d - p)} \right) \), where \( \overline{d - p} \) is the average log dividend-price ratio, and \( k \equiv -\log (\rho) - (1 - \rho) \log \left( \frac{1}{\rho} - 1 \right) \). Empirically, in US data the average dividend-price ratio has been approximately 4% annually, implying that \( \rho \) should be about 0.997 for monthly data. Furthermore, as summarized by Campbell, Lo, and MacKinlay (1997), the approximation error in equation (A.1) is quite small, particularly when it is applied to monthly data.

As discussed in Campbell (1991), the log-linear present value model given in equation (A.1) can be rearranged to show that a realized return for the fundamental component consists of three components: (i) the expected return for the fundamental component; (ii) revisions in its expected returns; and (iii) another revision component in future dividends:

\[
\Delta p_j^* = E[\Delta p_j^* | I_{t-1}] + f_t + e_t,
\]

where

\[
f_t \equiv \left( E \left[ \sum_{j=0}^\infty \rho^j \Delta p_{j+1}^* | I_t \right] - E \left[ \sum_{j=0}^\infty \rho^j \Delta p_j^* | I_{t-1} \right] \right),
\]

\[
e_t \equiv \left( E \left[ \sum_{j=0}^\infty \rho^j \Delta d_{j+1} | I_t \right] - E \left[ \sum_{j=0}^\infty \rho^j \Delta d_j | I_{t-1} \right] \right),
\]

in which revisions are made with the additional information during period \( t \), which is collected in the information set \( I_t' \); that is, the information set \( I_t' \) contains all elements of \( I_t \), except the final realized value of \( r_t \). Note that \( e_t \) denotes “new information” about future dividends that arrives during trading
Meanwhile, in order to find a tractable expression for our information revision term in equation (A.3), we rewrite equation (8) using equations (3)-(5), as in Hamilton (1989):

\[
E[\Delta p_{t}^{*} | I_{t}] = \beta_{1}(\sigma_{e,0}^{2} + (\sigma_{e,1}^{2} - \sigma_{e,0}^{2}) \Pr[S_{1,t} = 1]) + \beta_{2}(\sigma_{e,0}^{2} + (\sigma_{e,1}^{2} - \sigma_{e,0}^{2}) \Pr[S_{2,t} = 1])
\]

\[+ \beta_{i} \lambda_{i}(\sigma_{e,1}^{2} - \sigma_{e,0}^{2})(\Pr[S_{1,t} = 1| I_{t}] - \Pr[S_{1,t} = 1]) \]

\[+ \beta_{2} \lambda_{2}(\sigma_{e,1}^{2} - \sigma_{e,0}^{2})(\Pr[S_{2,t} = 1| I_{t}] - \Pr[S_{2,t} = 1]),
\]

where \( \lambda_{i} = p_{i} + q_{i} - 1, i = 1, 2 \). Then, given recurring volatility regimes (i.e., \( |\lambda_{i}| < 1, i = 1, 2 \)), it is straightforward to show that the discounted sum of the future expected return is given by:

\[
E\left[ \sum_{j=0}^{\infty} \rho^{j} \Delta p_{t+j}^{*} | I_{t} \right] = \beta_{1} \frac{\sigma_{e,0}^{2} + (\sigma_{e,1}^{2} - \sigma_{e,0}^{2}) \Pr[S_{1,t} = 1]}{1 - \rho} + \beta_{2} \frac{\sigma_{e,0}^{2} + (\sigma_{e,1}^{2} - \sigma_{e,0}^{2}) \Pr[S_{2,t} = 1]}{1 - \rho} \]

\[+ \beta_{i} \frac{\sigma_{e,1}^{2} - \sigma_{e,0}^{2} + (\Pr[S_{1,t} = 1| I_{t}] - \Pr[S_{1,t} = 1])}{1 - \rho \lambda_{i}} \]

\[+ \beta_{2} \frac{\sigma_{e,1}^{2} - \sigma_{e,0}^{2} + (\Pr[S_{2,t} = 1| I_{t}] - \Pr[S_{2,t} = 1])}{1 - \rho \lambda_{2}},
\]

This, in turn, implies that the information revision term \( f_{i} \) (i.e., volatility feedback term) can be rewritten as:

\[
f_{i} = \delta_{i} \left( E[\sigma_{e,0}^{2} | I_{t}'] - E[\sigma_{e,0}^{2} | I_{t-1}] \right) + \delta_{2} \left( E[\sigma_{e,1}^{2} | I_{t}'] - E[\sigma_{e,1}^{2} | I_{t-1}] \right),
\]

where \( \delta_{i} = -\frac{\beta_{i}}{1 - \rho \lambda_{i}}, i = 1, 2 \). Substituting equation (A.7) into equation (A.2), we obtain the following expression for \( \Delta p_{t}^{*} \):

18
\[ \Delta \rho_i = \beta_1 E[\sigma_{x,i}^2 | I_{t-1}] + \beta_2 E[\sigma_{y,i}^2 | I_{t-1}] \]
\[ + \delta_1 \left( E[\sigma_{x,i}^2 | I_{t}'] - E[\sigma_{x,i}^2 | I_{t-1}] \right) + \delta_2 \left( E[\sigma_{y,i}^2 | I_{t}'] - E[\sigma_{y,i}^2 | I_{t-1}] \right) + \epsilon_i, \]  

where \( \delta_i = -\frac{\beta_i}{1 - \rho \lambda_i} \) and \( \lambda_i = p_i + q_i - 1, \ i = 1, 2. \)

Finally, substituting equation (A.8) into equation (2), we obtain the following model of stock return:
\[ r_i = \beta_1 E[\sigma_{x,i}^2 | I_{t-1}] + \beta_2 E[\sigma_{y,i}^2 | I_{t-1}] \]
\[ + \delta_1 \left( E[\sigma_{x,i}^2 | I_{t}'] - E[\sigma_{x,i}^2 | I_{t-1}] \right) + \delta_2 \left( E[\sigma_{y,i}^2 | I_{t}'] - E[\sigma_{y,i}^2 | I_{t-1}] \right) + \Delta z_i + \epsilon_i. \]

which is reproduced in equation (9) in Section 1.2.
REFERENCES


Table 1. Estimates of Kim, Morley, and Nelson’s (2004) Model

\[ r_i = \beta_i E[\sigma_{e,i}^2 | I_{I-1}, t] + \delta_i (\sigma_{\epsilon,i}^2 - E[\sigma_{\epsilon,i}^2 | I_{I-1}, t]) + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma_i^2), \]
\[ \sigma_{\epsilon,i}^2 = \sigma_{\epsilon,0}^2 (1 - S_{I,i}) + \lambda_{i} S_{I,i}, \]
\[ \Pr[S_{I,i} = 1 | S_{I,i-1} = 1] = p_i, \quad \Pr[S_{I,i} = 0 | S_{I,i-1} = 0] = q_i. \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_i)</td>
<td>0.028 (0.007)</td>
</tr>
<tr>
<td>(\sigma_{\epsilon,0})</td>
<td>3.265 (0.100)</td>
</tr>
<tr>
<td>(\sigma_{\epsilon,1})</td>
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</tr>
<tr>
<td>(p_i)</td>
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<tr>
<td>(q_i)</td>
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<tr>
<td>Log-Likelihood Value</td>
<td>-1812.992</td>
</tr>
</tbody>
</table>

Note: 1. Standard errors in parentheses.
2. \(\delta_i = -\beta_i / (1 - \rho \lambda_i), \quad \lambda_i = p_i + q_i - 1, \quad \rho = 0.997.\)
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Business Cycle-Related Component of Stock Price</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.045 (0.012)</td>
</tr>
<tr>
<td>$\beta_2$</td>
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<tr>
<td>$\sigma_{\varepsilon,0}$</td>
<td>2.688 (0.322)</td>
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<tr>
<td>$\sigma_{\varepsilon,1}$</td>
<td>4.544 (0.312)</td>
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<tr>
<td>$\rho_1$</td>
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<td>$\rho_2$</td>
<td>0.966 (0.007)</td>
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<tr>
<td><strong>Business Cycles-Unrelated Component of Stock Price</strong></td>
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</tr>
<tr>
<td>$\phi_1$</td>
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<td>$\phi_2$</td>
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<tr>
<td>$\tau$</td>
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</tr>
<tr>
<td>$\sigma_{\varepsilon,0}$</td>
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<tr>
<td>$\sigma_{\varepsilon,1}$</td>
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<tr>
<td>$\rho_1$</td>
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<tr>
<td>$\rho_2$</td>
<td>0.985 (0.008)</td>
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<tr>
<td><strong>Output Growth Equation</strong></td>
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<tr>
<td>$\mu_0$</td>
<td>0.355 (0.033)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-0.096 (0.057)</td>
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<tr>
<td>$\mu_{0,\text{dummy, pre-2008}}$</td>
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<tr>
<td>$\mu_{1,\text{dummy, post-2008}}$</td>
<td>-0.712 (0.163)</td>
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<tr>
<td>$\psi_1$</td>
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<tr>
<td>$\psi_2$</td>
<td>0.055 (0.038)</td>
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<td>$\sigma_{\varepsilon,\text{pre-1984}}$</td>
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<tr>
<td>$\rho_{\varepsilon}$</td>
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Log-Likelihood Value -2302.849

**Note:** 1. Standard errors in parentheses.
2. $\delta_i = -\beta_i / (1 - \rho \lambda_i), \lambda_i = p_i + q_i - 1, \rho = 0.997, i = 1,2.$
Table 3. Estimates of the Proposed Bivariate Model using Consumption Growth

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Growth-Related Component of Stock Price</td>
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<tr>
<td>$\beta_1$</td>
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<tr>
<td>$\beta_2$</td>
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<td>$\sigma_{x0}$</td>
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<tr>
<td>$\sigma_{x1}$</td>
<td>4.788 (0.259)</td>
</tr>
<tr>
<td>$p_1$</td>
<td>0.939 (0.016)</td>
</tr>
<tr>
<td>$q_1$</td>
<td>0.970 (0.005)</td>
</tr>
<tr>
<td>Economic Growth-Unrelated Component of Stock Price</td>
<td></td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.831 (0.141)</td>
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<tr>
<td>$\phi_2$</td>
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<tr>
<td>$\tau$</td>
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<tr>
<td>$\sigma_{x0}$</td>
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<tr>
<td>$\sigma_{x1}$</td>
<td>2.967 (3.107)</td>
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<tr>
<td>$p_2$</td>
<td>0.623 (0.187)</td>
</tr>
<tr>
<td>$q_2$</td>
<td>0.993 (0.003)</td>
</tr>
<tr>
<td>Consumption Growth Equation</td>
<td></td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.394 (0.022)</td>
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<tr>
<td>$\mu_1$</td>
<td>0.186 (0.024)</td>
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<tr>
<td>$\psi_1$</td>
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<td>$\psi_2$</td>
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<td>$\sigma_{u,pre-1984}$</td>
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<td>$\sigma_{u,post-1984}$</td>
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<td>$\rho_{x,u}$</td>
<td>0.143 (0.042)</td>
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<td>Log-Likelihood Value</td>
<td>-1966.876</td>
</tr>
</tbody>
</table>

Note: 1. Standard errors in parentheses.
2. $\delta_i = -\beta_i / (1 - \rho \lambda_i), \lambda_i = p_i + q_i - 1, \rho = 0.997, i = 1, 2.$
Figure 1. CBOE’s S&P 500 Implied Volatility Index (VIX)

Note: The shaded areas indicate the NBER recession dates.

Figure 2. Excess Stock Returns and Output Growth

Note: The shaded areas indicate the NBER recession dates.
Figure 3. Probability of High-Volatility Regime

[Kim, Morley, and Nelson’s (2004) Model]
Figure 5. Estimates of Non-Cyclical Component of Stock Price