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The role of constant instruments in dynamic panel estimation☆

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HIGHLIGHTS

- In dynamic panel estimation, constants are often omitted from the instrument sets.
- We examine the effect of this omittance when the instrument means are large.
- We show that the properties of the estimator depend on the instrument means.
- We show that existing estimators can be more biased and much less efficient.
- We provide simple solutions for practitioners.

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ABSTRACT

In the standard generalized method of moments estimation of dynamic panel data models, the constant term is usually omitted from instrument sets. As a result, adding a constant to the dependent variable affects the estimates for models without full period dummies. Omitting the constant term from instrument sets may also result in substantial bias and efficiency loss if the mean of the variable is large in magnitude. In this note, we provide analytical and numerical results and propose convenient solutions for practitioners. We suggest that full period dummies be included as extra exogenous instruments even for models without time effects on the right-hand side.

1. Introduction

Consider the linear structural equation $y_i = \alpha + x_i\beta + u_i$ and the reduced form equation $x_i = \pi_0 + z_i\pi_1 + v_i$, where z_i represents non-constant instruments and u_i and v_i are arbitrarily correlated. In this framework, it is natural to use $(1, z_i)$ as instruments for the collective regressor vector $(1, x_i)$. Even for the demeaned equation $y_i - \bar{y} = (x_i - \bar{x})\beta + (u_i - \bar{u})$, where the intercept is eliminated, the constant term should still be used as an instrument because it is non-redundant (see Breusch et al., 1999, for redundancy of moment conditions) and deals with the nonzero unconditional mean of z_i.

We econometricians are, however, sometimes so preoccupied with dealing with the nuisance intercept that we fail to explicitly use the constant term as an instrument. A notable example is the generalized method of moments (GMM) estimation of dynamic panel data models. For the model $y_{it} = \alpha_i + \beta y_{it-1} + u_{it}$, where u_{it} is serially independent, first differences have the relationship $\Delta y_{it} = \beta\Delta y_{it-1} + \Delta u_{it}$, and lagged endogenous variables are used as instruments (see Anderson and Hsiao, 1981; Arellano and Bond, 1991; Ahn and Schmidt, 1995; Blundell and Bond, 1998; Baltagi, 2013, among others). Unfortunately, no constant is mentioned clearly as a valid instrument in the literature (with the exception of Ahn and Schmidt, 1995, who give a short remark on the nonzero mean of α_i), and some popular econometric packages do not include constant terms in the instrument sets when they are not

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present in the model. (For example, STATA's "xtabond" command does not use it by default unless they appear on the right-hand side.) Attention is being paid to this issue only recently; see Ahn and Kitazawa (2014), and Gørgens et al. (2014).

The consequence of excluding constant instruments can be serious. First, adding a constant to a variable may change the slope estimate. For example, STATA's difference generalized method of moments (GMM) and system GMM commands ("xtabond" and "xtpdpsys") invoked with default options give different results when applied to y_{it} and $y_{it}+10$, unless full period dummies are included as exogenous regressors. This is an undesirable move since shifting variables does not affect correlation or causality. Second, the failure to use constant instruments can lead to substantial efficiency loss. For some models, it may weaken the strength of the instruments so severely that the resulting estimator may be inconsistent along some asymptotic expansion paths for the unconditional means of instrumental variables. The following example is illustrative.[1]

Example 1. Consider the two-period model $y_{it} = \alpha_i + \beta x_{it} + u_{it}$ for $t = 1, 2$ and $i = 1, \ldots, n$. We may estimate the differenced equation $\Delta y_i = \beta\Delta x_i + \Delta u_i$, where $\Delta y_i = y_{i2} - y_{i1}$, and Δx_i and Δu_i are similarly defined. Suppose that the regressor x_{it} is generated by $x_{it} = \mu + \varepsilon_{it} + \gamma u_{it-1}$, where u_{it} and ε_{it} are mutually and serially *i.i.d.* with $E(u_{it}^2) = \sigma_u^2$ and $E(\varepsilon_{it}^2) = \sigma_\varepsilon^2$. The regressor is predetermined with $E(x_{it}u_{it}) = 0$ but $E(x_{i2}u_{i1}) \neq 0$ unless $\gamma = 0$. Because Δx_i is correlated with Δu_i, one may consider instrumental variable (IV) estimation using x_{i1} as an instrument.

For this simple model, it is easy to derive the asymptotic distribution of the IV estimators. Since $E(x_{i1}\Delta x_i) = -\sigma_x^2$ and $E[(x_{i1} - Ex_{i1})^2(\Delta u_i)^2] = 2\sigma_u^2\sigma_x^2$, where $\sigma_x^2 = \sigma_\varepsilon^2 + \gamma^2\sigma_u^2$, the asymptotic distribution of the IV estimator $\hat\beta$ using $(1, x_{i1})$ as instruments is

$$\sqrt{n}(\hat\beta - \beta) = \frac{n^{-1/2}\sum_{i=1}^n (x_{i1} - \bar x_1)\Delta u_i}{n^{-1}\sum_{i=1}^n (x_{i1} - \bar x_1)\Delta x_i} \Rightarrow \frac{N(0, 2\sigma_u^2\sigma_x^2)}{-\sigma_x^2} \sim N\left(0, \frac{2\sigma_u^2}{\sigma_x^2}\right),$$

where $\bar x_1 = n^{-1}\sum_{i=1}^n x_{i1} \to_p \mu$. Due to the demeaning of x_{1i}, the estimator $\hat\beta_1$ does not depend on μ nor does the asymptotic distribution. In contrast, since $E[x_{i1}^2(\Delta u_i)^2] = 2\sigma_u^2(\sigma_x^2 + \mu^2)$, the IV estimator $\tilde\beta$ omitting the constant term from the instrument set has the asymptotic distribution

$$\sqrt{n}(\tilde\beta - \beta) = \frac{n^{-1/2}\sum_{i=1}^n x_{i1}\Delta u_i}{n^{-1}\sum_{i=1}^n x_{i1}\Delta x_i} \Rightarrow \frac{N(0, 2\sigma_u^2(\mu^2 + \sigma_x^2))}{-\sigma_x^2} \sim N\left(0, \frac{2\sigma_u^2}{\sigma_x^2} + \frac{2\sigma_u^2\mu^2}{\sigma_x^4}\right)$$

as $n \to \infty$. The estimator depends on μ, and the asymptotic variance has an additional positive term $2\sigma_u^2\mu^2/\sigma_x^4$. If μ^2 is comparable to the sample size, the variance of $\tilde\beta$ may be substantial even for a large n, and it is not hard to see that $\tilde\beta$ is inconsistent along asymptotic paths with $\mu = \mu_n = n^{1/2}\tilde\mu$ for some fixed nonzero $\tilde\mu$.

The comparatively large variance of $\tilde\beta$ is associated with the omitted intercept in the first-stage regression $\Delta x_i = \pi_1 x_{i1} + \text{error}_i$, which is "unbalanced" as the mean-zero dependent variable is explained by a variable with a non-zero mean. A poor fit of the first-stage regression due to nonzero $E(x_{i1})$ leads to a large sampling variability in the final estimator. □

The $\tilde\beta$ estimator in Example 1 is merely the result of a mistake. The constant term is important and should not be omitted because the mean of the instrument x_{i1} is nonzero even though the regressor Δx_i has zero mean.

Similar results are obtained for panel dynamic models without period dummies. For a simple model $y_{it} = \alpha_i + \beta y_{it-1} + u_{it}$, when the time period is just sufficient to identify the persistency parameter (that is, when y_{it} is observed for $t = 0, 1, 2$), the estimator will be inconsistent and asymptotically random if the mean of the dependent variable is of the order $\sqrt{n}$, where n is the cross-sectional dimension. For larger time dimensions, the estimator is $\sqrt{n}$-consistent but may suffer from substantial efficiency loss. In the following section, we derive relevant results and discuss practically convenient methods to include constant instruments.

2. Role of constant instruments in difference GMM

In this section, we first discuss the effect of omitting the constant term from the instrument sets used by Arellano and Bond's (1991) difference GMM (DGMM) estimation for the simple panel dynamic model $y_{it} = \alpha_i + \beta x_{it} + u_{it}$ with $x_{it} = y_{it-1}$. We are particularly interested in the one-step efficient DGMM estimator assuming that u_{it} is *i.i.d.* over i and t. The DGMM estimator is also obtained as the GMM estimator based on the moment restrictions $E(z_{it}\ddot u_{it}) = 0$ for $t = 1, 2, \ldots, T-1$, where $z_{it} = (y_{i0}, y_{i1}, \ldots, y_{it-1})'$ and $\ddot u_{it}$ are the forward orthogonal deviations (Arellano and Bover, 1995):

$$\ddot u_{it} = c_{T-t}\left(u_{it} - \frac{1}{T-t}\sum_{s=t+1}^T u_{is}\right), \qquad c_m = \left(\frac{m}{m+1}\right)^{1/2}.$$

Note that z_{it} does not contain a constant. For balanced panels, the one-step efficient DGMM estimator equals

$$\tilde\beta = \left[\sum_{t=1}^{T-1} \ddot{\boldsymbol{x}}_t'\boldsymbol{z}_t\left(\boldsymbol{z}_t'\boldsymbol{z}_t\right)^{-1}\boldsymbol{z}_t'\ddot{\boldsymbol{x}}_t\right]^{-1}\sum_{t=1}^{T-1}\ddot{\boldsymbol{x}}_t'\boldsymbol{z}_t(\boldsymbol{z}_t'\boldsymbol{z}_t)^{-1}\boldsymbol{z}_t'\ddot{\boldsymbol{y}}_t = \beta + \left[\sum_{t=1}^{T-1} \ddot{\boldsymbol{x}}_t'\boldsymbol{z}_t\left(\boldsymbol{z}_t'\boldsymbol{z}_t\right)^{-1}\boldsymbol{z}_t'\ddot{\boldsymbol{x}}_t\right]^{-1}\sum_{t=1}^{T-1}\ddot{\boldsymbol{x}}_t'\boldsymbol{z}_t(\boldsymbol{z}_t'\boldsymbol{z}_t)^{-1}\boldsymbol{z}_t'\ddot{\boldsymbol{u}}_t,$$

where $\ddot{\boldsymbol{x}}_t$ and $\ddot{\boldsymbol{y}}_t$ are the n-vectors of forward orthogonal deviations $\ddot x_{it}$ and $\ddot y_{it}$, respectively, for $i = 1, \ldots, n$ and $\boldsymbol{z}_t = (z_{1t}, \ldots, z_{nt})'$.

For asymptotic analysis, assume that $|\beta| < 1$ and that the system is initialized at $t = -\infty$ such that y_{it} is stationary over t. Thus, $y_{it} = \mu + \eta_i + v_{it}$, where $\mu = E(\alpha_i)/(1-\beta)$, $\eta_i = \alpha_i/(1-\beta) - \mu$, and $v_{it} = \sum_{j=0}^\infty \beta^j u_{it-j}$. We will consider the case of $\mu \to \infty$ as $n \to \infty$, especially when $\mu = \mu_n = \sqrt{n}\tilde\mu$ for some fixed $\tilde\mu \neq 0$. This asymptotic framework is employed just to analyze the situation where the mean of the dependent variable is large relative to its variability and the cross-sectional dimension.

When $T = 2$, the DGMM estimator is inconsistent and asymptotically random as $n \to \infty$ in this case, because $\tilde\beta = \beta + (n^{-1}\boldsymbol{z}_1'\ddot{\boldsymbol{x}}_1)^{-1}n^{-1}\boldsymbol{z}_1'\ddot{\boldsymbol{u}}_1$ and both $n^{-1}\boldsymbol{z}_1'\ddot{\boldsymbol{x}}_1$ and $n^{-1}\boldsymbol{z}_1'\ddot{\boldsymbol{u}}_1$ are asymptotically non-degenerate when the global mean of $\boldsymbol{z}_t$ is of the order of $\sqrt{n}$ and dominates the random component. (Note that $n^{-1}\boldsymbol{z}_1'\ddot{\boldsymbol{u}}_1 = \tilde\mu \cdot n^{-1/2}\sum_{i=1}^n \ddot u_{i1} + o_p(1)$.)

When $T > 2$, the DGMM estimator recovers root-n consistency. To show this, transform the instruments to $\tilde z_{it} = (y_{i0}, \Delta y_{i1}, \ldots, \Delta y_{it-1})'$, which does not change the estimator. Letting $\tilde{\boldsymbol{z}}_t = (\tilde z_{1t}, \ldots, \tilde z_{nt})'$, we can derive the consistency result by defining $A_{nt} = \mathrm{diag}(n, n^{1/2}I_{t-1})$ and showing that $Q_{nt} = A_{nt}^{-1}\tilde{\boldsymbol{z}}_t'\tilde{\boldsymbol{z}}_t A_{nt}^{-1}$ is

[1] This example was suggested by the anonymous reviewer.

Table 1
Comparison of AB and AB1 with 5000 replications.

$\beta = 0.5$		$\mu = 1$				$\mu = \sqrt{n}$			
n	T	Mean		Variance		Mean		Variance	
		AB	AB1	AB	AB1	AB	AB1	AB	AB1
100	2	0.616	0.495	5.1944	0.1698	−0.011	0.495	568.61	0.1698
200	2	0.568	0.506	0.1442	0.0750	−0.812	0.506	2939.6	0.0750
400	2	0.525	0.498	0.0539	0.0327	0.038	0.498	2775.3	0.0327
100	3	0.464	0.442	0.0538	0.0349	0.401	0.442	0.1734	0.0349
200	3	0.485	0.475	0.0263	0.0190	0.463	0.475	0.1018	0.0190
400	3	0.495	0.489	0.0127	0.0093	0.483	0.489	0.0511	0.0093
100	5	0.457	0.454	0.0124	0.0097	0.426	0.454	0.0218	0.0097
200	5	0.478	0.477	0.0063	0.0049	0.460	0.477	0.0117	0.0049
400	5	0.491	0.489	0.0031	0.0025	0.481	0.489	0.0062	0.0025
100	10	0.468	0.468	0.0026	0.0023	0.460	0.468	0.0034	0.0023
200	10	0.484	0.484	0.0014	0.0012	0.479	0.484	0.0018	0.0012
400	10	0.493	0.492	0.0007	0.0006	0.490	0.492	0.0009	0.0006

Notes: y_{it} is generated from $y_{it} = \mu + \eta_i + v_{it}$, where $\eta_i \sim N(0, 1)$ and $v_{it} = 0.5v_{it-1} + u_{it}$, with $u_{it} \sim i.i.d.\ N(0, 1)$. $v_{i0} \sim N(0, 1/(1 - 0.5^2))$. No time effects are present in the data generating process. y_{it} is observed for $t = 0, \ldots, T$. For $T > 2$, AB1 can also be obtained with the STATA command "`xtabond y, inst(tdum3-tdum6)`" for $T = 5$, for example, after generating the period dummies by "`tab year, gen(tdum)`", where `year` is the period variable.

asymptotically nonsingular, that $h_{nt} = A_{nt}^{-1}\tilde{\boldsymbol{z}}_t'\ddot{\boldsymbol{u}}_t = O_p(1)$, and that $\xi_{nt} = A_{nt}^{-1}\tilde{\boldsymbol{z}}_t'\ddot{\boldsymbol{x}}_t n^{-1/2}$ has a nonzero limit for $T > 2$, so that

$$n^{1/2}(\tilde{\beta} - \beta) = \left(\sum_{t=1}^{T-1} \xi_{nt}' Q_{nt}^{-1} \xi_{nt}\right)^{-1} \sum_{t=1}^{T-1} \xi_{nt}' Q_{nt}^{-1} h_{nt} = O_p(1).$$

Importantly, plim ξ_{nt} is nonzero for $t > 1$, which accounts for the root-n consistency for $T > 2$. The extra instruments Δy_{is} for $s \geq 1$, which are available for $T > 2$, are free of μ and hence strong, while y_{i0} remains weak due to its large mean. Alternatively, the instrument set z_{it} can be transformed to $(y_{i0}, \tilde{y}_{i1}, \ldots, \tilde{y}_{it-1})'$, where $\tilde{y}_{is}$ are the residuals from the OLS regression of y_{is} on y_{i0} without the intercept. This transformation suggests that one instrument y_{i0} is used to demean the other instruments, and thus the residuals $\tilde{y}_{is}$ do not have large means, which explains the consistency. Although $\tilde{\beta}$ is $\sqrt{n}$-consistent, the signal is weaker when the constant term is mistakenly omitted from the instrument set because one instrument is asymptotically irrelevant and only differences (Δy_{is}) provide information for identification.

The problem explained thus far can be resolved by simply including the constant term in every instrument set. In some implementations (e.g., STATA), this is done by specifying the period dummies as extra exogenous instruments. (Readers are referred to the notes to Table 1, where a convenient method is presented.) Alternatively, one may just globally demean all the variables (y_{it} and all the regressors and instruments in more general models) before estimating the model. If full period dummies are present in the model, they are usually added to the instrument set automatically and no special treatment is required. If period dummies do not appear on the right-hand side, it is still recommended to use them as instruments.[2]

We carried out GAUSS simulations for the simple panel dynamic model $y_{it} = \alpha_i + \beta y_{it-1} + u_{it}$. The data are generated from $y_{it} = \mu + \eta_i + v_{it}$, $v_{it} = \beta v_{it-1} + u_{it}$ with $\beta = 0.5$, where $\eta_i \sim N(0, 1)$, $u_{it} \sim i.i.d.\ N(0, 1)$, $v_{i0} \sim N(0, 1/(1 - \beta^2))$, and all the random variables are *i.i.d.* across i. Thus, y_{it} is *i.i.d.* across i and stationary over t. We consider $n = 100, 200, 400$ and $T = 2, 3, 5, 10$ for each of $\mu = 1$ and $\mu = \sqrt{n}$. The usual DGMM estimator is labeled AB (for Arellano–Bond), and the modified estimator (with the constant term included in every instrument set) is denoted as AB1. We expect that AB1 is always $\sqrt{n}$-consistent, and that AB is $\sqrt{n}$-consistent when μ is fixed or $T > 2$. If $\mu = \sqrt{n}$ and $T = 2$, then AB is expected to be inconsistent as $n \to \infty$. The efficiency gain by AB1 over AB would be large for $\mu = \sqrt{n}$.

Table 1 gives the simulation results from 5000 replications. The AB estimator obviously depends on the global mean (μ), while AB1 remains unaffected by a change in μ. For $T = 2$, AB does not appear consistent when $\mu = \sqrt{n}$, as theoretically predicted. For larger T, the performance of AB still depends on μ. When μ is large in magnitude ($\mu = \sqrt{n}$), the efficiency gain of AB1 (the estimator including the constant term in the instrument set) is considerable. For example, when $n = 100$ and $\mu = 10$, the simulated variance of AB1 is about 67% of that of AB.

The system GMM estimator (Arellano and Bover, 1995; Blundell and Bond, 1998) also suffers from the problem of not being invariant under the translation of the dependent variable when constant instruments are omitted. However, the inclusion of the global intercept in the levels equation enables dealing with any large mean and resolves most of the efficiency loss problem.

3. Concluding remarks

In the DGMM estimation of dynamic panel data models, the constant term is often omitted from the instrument set. As a result, the value and statistical properties of the estimator depend on the global mean of the data, and the estimator may exhibit considerable bias and efficiency loss. This problem can be resolved by including the constant term in the instrument set, that is, by including the period dummies as extra exogenous instruments, or by globally demeaning all the relevant variables. Our simulation results suggest that the bias reduction and efficiency gain by using the constant term can be substantial.

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References

Ahn, S.C., Kitazawa, Y., 2014. A non-invariant problem in panel GMM estimators when level instruments are used for difference equations. Sogang University, Mimeo.

[2] When y_{it} exhibits a linear trend or other deterministic trends, they should also be used as instruments to fit the unconditional means of the other instruments. However, these deterministic trends would usually be specified as exogenous regressors in the model, when software packages automatically include them in the instrument sets.

Ahn, S.C., Schmidt, P., 1995. Efficient estimation of dynamic panel data models. J. Econometrics 68, 5–27.
Anderson, T.W., Hsiao, C., 1981. Estimation of dynamic models with error components. J. Amer. Statist. Assoc. 76, 598–606.
Arellano, M., Bond, S., 1991. Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. Rev. Econom. Stud. 58, 277–297.
Arellano, M., Bover, O., 1995. Another look at the instrumental variable estimation of error-components models. J. Econometrics 68, 29–51.
Baltagi, B.H., 2013. Econometric Analysis of Panel Data, fifth ed. Wiley.
Blundell, R., Bond, S., 1998. Initial conditions and moment restrictions in dynamic panel data models. J. Econometrics 87, 115–143.
Breusch, T., Qian, H., Schmidt, P., Wyhowski, D., 1999. Redundancy of moment conditions. J. Econometrics 91, 89–111.
Gørgens, T., Han, C., Xue, S., 2014. Moment restrictions and identification in linear dynamic panel data models. The Australian National University, Mimeo.
Han, C., Kim, H., 2013. Parametrically weighted GMM of dynamic panel estimation. Korea University, Mimeo.