WORKING PAPER SERIES

Nonlinearity and the Permanent Effects of Recessions

Chang-Jin Kim
James Morley
and
Jeremy Piger

Working Paper 2002-014D

October 2002
Revised June 2003

FEDERAL RESERVE BANK OF ST. LOUIS
Research Division
411 Locust Street
St. Louis, MO 63102

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

Photo courtesy of The Gateway Arch, St. Louis, MO. www.gatewayarch.com
Nonlinearity and the Permanent Effects of Recessions*

Chang-Jin Kim  
*Korea University*

James Morley  
*Washington University in St. Louis*

Jeremy Piger  
*Federal Reserve Bank of St. Louis*

May 27, 2002

ABSTRACT: This paper presents a new nonlinear model of economic growth dynamics that captures a post-recession “bounce-back” in the level of output. While a number of studies have attempted to model such an effect using ad hoc recession-based dummy variables, we estimate this business cycle asymmetry using a Markov-switching model. When the model is applied to U.S. GDP, we find a large “bounce-back” effect that is statistically significant according to our Monte Carlo analysis and, in contrast to a vast literature based on linear ARMA models, implies small permanent effects of recessions. Once the business cycle asymmetry is taken into account, there is little or no remaining serial correlation, suggesting that the nonlinearity in the model is sufficient to capture the defining features of U.S. GDP dynamics. When the model is applied to other countries, we generally find larger permanent effects of recessions.

*Keywords:* Asymmetry, Business Cycle, Economic Fluctuations, Markov-Switching  
*JEL Classification:* C22, C5, E32

* * *

* We would like to thank for helpful comments the seminar participants at the May 2002 Workshop on State-Space Models, Regime Switching and Identification held at Washington University and the April 2003 International Conference "On the Wealth of Nations - Extending the Tinbergen Heritage" held at Erasmus University. Mralini Lhila provided research assistance. Responsibility for any errors is our own. Morley acknowledges support from the Weidenbaum Center on the Economy, Government, and Public Policy. The views expressed in this paper should not be interpreted as those of the Weidenbaum Center, the Federal Reserve Bank of St. Louis, or the Federal Reserve System.
1. Introduction

In his seminal paper, Hamilton (1989) captures asymmetry in U.S. business cycles using an endogenous regime-switching model of real output. His model portrays the short, violent nature of recessions relative to expansions. However, other studies emphasize another distinctive feature of U.S. business cycles not captured by Hamilton’s model: output growth tends to be relatively strong following recessions. This feature has traditionally been modeled in an *ad hoc* way by allowing growth dynamics to change in the quarters immediately after a decline in output below its historical maximum (see, for example, Beaudry and Koop, 1993).

In this paper we show that Hamilton’s model can be extended to allow for a post-recession “bounce-back” in the level of output, while maintaining endogenous estimation of the underlying business cycle regimes. Our model provides a test of the “bounce-back” effect and, unlike the *ad hoc* approaches, produces a straightforward estimate of the permanent effects of recessions on the level of output. We find that a post-recession “bounce-back” is an important feature of U.S. business cycle dynamics and that the permanent effects of recessions are substantially less than suggested by both Hamilton and most linear models (e.g. Nelson and Plosser, 1982, Campbell and Mankiw, 1987, and Stock and Watson, 1988). In addition, once the business cycle asymmetry is taken into account, there is little or no remaining serial correlation, suggesting the nonlinearity in the model is sufficient to capture the defining features of U.S. GDP dynamics. The results are robust to allowing for a one-time structural break in business cycle volatility in the mid-1980s and relating the size of the post-recession “bounce-back” to the depth of the
preceding recession. Meanwhile, when we examine international evidence by applying our model to output data for Australia, Canada, Germany, and the United Kingdom, we generally find larger permanent effects of recessions.

2. Background

The idea of inherently different dynamics in expansions and recessions has a long history in business cycle analysis, dating back at least to Mitchell (1927) and Keynes (1936). Recent advances in econometrics have allowed this idea to be formally modeled and tested. Hamilton (1989) captures asymmetric dynamics using a Markov-switching model that estimates two regimes in U.S. GNP growth behaviour. Notably, even though the timing of the regimes is endogenously estimated, he finds that the regimes correspond closely to NBER-dated recessions and expansions.

While the statistical significance of the Markov-switching behaviour in output is clouded by nonstandard test conditions (see Hansen, 1992, and Garcia, 1998), one implication of Hamilton’s estimates is clear: recessions have large permanent effects on the level of output. By one measure discussed in his paper and employed here, the expected level of output is permanently lowered by as much as 4.5% as a result of a transition into recession. However, one reason this estimate may be so large is that Hamilton’s original model is unable to capture the high growth recovery phase typical of post-recession dynamics in the United States. This apparent “bounce-back” in output is evident in Table 1, which reports average growth of real U.S. GDP in the quarters following the troughs of NBER-dated postwar recessions.

One approach to modeling the high growth recovery phase is to add a third regime
to Hamilton’s model, as in Sichel (1994). However, there is much evidence that recoveries are not independent of the preceding recession, as would be implied by a three regime model, but rather the magnitude of the “bounce-back” is related to the length and severity of the recession (see Friedman, 1964, 1993, and Wynne and Balke, 1992, 1996). Kim and Nelson (1999a) allow for this type of business cycle asymmetry by modeling regime switching in the cyclical component of output only. While this relates the “bounce-back” to the preceding recession, it constrains the effects of recessionary shocks to be completely transitory, *a priori*. Thus, we cannot use this approach to examine the permanent effects of recessions on the level of output. Kim and Murray (2002) combine the Hamilton (1989) and Kim and Nelson (1999a) approaches in a multivariate model with regime switching in both the trend and cyclical component of output. While this approach is capable of providing a measure of the permanent effects of recessions, it comes at the price of considerable added complexity and the need for strong identification assumptions.

A related literature models the “bounce-back” effect using nonlinear ARMA processes in which dynamics change when an observed indicator variable exceeds a given threshold. In an important paper, Beaudry and Koop (1993) augment a standard ARMA model of output growth with a “current-depth-of-recession” dummy variable that measures the distance output has fallen below its previous historical maximum. They find that this additional variable is highly significant using a standard *t*-test and recessions typically have no significant permanent effect on the level of U.S. real GDP. However, Hess and Iwata (1997) argue that the dummy variable is nonstationary and the *t*-test overstates the significance of the “bounce-back” effect. The Beaudry and Koop model
has been extended and modified by several authors, most notably Pesaran and Potter (1997) who endogenize the threshold.

Our approach in this paper is to augment Hamilton’s original model with a “bounce-back” term that is scaled by the length of each recession and can generate faster growth in the quarters immediately following a recession. In this way, our model is much like Beaudry and Koop’s (1993). However, unlike the “current-depth-of-recession” variable used in their paper, our “bounce-back” term is directly related to the underlying recessionary regimes and is, therefore, endogenously estimated. It is also stationary by construction and so does not suffer from the Hess and Iwata (1997) critique. Meanwhile, our model places no constraints on the permanent effects of recessions and, like Hamilton’s original model, yields a straightforward measure of the expected long-run effect.

3. Model

Our model is given as follows:

$$
\phi(L) \left( \Delta y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j} \right) = \varepsilon_t, \quad \varepsilon_t \sim i.i.d. N(0, \sigma^2),
$$

where the lag operator $\phi(L)$ is $k$-th order with roots outside the unit circle, $\Delta y_t$ is the first difference of the logarithm of real GDP, and $S_t$ is an unobserved Markov-switching state variable that takes on discrete values of 0 or 1 according to transition probabilities.
Pr[\(S_t = 0 \mid S_{t-1} = 0\)] = q \text{ and } Pr[\(S_t = 1 \mid S_{t-1} = 1\)] = p. \text{ We normalize the states by restricting } \mu_t < 0. \text{ That is, } S_t = 1 \text{ corresponds to a “lower growth” regime or, if } \mu_0 + \mu_1 < 0, \text{ a “contractionary” regime.}

The key variable in our model is the summation term \(\sum_{j=1}^{m} S_{t-j}\), which we denote as \(\tilde{S}_t\) hereafter. This term implies a “bounce-back” effect if \(\lambda > 0\), while Hamilton’s (1989) model obtains if \(\lambda = 0\). Given \(\lambda > 0\), \(\tilde{S}_t\) implies that growth will be above average for the first \(m\) periods of an “expansionary” regime.

To see how the “bounce-back” effect works, consider Figure 1, which shows the simulated effect of a recession for both our model and Hamilton’s original model. For both models, we set the underlying growth rate parameters to be \(\mu_0 = 1\) and \(\mu_1 = -2\). For our model, we set the “bounce-back” coefficient to be \(\lambda = 0.2\) (versus \(\lambda = 0\) for Hamilton’s model) and the length of the post-recession “bounce-back” \(m = 6\) periods.

We ignore the autoregressive parameters since, for simplicity of presentation, we abstract from the regular linear \(\varepsilon_t\) shocks in simulating the effects of a recession on output. In the bottom of the figure, the thick line represents a hypothetical time path for the state variable \(S_t\). The shift in \(S_t\) from 0 to 1 represents a movement of the economy into a “contractionary” regime for 4 periods, denoted by the shading. As the regime hits in period 0 and persists until period 4, output falls for both our model and Hamilton’s model. Meanwhile, the summation term \(\tilde{S}_t\) increases each period up to the length of the recession, which is 4 periods in this example. For our model, the effect of the \(\tilde{S}_t\) term
begins to offset the effect of the $S_t$ term as the recession persists, and output starts to level off.\textsuperscript{1} After the recession ends and $S_t$ returns to 0, the summation term $\tilde{S}_t$ reaches its maximum, and the level of output rises faster than average since $\lambda > 0$. This “bounce-back” in the level of output continues for $m = 6$ periods, but its effect diminishes as the expansion persists and the $\tilde{S}_t$ term decreases until it reaches its minimum value of 0. By contrast, Hamilton’s model with $\lambda = 0$ has output rise from its trough at its regular “expansionary” growth rate only, implying a much larger permanent effect of the recession on the level of output.

We estimate the model via maximum likelihood using the filter presented in Hamilton (1989). The main added complexity is that, due to the $\tilde{S}_t$ term, we need to keep track of more states ($2^{k+m}$ versus $2^k$, where $k$ is the number of autoregressive terms) when constructing the likelihood function in each period. Standard errors are based on numerical second derivatives.

4. Estimates for U.S. GDP

The data for $y_t$ are 100 times the log of quarterly real U.S. GDP over the sample period of 1947:Q1 to 2003:Q1. We use the Schwarz Information Criterion to select the lag length $k$ for the autoregressive polynomial and the length $m$ of the post-recession “bounce-back”. We consider upper-bounds of $k = 4$ and $m = 9$. For the autoregressive

\textsuperscript{1} As an alternative specification, we could have allowed the recession state variable and the “bounce-back” term to interact such that the dynamics in the Hamilton model and the augmented model differ only following a recession. However, when we estimated a more general model that included both the “bounce-back” term and the interaction term, we found that the latter was highly insignificant (the $t$-statistic was -0.12). Thus, the leveling off of output during a prolonged recession appears to be an important aspect of the “bounce-back” dynamics.
polynomial, we find that $k = 0$, suggesting that the nonlinear dynamics in our model are sufficient to capture most or all of the serial correlation in the data. For the post-recession “bounce-back”, we find that $m = 6$ quarters, which is consistent with the results in Table 1. Table 2 reports maximum likelihood estimates for the $k = 0$ and $m = 6$ case. The results are robust for similar values of $m$ and $k$.

The first result to notice in Table 2 is that $\mu_0 + \mu_1 < 0$, implying that $S_t = 1$ corresponds to a “contractionary” regime. The transition probabilities also suggest that expansions are more persistent than contractions, much like the NBER reference cycle. Indeed, the top panel of Figure 2 reveals a strong correspondence between the smoothed probability of being in a contractionary regime and the NBER recession dates represented by shading. For eight of the ten NBER recessions in the sample, the smoothed probability spikes up above 50% around the time of the business cycle peak date established by the NBER. The 1970 and 2001 recessions are the exceptions, for these recessions the smoothed probability moves up during the NBER recession dates, but remains below 50%. Also, for seven of these eight recessions, the smoothed probability falls to close to zero around the trough date established by the NBER. Here the exception is the 1990-1991 recession, for which the smoothed probability only returns to low levels after the end of the NBER trough date. In Section 6, we discuss modifications of the model that improve its ability to capture the 1970, 1990-1991 and 2001 recessions, although at the cost of some additional complexity.

---

2 At the time of writing this paper, the NBER had not yet established the trough date for the recession that began in March 2001. For the purposes of shading, we use the model-based estimate of the trough date obtained by Chauvet and Piger (2003).
The bottom panel of Figure 2 displays the smoothed estimate of $\tilde{S}_t$. As in Figure 1, this term increases as the length of each contraction progresses, and declines soon after the recession is over. Again, this term and its coefficient $\lambda$ determine the size of the “bounce-back” effect. Our estimate of $\lambda$ is positive, corresponding to faster growth during post-recession recoveries. The $t$-statistic for $H_0 : \lambda = 0$ is $6.4$, which is highly significant using standard asymptotic critical values.

A possible concern is whether relying on the standard critical values for a $t$-test of $H_0 : \lambda = 0$ is appropriate. Hess and Iwata (1997) argue that Beaudry and Koop’s (1993) “current-depth-of-recession” variable is nonstationary. Thus, the estimate for its coefficient has a nonstandard distribution. In our case, however, the $\tilde{S}_t$ term will be stationary since $S_t$ is stationary and $\tilde{S}_t$ is the sum of a finite number of lags of $S_t$. Of course, given the persistence of the $\tilde{S}_t$ term, the small sample distribution may be very different to the asymptotic distribution.

To address this concern, we conduct a Monte Carlo experiment for sample sizes of $T=200$ and $T=500$. For our data generating process, we use Hamilton’s (1989) original estimated model, for which $\lambda = 0$. We estimate our model allowing $\lambda \neq 0$ for each simulation and calculate $t$-statistics for the null hypothesis $H_0 : \lambda = 0$. Table 3 reports critical values for our experiment based on 1000 simulations.\(^3\) The critical values are

\(^3\) Only recent advances in computing power make this experiment practicable. With a 1.9 GHz processor, it takes well over 400 hours to complete 1000 simulations.
larger than the standard normal case, reflecting a small-sample distortion. However, our estimate of $\lambda$ remains significant at the 1% level, even using the $T=200$ results.

5. Are U.S. Recessions Permanent?

Given the presence of a “bounce-back” effect, there is a question of whether recessions have permanent effects on the level of output. Hamilton (1989) provides a useful measure of the long-run effects of recessions in the context of regime switching models. He considers the expected difference in the long-run level of output given that the economy is currently in a “contractionary” regime versus an “expansionary” regime:

$$\lim_{j \to \infty} \{E[y_{t+j} \mid S_t = 1, \Omega_{t-1}] - E[y_{t+j} \mid S_t = 0, \Omega_{t-1}]\},$$

where $\Omega_{t-1} = \{S_{t-1} = 0, S_{t-2} = 0; \ldots; y_{t-1}, y_{t-2}, \ldots\}$. For our model, this limit, which we denote as $\Lambda$ hereafter, is a closed-form expression:

$$\Lambda = (\mu_s + m\lambda)/(2 - q - p).$$

The estimated value for $\Lambda$ reported in Table 1 is –0.412, or just under a 0.5% permanent drop in the level of GDP. By contrast, Hamilton’s estimates imply a 4.5% permanent drop.

It is interesting to compare our finding to what has been reported in classic studies, including Nelson and Plosser (1982), Campbell and Mankiw (1987), and Stock
and Watson (1988), on the long-run effects of shocks to the level of output using linear ARIMA models. For example, consider the following linear autoregressive model of the first differences of output:

\[
\phi(L)(\Delta y_t - \mu) = \varepsilon_t, \quad \varepsilon_t \sim i.i.d.(0, \sigma^2),
\]

where the lag operator \( \phi(L) \) is \( k \)-th order with roots outside the unit circle. For this model, the long-run response of output to a unit shock, \( \varepsilon_t = 1 \), is the following closed-form expression:

\[
\lim_{j \to \infty} \left\{ E[y_{t+j} \mid \varepsilon_t = 1] - E[y_{t+j} \mid \varepsilon_t = 0] \right\} = \frac{1}{\phi(1)}.
\]

The literature reports estimates of this expression that are uniformly large. For example, Stock and Watson (1988) survey the literature and report a range of estimates between 1.6 for lag order \( k = 1 \) and 0.9 for lag order \( k = 24 \). These estimates are consistent with what we find for the regular linear \( \varepsilon_t \) shocks since our preferred lag order \( k = 0 \) corresponds to an implied long-run multiplier of 1. However, since linear models restrict the dynamics to be the same for all shocks, these estimates imply that all shocks, even recessionary shocks, have large permanent effects on output. By contrast, without this restriction, we find that not all shocks have the same dynamics. Again, recessionary shocks have nonlinear dynamics that imply much smaller long-run effects on output.
6. A Structural Break in Business Cycle Volatility and the Role of Depth

As Figure 2 demonstrates, the recession dates established by the NBER are closely matched by the contractionary regimes identified by the model. The exceptions to this are the 1970 and 2001 recessions, for which there is little evidence of a contractionary regime, and the 1990-1991 recession, for which the end of the contractionary regime is after the trough established by the NBER. In this section, we discuss some possible reasons for these exceptions, and present two modifications to the model that improve its ability to capture the NBER-dated recessions.

One explanation for the model’s inability to match all NBER recessions is that it ignores a reduction in the volatility of the U.S. business cycle since the mid-1980s. Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000) have shown that Hamilton’s (1989) Markov-switching model is better able to detect NBER recessions once this structural change is accounted for. Thus, it is possible that the failure of our model to capture certain recessions is a consequence of ignoring this structural change.

To analyze the role of the apparent reduction in business cycle volatility in explaining our results, we consider a model that allows for a structural break in model parameters related to business cycle volatility. Specifically, as in Kim and Nelson (1999b), we allow for a one-time change in the drift parameters \( \mu_0 \) and \( \mu_1 \) and the variance parameter \( \sigma^2 \). The breakpoint is set at 1984:Q1, the date established by both Kim and Nelson (1999b) and McConnell and Perez-Quiros (2000). All other model parameters are assumed to be constant over the entire sample period.
Table 4 reports the parameter estimates for this model with a structural break. For comparison purposes, we set the autoregressive lag length $k = 0$ and the length of the “bounce-back” $m = 6$. The estimates suggest large changes in all three parameters corresponding to a reduction in volatility. The standard deviation of $\varepsilon_t$ shocks falls by half from 0.9% to 0.4% and there is a reduction in the gap between the drift parameters from 1.9% to 1.2%. Also, the average growth rate in recessions increases from –0.8% to –0.3%. Figure 3 shows that allowing for this structural break improves the ability of the model to capture both the 1970 and 2001 recession significantly, with the smoothed probability of recession now rising above 50% for both recessions. However, the model has a difficult time identifying the end of the two recessions that occur after the structural break in the mid-1980s.

In addition to being affected by a reduction in volatility, the two most recent recessions have been followed by relatively weak recoveries. It is likely that our model, which predicts rapid growth following the end of a recession, is overstating the length of the estimated contractionary regimes as a way to account for this weak growth. Indeed, according to Figure 3, the 1990s contractionary regime did not end until the fast growth in output in the late-1990s. One reason that our model might suggest such different timing than the NBER for the 1990s recession is that it implicitly scales the size of the “bounce-back” by the length of the preceding recession, while it may actually be more closely linked to the severity or depth of the recession. Length and depth of recessions are obviously related. However, the link between the two may have weakened since the structural break in GDP volatility reduced the depth of recessions.
To examine whether differences in the depth of specific recessions can explain our results, we consider the following modification of our model:

\[
\phi(L) \left( \Delta y_t - \mu_0 - \mu_1 S_t - \lambda \sum_{j=1}^{m} S_{t-j} \Delta y_{t-j} \right) = \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0, \sigma^2),
\]

where each lagged state \( S_{t-j} \) in the summation term interacts with the corresponding lagged change in output \( \Delta y_{t-j} \). This modification implicitly scales the size of the post-recession “bounce-back” by the depth of the recession. That is, given two recessions of equal length but different cumulative declines in output, the deeper recession is predicted to have the larger “bounce-back”.

Table 5 reports the results for the model with a structural break and depth as the determinant of the size of the post-recession “bounce-back”. Again for comparison purposes, we set the autoregressive lag length \( k = 0 \) and the length of the “bounce-back” \( m = 6 \). The parameter estimates are qualitatively similar to those reported in Tables 2 and 4. However, Figure 4 demonstrates that the model with a structural break and depth is able to capture the two most recent recessions. This finding suggests that the decreased

---

4 To be precise, our model captures length up to the upper-bound equal to \( m \), the length of the post-recession “bounce-back” period. However, since the longest postwar recession in the U.S. is six quarters and our model selection procedure picks \( m = 6 \), the summation term can be said to capture length.

5 Note, that this modification, while apparently simple, makes calculation of the long-run effects of recessions much more difficult due to the interaction of two random variables. A closed form solution for the long-run effect is not available.

6 Note that the “bounce-back” coefficient is not directly comparable. However, since the estimates in Table 2 suggest that average GDP growth is close to \(-1\%\) in a recession, the “bounce-back” parameter for the depth model should be of a similar magnitude, but opposite sign.
severity of the two most recent recessions helps explain their subsequent slow recoveries.

7. International Evidence

Are the nonlinear dynamics in the U.S. data a common feature of business cycle fluctuations for other countries? Although several authors, for example Mills and Wang (2002), have fit regime-switching models to international data, the focus of analysis has been on models in which the extent of a “bounce-back”, and thus the long run effect of a recession, is assumed rather than estimated. To investigate the degree to which a “bounce-back” effect is present outside of the United States, we apply our main model to real GDP data obtained from the OECD for Australia, Canada, Germany, and the United Kingdom. Due to data limitations and the observation that at least two of the countries (Canada and Germany) display significant slowdowns in trend productivity growth sometime in the early 1970s, we consider the international evidence for a shorter sample beginning in 1973. We also re-estimate the model for the United States over this shorter sample period to provide a benchmark for comparison. For all countries, we set the autoregressive lag length $k = 2$ and the length of the “bounce-back” $m = 6$. The lag order is sufficient to capture all of the significant autoregressive dynamics for every country and provides a common model specification for comparison purposes.

Figure 5 displays the smoothed probability of being in a “lower growth” regime and the output series for Australia, Canada, Germany, the United Kingdom, and the United States, respectively. The model appears to capture recessions, with the “lower growth” regime corresponding to contractions in the output series for every country
except Germany, for which the growth rate in that regime is essentially zero. For the United States, the smoothed probabilities in Figure 5 closely match those for the same part of the longer sample period displayed in Figure 2.7

Table 6 reports estimates of the average length of recessions $1/(1 - p)$, the “bounce-back” coefficient $\lambda$, and our measure of the permanent effect of recessions $\Lambda$ for each country. The estimates of $1/(1 - p)$ suggest that recessions have lasted longer in Canada, Germany, and the United Kingdom than in Australia and the United States. There is wide variation in the estimates of the “bounce-back” coefficient $\lambda$. Australia, Canada, and the United States have a positive “bounce-back” effect, while the estimates for Germany and the United Kingdom are essentially zero.8 The results for $\Lambda$, the permanent effects of recessions, generally reflect the presence of a “bounce-back” effect, although Canada suffers from lengthy and severe recessions. Thus, only Australia and the United States have small permanent effects of recessions, while Canada, Germany, and the United Kingdom have large effects.

One caveat for these results is that, given $m = 6$, our model cannot capture a post-recession “bounce-back” that occurs later in expansions. However, even if a delayed recovery is driving the results for Canada, Germany and the United Kingdom, it still suggests that the welfare costs of recessions are much higher than in Australia and the United States. Indeed, while many explanations for the differing dynamics are possible,

---

7 The slightly weaker probability for the 1990s recession appears to reflect the AR(2) specification.
8 The estimates are positive, but very small and highly insignificant. However, we normalize the “bounce-back” coefficient to be non-negative for these countries. The normalization is necessary because a model with a negative “bounce-back” coefficient and short-lived contractionary regimes is observationally equivalent to a model with no “bounce-back” effect and more persistent recessionary regimes. Given the normalization of the regimes, there is no similar observational equivalence for models with positive “bounce-back” coefficients.
the longer duration and persistence of recessions in Canada, Germany, and the United Kingdom is suggestive of theories of hysteresis used to explain higher levels of unemployment in these countries (see, for example, Blanchard and Summers, 1986). That is, the “bounce-back” effect in Australia and the United States could reflect greater flexibility in labour markets. Meanwhile, the small, but positive “bounce-back” effect for Canada could reflect the influence of U.S. recoveries on the Canadian economy.

8. Conclusions

In summary, we find that the permanent effects of recessions for the United States are substantially less than suggested by both Hamilton (1989) and most linear models (e.g. Nelson and Plosser, 1982, Campbell and Mankiw, 1987, and Stock and Watson, 1988). Instead, we find evidence of a large “bounce-back” effect during the recovery phase of the business cycle. Meanwhile, after accounting for the dynamics of recessions and their recoveries, our model implies that there is little or no remaining serial correlation in U.S. output growth during the regular expansion phase of the business cycle. A modified version of the model that allows for a structural break in business cycle volatility and relates the size of the “bounce-back” to the depth of the preceding recession produces nearly identical regimes to the NBER-dated recessions and expansions. Finally, when the main model is applied to international data, the “bounce-back” effect is generally less relevant outside the United States, corresponding to larger permanent effects of recessions for other countries.
References


Friedman, M., 1964, Monetary studies of the National Bureau, the National Bureau enters its 45th Year, 44th Annual Report, 7-25 (NBER, New York); Reprinted in Friedman, M., 1969, The optimum quantity of money and other essays (Aldine, Chicago).


Economic Perspectives 2, 147-174.

Wynne, M.A. and N.S. Balke, 1992, Are deep recessions followed by strong recoveries?

Wynne, M.A. and N.S. Balke, 1996, Are deep recessions followed by strong recoveries?
   Results for the G-7 countries, Applied Economics 28, 889-897.
Table 1

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Average Growth (%)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>0.83</td>
<td>224</td>
</tr>
<tr>
<td>1 quarter after recession</td>
<td>1.75</td>
<td>10</td>
</tr>
<tr>
<td>2 quarters after recession</td>
<td>1.59</td>
<td>10</td>
</tr>
<tr>
<td>3 quarters after recession</td>
<td>1.53</td>
<td>10</td>
</tr>
<tr>
<td>4 quarters after recession</td>
<td>1.51</td>
<td>9</td>
</tr>
<tr>
<td>5 quarters after recession</td>
<td>1.08</td>
<td>9</td>
</tr>
<tr>
<td>6 quarters after recession</td>
<td>1.09</td>
<td>8</td>
</tr>
<tr>
<td>7 quarters after recession</td>
<td>1.00</td>
<td>8</td>
</tr>
<tr>
<td>8 quarters after recession</td>
<td>0.83</td>
<td>7</td>
</tr>
</tbody>
</table>
### Table 2
**Maximum Likelihood Estimates**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.836</td>
<td>0.064</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-2.055</td>
<td>0.232</td>
</tr>
<tr>
<td>$\mu_0 + \mu_1$</td>
<td>-1.219</td>
<td>0.229</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.319</td>
<td>0.050</td>
</tr>
<tr>
<td>$q$</td>
<td>0.957</td>
<td>0.017</td>
</tr>
<tr>
<td>$p$</td>
<td>0.695</td>
<td>0.101</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.768</td>
<td>0.042</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>-0.412</td>
<td>0.898</td>
</tr>
</tbody>
</table>

**Log likelihood**  
-288.088
Table 3
Monte Carlo Results

<table>
<thead>
<tr>
<th>$p$-value</th>
<th>$T=200$</th>
<th>$T=500$</th>
<th>$N(0,1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5.00</td>
<td>3.16</td>
<td>2.57</td>
</tr>
<tr>
<td>0.05</td>
<td>3.09</td>
<td>2.19</td>
<td>1.96</td>
</tr>
<tr>
<td>0.10</td>
<td>2.45</td>
<td>1.85</td>
<td>1.64</td>
</tr>
</tbody>
</table>

*Based on 1000 simulations.*
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{0,1947;Q1–1983;Q4}$</td>
<td>1.084</td>
<td>0.114</td>
</tr>
<tr>
<td>$\mu_{0,1984;Q1–2003;Q1}$</td>
<td>0.873</td>
<td>0.069</td>
</tr>
<tr>
<td>$\mu_{1,1947;Q1–1983;Q4}$</td>
<td>-1.826</td>
<td>0.245</td>
</tr>
<tr>
<td>$\mu_{1,1984;Q1–2003;Q1}$</td>
<td>-1.123</td>
<td>0.160</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.154</td>
<td>0.035</td>
</tr>
<tr>
<td>$q$</td>
<td>0.936</td>
<td>0.024</td>
</tr>
<tr>
<td>$p$</td>
<td>0.862</td>
<td>0.055</td>
</tr>
<tr>
<td>$\sigma_{1947;Q1–1983;Q4}$</td>
<td>0.930</td>
<td>0.070</td>
</tr>
<tr>
<td>$\sigma_{1984;Q1–2003;Q1}$</td>
<td>0.428</td>
<td>0.037</td>
</tr>
</tbody>
</table>

*Log likelihood*  
-269.662
Table 5
Maximum Likelihood Estimates Given a Structural Break and Depth

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{2017:Q1-1983:Q4}$</td>
<td>1.191</td>
<td>0.141</td>
</tr>
<tr>
<td>$\mu_{2018:Q1-2003:Q1}$</td>
<td>0.927</td>
<td>0.063</td>
</tr>
<tr>
<td>$\mu_{1984:Q1-1983:Q4}$</td>
<td>-1.719</td>
<td>0.293</td>
</tr>
<tr>
<td>$\mu_{1984:Q1-2003:Q1}$</td>
<td>-0.963</td>
<td>0.167</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.303</td>
<td>0.105</td>
</tr>
<tr>
<td>$q$</td>
<td>0.926</td>
<td>0.025</td>
</tr>
<tr>
<td>$p$</td>
<td>0.759</td>
<td>0.076</td>
</tr>
<tr>
<td>$\sigma_{1947:Q1-1983:Q4}$</td>
<td>0.887</td>
<td>0.069</td>
</tr>
<tr>
<td>$\sigma_{1984:Q1-2003:Q1}$</td>
<td>0.436</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Log likelihood: -269.422
Table 6  
**International Comparison**

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimate of $1/(1 - p)$</th>
<th>Estimate of $\lambda$</th>
<th>Estimate of $\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>2.8</td>
<td>0.153</td>
<td>-1.573</td>
</tr>
<tr>
<td>Canada</td>
<td>5.9</td>
<td>0.120</td>
<td>-3.878</td>
</tr>
<tr>
<td>Germany</td>
<td>7.9</td>
<td>0.000</td>
<td>-3.476</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>4.5</td>
<td>0.000</td>
<td>-5.337</td>
</tr>
<tr>
<td>United States</td>
<td>3.1</td>
<td>0.241</td>
<td>-1.268</td>
</tr>
</tbody>
</table>
Fig. 1
The “Bounce-Back” Effect (Recession is shaded)
Fig. 2
Smoothed Inferences for $S_t$ and $\tilde{S}_t$ (NBER recessions are shaded)
Fig. 3
Smoothed Inferences for $S_t$ Given a Structural Break (NBER recessions are shaded)
Pr[$S_t \mid y_1, \ldots, y_T$]

Fig. 4
Smoothed Inferences for $S_t$ Given a Structural Break and Depth (NBER recessions are shaded)
Fig. 5
Smoothed Probability of $S_t$ (right axis) and Log Real GDP (left axis)
Fig. 5 continued
Smoothed Probability of $S_t$ (right axis) and Log Real GDP (left axis)
Fig. 5 continued
Smoothed Probability of $S_t$ (right axis) and Log Real GDP (left axis)