Does an Intertemporal Tradeoff between Risk and Return Explain Mean Reversion in Stock Prices?

Chang-Jin Kim*

Korea University

James C. Morley*†

Washington University

and

Charles R. Nelson*

University of Washington

December 5, 1999

* We have received helpful comments from Charles Engel, Dick Startz, Eric Zivot, participants in seminars at the Federal Reserve Bank of Dallas, the Federal Reserve Bank of St. Louis, Washington University, and the University of Washington, and two anonymous referees, but responsibility for any errors is entirely our own. Support from the National Science Foundation under grants SBR-9711301 and SES-9818789 and the Ford and Louisa Van Voorhis endowment at the University of Washington is gratefully acknowledged. This paper is based in part on Morley's doctoral dissertation at the University of Washington.

† Corresponding author. Department of Economics, Box 1208, Washington University, One Brookings Drive, St. Louis, MO 63130-4899, USA. Email: morley@wueconc.wustl.edu.

ABSTRACT: When volatility feedback is taken into account, there is strong evidence of a positive tradeoff between stock market volatility and expected returns on a market portfolio. In this paper, we ask whether this intertemporal tradeoff between risk and return is responsible for the reported evidence of mean reversion in stock prices. There are two relevant findings. First, price movements not related to the effects of Markov-switching market volatility are largely unpredictable over long horizons. Second, time-varying parameter estimates of stock return predictability reject any inherent mean reversion in favour of behaviour implicit in the historical tradeoff between risk and return.

JEL classification: G12; G14

Keywords: Volatility Feedback; Mean Reversion; Markov Switching; Time-Varying Parameter

1. Introduction

More than a decade has passed since Fama and French (1988) and Poterba and Summers (1988) reported that price movements for market portfolios of common stocks tend to be at least partially offset over long horizons. This behaviour, labeled "mean reversion," runs contrary to the random walk hypothesis of stock prices. Subsequent papers by Richardson and Stock (1989), Kim, Nelson, and Startz (1991), and Richardson (1993) have challenged the statistical significance of the mean reversion evidence. But, as Summers (1986) points out, statistical tests employed in studies of the random walk hypothesis are ultimately constrained by their low power against mean-reverting alternatives. Thus, point estimates, which according to Fama and French (p. 247) imply "25-45 percent of the variation of 3–5-year stock returns is predictable from past returns," may require some behavioural explanation.

In this paper, we take the economic magnitude of the reported evidence of mean reversion at face value and ask whether it can be explained by an intertemporal tradeoff between risk and return. We add empirical content to our explanation by limiting our definition of risk to the general level of stock market volatility. Specifically, we consider an empirical model of the tradeoff between market volatility and expected returns on a market portfolio, originally due to Turner, Startz, and Nelson (1989) (hereafter, "the TSN model"). The model uses Markov-switching regimes to capture the effects of large changes in market volatility. This approach has been used elsewhere, including Schwert (1989a) and Schaller and van Norden (1997). But the formulation used in Turner, Startz, and Nelson has the distinctive feature that it implicitly accounts for volatility feedback in measuring the intertemporal tradeoff between risk and return. Volatility feedback is

simply the idea that an unanticipated change in the level of market volatility will have an immediate impact on stock prices as investors react to new information about future discounted expected returns. In particular, if the level of market volatility is persistent, then the current price index and future discounted expected returns should move in opposite directions. Thus, it is important to account for volatility feedback in order to avoid obscuring any underlying positive tradeoff between volatility and expected returns. As discussed in Kim, Morley, and Nelson (1999), estimates for the TSN model provide strong support for both volatility feedback and a positive tradeoff between volatility and expected returns. French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992) find similar results for alternative specifications of volatility. The conclusion of these papers is that the predictable behavioural response of risk-averse investors to large changes in market volatility explains a statistically significant portion of stock price movements. The question in this paper, then, is whether these movements are responsible for the reported evidence of mean reversion.

To test our explanation, we incorporate the TSN model within a regression test of stock return predictability due to Jegadeesh (1991) (hereafter, "the Jegadeesh test"). This allows us to estimate the remaining predictability of stock price movements that are not directly related to the effects of Markov-switching market volatility. Using CRSP data on NYSE-listed stocks for the period of 1926-96, we find that the unexplained price movements display much less predictability than overall returns. For example, the largest estimate of predictability for the value-weighted market portfolio is reduced from implying four-year mean reversion of more than 30 percent to implying four-year

reversion of only 2 percent. Meanwhile, the estimates of predictability are always statistically insignificant for even the most generous distributional assumptions.

To further test our explanation, we develop a time-varying parameter version of the Jegadeesh test. This allows us to estimate changes in overall stock return predictability over the 1926-96 sample. Such changes are of interest because of the stock price behaviour implicit in the actual historical tradeoff between risk and return. In particular, the probability inferences for the TSN model depict seemingly periodic 3–5-year regime shifts during the 1930s and 1940s, followed by much less regular shifts during the postwar period. A broadly similar historical pattern for market volatility is portrayed in classic studies by Officer (1976), French, Schwert, and Stambaugh (1987), and Schwert (1989a,b). Thus, given our finding for the previous test that unexplained price movements are largely unpredictable, only 1930s and 1940s price movements should be consistent with mean reversion over 3–5-year horizons. By contrast, postwar price movements should be more consistent with the random walk hypothesis than mean reversion. This change is precisely what we find. In particular, time-varying estimates of return predictability reflect both the historical tendency for price movements to be offset over 3–5-year horizons during the 1930s and 1940s and the disappearance of any such tendency during the postwar period. This result argues against any inherent mean reversion and provides further support for our explanation of the reported mean reversion evidence. It also explains why Fama and French (1988), Poterba and Summers (1988), and Kim, Nelson, and Startz (1991) find that the mean reversion evidence is extremely sensitive to the inclusion of 1930s and 1940s data in estimation.

The rest of this paper is organized as follows. Section 2 presents the details behind the tests employed in this paper. Section 3 reports empirical results for monthly data from CRSP. Section 4 concludes. Tables and figures follow the appendix and a list of references.

2. Tests of Predictability

Following Jegadeesh (1991), we employ regression tests of stock return predictability where the dependent variable is a one month return, the independent variable is a lagged multiple month return, and the coefficient on the lagged return is negative given mean reversion. Richardson and Smith (1994) demonstrate the similarity of the Jegadeesh test to the overlapping autoregression test used in Fama and French (1988) and the variance ratio test used in Poterba and Summers (1988). They show that each test statistic can be represented as a weighted-average of sample autocorrelations for returns, with the difference between each statistic being the weights. Given the rough equivalence of these three tests, there are two reasons why we choose the Jegadeesh test in particular. First, Jegadeesh shows that, within a class of regression tests that also includes the overlapping autoregression test, his test has the highest asymptotic power against Summers' (1986) fads model.[1] Second, implementation of the Kalman filter for time-varying parameter analysis is most straightforward for the Jegadeesh test since it avoids the imposed MA error structure of an overlapping autoregression.

[1] See Jegadeesh (1991) for details. Briefly, he uses the approximate slope criterion to determine the optimal aggregation intervals for the dependent and independent variables in terms of power against mean reversion. For Summers' (1986) fads model with a variety of parameter values, Jegadeesh finds that the optimal aggregation interval for the dependent variable is always one month.

We consider three variations on the Jegadeesh test. Each variation is designed to address a different issue and can be thought of in terms of what is assumed about the specification of the mean return and coefficient in the regression equation. The first variation has a constant mean and fixed coefficient and provides a formal benchmark for our extensions. The second variation has a time-varying mean and fixed coefficient and tests for the predictability of stock price movements that are not directly related to the effects of Markov-switching market volatility. The third variation has a constant mean and time-varying coefficient and tests for changes in stock return predictability.

2.1 Constant Mean and Fixed Coefficient Specification

The first variation is actually the original specification employed in Jegadeesh (1991). The regression equation is given as follows:

$$r_t - m = b(k)\sum_{j=1}^{k}(r_{t-j} - m) + e_t \ , \tag{1}$$

where r_t is the one month continuously compounded return on a market portfolio, m is the mean of r_t, k is the holding period in months for lagged returns, and e_t is a serially uncorrelated error term. Under the null hypothesis $H_0 : b(k) = 0$, sometimes referred to as the "random walk hypothesis," the market return is serially uncorrelated, with constant expected value m.[2] Under the alternative hypothesis $H_A : b(k) \neq 0$, the market return is

[2] We refer to the broad version of the "random walk hypothesis" that allows for a positive drift and time dependence for higher moments, including the variance.

predictable using past returns, with $b(k) < 0$ corresponding to mean reversion. Note that the coefficient $b(k)$ is most comparable to the regression coefficient for a *k/2*-month overlapping autoregression since both reflect an almost identical set of sample autocorrelations (Richardson and Smith, 1994). Therefore, given that previous studies find the strongest evidence of mean reversion for 2–5-year overlapping autoregressions, we should expect to find the strongest evidence of mean reversion for holding periods between 4–10-years. Since we take the reported evidence of mean reversion at face value, we intentionally focus on holding periods in this range, even though this stacks the evidence in favour of finding mean reversion (Richardson, 1993). For the same reason, we purposely do not adjust reported estimates for a negative bias in $b(k)$ under the null hypothesis (Jegadeesh, 1991). For estimation, we use ordinary least squares (OLS).

2.2 Time-Varying Mean and Fixed Coefficient Specification

The second variation extends the Jegadeesh test by nesting the TSN model of the intertemporal tradeoff between risk and return. The regression equation is given as follows:

$$r_t - m_t = b(k)\sum_{j=1}^{k}(r_{t-j} - m_{t-j}) + e_t \,, \tag{2}$$

where e_t has a two-state Markov-switching variance:

$$e_t \sim i.i.d.\, N(0, s_{et}^2)\,,$$

$$s_{et}^2 = s_{e0}^2(1 - S_t) + s_{e1}^2 S_t\,,\ s_{e1}^2 > s_{e0}^2\,,$$

$$S_t = \{0,1\}\,,\ \Pr[S_t = 0 | S_{t-1} = 0] = q\,,\ \Pr[S_t = 1 | S_{t-1} = 1] = p\,.$$

That is, the conditional variance s_{et}^2 switches between "high" and "low" volatility regimes according to an unobserved Markov-switching state variable S_t with transition probabilities q and p.[3] The time-varying mean m_t has the following three terms:

$$m_t = m_0 + m_1 \Pr[S_t = 1 | r_{t-1}, r_{t-2}, ...] + d\,(S_t - \Pr[S_t = 1 | r_{t-1}, r_{t-2}, ...])\,, \tag{3}$$

where the parameters m_0 and m_1 and the conditional probability $\Pr[S_t = 1 | r_{t-1}, r_{t-2}, ...]$ determine the expected return when $b(k) = 0$, while the parameter d and the difference between the true state and its conditional probability $(S_t - \Pr[S_t = 1 | r_{t-1}, r_{t-2}, ...])$ determine the volatility feedback effect. The full theoretical motivation for this interpretation of the parameters is presented in Kim, Morley, and Nelson (1999). Meanwhile, the specification in equation (3) represents a simple linear transformation of the "learning" specification developed in Turner, Startz, and Nelson (1989). For estimation, we use maximum likelihood and a modified version of the filter for Markov switching presented in Hamilton (1989). Details can be found in Appendix A.1.

[3] Numerous other studies, including Schwert (1989a) and Schaller and van Norden (1997), have used Markov switching to capture large changes in market volatility. The best justification for Markov-switching volatility, however, comes from a paper by Hamilton and Susmel (1992). They develop a Markov-switching autoregressive conditional heteroskedasticity (SWARCH) model of weekly stock returns and find that, once Markov-switching regime changes are accounted for, most ARCH effects die out at the monthly return horizon considered in this paper.

The components of the time-varying mean m_t warrant further discussion. First, consider the expected return component: $m_0 + m_1 \Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...]$. We assume the expected return is a simple linear function of expectations about level of market volatility. Then, given a positive tradeoff between volatility and expected return, both m_0 and m_1 should be positive. That is, a positive and increasing conditional expectation of the level of volatility should correspond to a positive and increasing return, ceteris paribus. Note that we avoid imposing a strict proportionality on the relationship between expected return and expected volatility. In particular, the marginal impact of an increase in the expectation of market volatility can be different than the overall impact of having a positive level of volatility. Second, consider the volatility feedback component: $d(S_t - \Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...])$. Volatility feedback can arise whenever investors acquire new information about volatility. In this paper, we follow Turner, Startz, and Nelson (1999) and proxy this new information by the difference between the true unobserved volatility regime S_t and its conditional probability $\Pr[S_t = 1 \mid r_{t-1,} r_{t-2}, ...]$. Then, if volatility regimes are persistent (i.e., the sum of the transition probabilities is greater than one: $q + p > 1$), the new information embodied in $(S_t - \Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...]) \neq 0$ produces a corresponding change in the discounted sum of future expected returns on the market portfolio. From Campbell and Shiller's (1989) log-linear approximate present-value identity, this change in the discounted sum of future expected returns is equivalent to an opposite movement in the market price index. Thus, given a positive tradeoff between volatility and expected returns, the volatility feedback coefficient d should be negative. That is, news about higher future volatility should correspond to an immediate

decline in the price index, producing a lower return, ceteris paribus. Finally, note that the volatility feedback effect d should be easier to detect than the partial effect m_1 since volatility feedback embodies a change in the discounted sum of all future expected returns.

2.3 Constant Mean and Time-Varying Coefficient Specification

The third variation extends the Jegadeesh test by allowing stock return predictability to change over time. The regression equation is given as follows:

$$r_t - m = b_t(k)\sum_{j=1}^{k}(r_{t-j} - m) + e_t \ , \tag{4}$$

where e_t is a serially uncorrelated error term. To identify $b_t(k)$, we must impose structure on its evolution. In this paper, we choose a random walk process:

$$b_t(k) = b_{t-1}(k) + v_t \ , \tag{5}$$

where v_t is a serially uncorrelated error term, independent of e_t. Garbade (1977) and Engle and Watson (1987) argue that a random walk provides a good empirical model of the univariate behaviour of regression coefficients in many situations by allowing for permanent changes in regression coefficients. At the same time, it is fairly robust to

misspecification.[4] The random walk process also allows a constant coefficient as a special case when the variance of v_t collapses to zero. For estimation, we use maximum likelihood and the Kalman filter. Details can be found in Appendix A.2.

3. Empirical Results

3.1 Data

To test for predictability, we use stock return data from the CRSP file. The data, available for the sample period of January 1926 to December 1996, are the total monthly returns on the value-weighted portfolio and the equal-weighted portfolio of all NYSE-listed stocks, where "total" denotes capital gain plus dividend yield as calculated by CRSP. Following Fama and French (1988), we deflate nominal returns by the monthly CPI (not seasonally adjusted) for all urban consumers from Ibbotson Associates to get a measure of real returns. We convert to continuously compounded returns by taking natural logarithms of simple gross returns.

3.2 Constant Mean and Fixed Coefficient Results

Table 1 reports OLS estimates for the constant mean and fixed coefficient specification and holding periods of 48, 72, 96, and 120 months.[5] The results confirm what has been previously reported in the literature and are reported here to provide a

[4] See Garbade (1977) for a Monte Carlo investigation of the consequences of misspecification. Briefly, he shows that a random walk process detects parameter instability even when the truth is either a one-time discrete jump in the parameter or a persistent, but stationary, first-order autoregressive process. In addition, he points out that the graphical representations of parameter estimates tend to reflect the true nature of parameter instability, not just its presence.

[5] All OLS estimates were calculated in EViews. Following Jegadeesh (1991), we use White's (1980) heteroskedasticity consistent standard errors.

benchmark for our extensions. First, for the full 1926-96 sample, the reported economic magnitude of stock return predictability is large in most cases, although the estimates are statistically significant at conventional levels in only a few cases. To help think about the "economic magnitude" of the reported estimates, consider the implied reversion of a price shock over a four-year horizon (see Appendix A.3 for calculation details) for some of the larger estimates. For example, the statistically insignificant point estimate of –0.0076 for the value-weighted portfolio (k=48) implies four-year reversion of as much as 30 percent. Meanwhile, the statistically significant point estimate of –0.0172 for the equal-weighted portfolio (k=72) implies four-year reversion of over 55 percent.[6] Second, the reported economic magnitude is extremely sensitive to the sample period. For example, the point estimate of –0.0373 for the equal-weighted portfolio (k=96) implies four-year reversion of as much as 85 percent for the 1926-46 sample period, while the corresponding point estimate of –0.0034 implies four-year reversion of only 15 percent for the 1947-96 sample period. The Chow statistics for a breakpoint in January 1947 suggest that the postwar reduction in the economic magnitude of predictability is statistically significant.

3.3 Time-Varying Mean and Fixed Coefficient Results

Table 2 reports the maximum likelihood estimates for the time-varying mean and fixed coefficient specification and holding periods of 48, 72, 96, and 120 months.[7] The

[6] While we purposely shy away from explicitly defining the particular threshold level of mean reversion that should be considered "large," we believe that four-year reversion of between 30-55 percent warrants the description.

[7] All maximum likelihood estimation was conducted using the OPTMUM procedure for the GAUSS programming language. Numerical derivatives were used in estimation, as well as for calculation of

first thing to notice is that, when we account for the intertemporal tradeoff between risk and return, the reported economic magnitude of the unexplained predictability is much smaller than before. The point estimate of –0.0004 for the value-weighted portfolio (k=48) implies four-year reversion of only 2 percent, compared to reversion of 30 percent. Meanwhile, the point estimate of –0.0049 for the equal-weighted portfolio (k=72) implies four-year reversion of 20 percent, compared to reversion of over 55 percent for the previous specification. Furthermore, the estimates of predictability are all statistically insignificant.

Testing the Markovian specification of regime switching is hindered by the failure of several assumptions of standard asymptotic distribution theory. Notably, as discussed in Hansen (1992) and Garcia (1995), the transition probabilities q and p are not identified under a null hypothesis of a constant mean and variance. Since the distribution of test statistics are model and data dependent, Hansen argues for the use of computationally intensive simulations to determine the small sample distributions. Garcia, however, derives asymptotic distributions of a likelihood ratio test for different two-state Markov-switching models. The largest asymptotic critical value he reports is 17.52, corresponding to a 1 percent significance level for a test of a two-state Markov-switching mean and variance model with an uncorrelated and heteroskedastic noise function. If we use this critical value as a rough guide, regime switching appears quite significant for stock returns. Likelihood ratio statistics for the null hypothesis $H_0: m_1 = g = 0, s_{e0}^2 = s_{e1}^2$ of no switching are between 123.06 and 288.94 for the value-weighted portfolio (k=120 and

asymptotic standard errors. Parameters were appropriately constrained (e.g., variances were constrained to be non-negative). Inferences appear robust to a variety of starting values.

k=48, respectively) and between 184.58 and 381.24 for the equal-weighted portfolio (k=120 and k=48, respectively). Furthermore, the strong persistence of the regimes (i.e., $q + p > 1$) provides support for the Markovian specification of regime switching (Engel and Hamilton, 1990).

The estimates for m_0, m_1, and d, which correspond to the expected return and the volatility feedback effect, are also of considerable interest. Contrary to the findings in Turner, Startz, and Nelson (1989), the estimated partial effect m_1 of an increase in expected volatility is actually positive in most cases, although it is never statistically significant. For the case that corresponds to strongest evidence of mean reversion in Table 1 (i.e., the equal-weighted portfolio with k=72), the expected monthly return more than doubles from 0.91 percent in a perfectly anticipated low volatility regime to 1.96 percent in a perfectly anticipated high volatility regime. Meanwhile, the estimated volatility feedback effect d of an unanticipated transition into a high volatility regime on the market return is always negative, corresponding to a positive tradeoff between volatility and expected returns. The t-statistics for the null hypothesis $H_0 : d = 0$ of no volatility feedback are between –2.56 and –3.87 for the value-weighted portfolio (k=72 and k=96, respectively) and between –0.85 and –3.04 for the equal-weighted portfolio (k=72 and k=120, respectively). In terms of economic magnitude, even the smallest point estimates suggest that a completely unanticipated transition into a high volatility regime produces an immediate 2.99 percent decline in the value-weighted portfolio (k=72) and an immediate 1.46 percent decline in the equal-weighted portfolio (k=72).

Figure 1 displays the filtered and smoothed probabilities of the high volatility regime for holding periods of 48, 72, 96, and 120 months. The filtered, or one-sided,

probabilities are conditional on returns up to time t and maximum likelihood estimates of the hyper-parameters presented in Table 2. The smoothed, or two-sided, probabilities are conditional on all available returns and the maximum likelihood estimates. The main thing to notice about the probabilities is that, for both portfolios (k=48 and k=72), there are seemingly periodic 3–5-year regime shifts during the 1930s and 1940s. While there are also regime shifts in the postwar period, they come at much less regular intervals. This historical pattern of regime changes is important because, given our finding that unexplained movements are largely unpredictable, it suggests that volatility regime changes in 1930s and 1940s are wholly responsible for the reported evidence of mean reversion. In particular, given this pattern, the implicit changes in the mean return during the 1930s and 1940s should have produced negative sample autocorrelations at the appropriate horizons for the mean reversion evidence. A dramatic implication, then, is that there is no inherent mean reversion and the apparent predictability of stock returns should disappear in the postwar period as the regime shifts become less regular. We use the constant mean and time-varying coefficient specification to test this implication.

3.4 Constant Mean and Time-Varying Coefficient Results

Table 3 reports maximum likelihood estimates for the constant mean and time-varying coefficient specification and holding periods of 48, 72, 96, and 120 months. The estimates for the variance of the time-varying parameter suggest changes in the apparent predictability of stock returns. The likelihood ratio statistics for the null hypothesis of constant predictability $H_0 : s_v^2 = 0$ are as high as 4.3122 (p-value=0.038) for the value-weighted portfolio (k=72) and 9.3183 (p-value=0.002) for the equal-weighted portfolio

(k=72).[8] These results hold in spite of the fact that the maximum likelihood estimate of a time-varying parameter variance has a point mass at zero when the true variance is small (Stock and Watson, 1998). Meanwhile, the estimates are generally consistent with the Chow statistics reported in Table 1, but avoid biases associated with the assumption of a known breakpoint (Zivot and Andrews, 1992, and Andrews, 1993),

Figure 2 displays the filtered and smoothed inferences about the time-varying parameter. The point estimates of predictability start out negative and significant for both portfolios (k=72 and k=96), but are updated in the postwar period to reflect no apparent predictability. The filtered inferences suggest that the evidence against mean reversion became overwhelming in the mid-1950s for the value-weighted portfolio and in the mid-1970s for the equal-weighted portfolio. Interestingly, there is a small downward revision in the point estimates following the 1987 stock market crash. However, estimates remain both statistically insignificant and economically small by the end of the sample.

It should be noted that we do not account for the effects of heteroskedasticity on our statistical inferences for this specification. However, Morley (1999) estimates an extended version of the constant mean and time-varying coefficient specification that allows for a Markov-switching variance for e_t and finds that the evidence of mean reversion is weaker, especially in the early part of the sample. Furthermore, a weakening of the mean reversion evidence has been found in numerous other studies that account for

[8] Although we report standard errors for all the parameters in the tables, we emphasize the likelihood ratio statistic for testing this hypothesis since Garbade (1977) shows that it has good finite sample properties in detecting a variety of forms of parameter instability. To be clear, the likelihood ratio test does not have the highest local asymptotic power against specific forms of parameter instability such as a random walk coefficient (Nyblom, 1989). However, our interest is in more general forms of instability, as well as in the actual time path of the coefficient through time, which the time-varying parameter approach provides. In addition, the asymptotic distribution of the likelihood ratio statistic is concentrated towards the origin under the null hypothesis, making the likelihood ratio test conservative in the sense that reported p-values understate the true level of significance (Garbade, 1977; also see Kendall and Stuart, 1973).

heteroskedasticity in estimation, including Kim, Nelson, and Startz (1991), McQueen (1992), Kim and Nelson (1998), and Kim, Nelson, and Startz (1998).

4. Conclusions

An intertemporal tradeoff between risk and return explains the reported evidence of mean reversion in Fama and French (1988) and Poterba and Summers (1988). In particular, the historical timing of large changes in the level of stock market volatility produced changes in expected returns that are responsible for the apparent tendency of price movements to be offset over long horizons. Meanwhile, the absence of periodic changes in volatility during the postwar period corresponds to a disappearance of any apparent predictability for the postwar data. We arrive at our conclusions in two ways. First, when we consider an empirical model of stock returns that captures volatility feedback in the presence of a positive tradeoff between market volatility and expected returns, we find that unexplained price movements are largely unpredictable. Second, when we allow the apparent predictability of overall stock returns to change over time with a time-varying parameter model, we find that postwar price movements are more consistent with the behaviour implicit in the historical tradeoff between risk and return than any inherent mean reversion.

We conclude this paper by noting that our results provide strong support for market efficiency. To be clear, we do not provide a decisive test of market efficiency since any failure to explain the mean reversion evidence could have reflected our limited definition of risk as much as it might have reflected a failure of market efficiency due to a tendency for investors to overreact to news about fundamentals. But, given that a tradeoff between

risk and return does appear to explain the mean reversion evidence, our results provide a serious challenge to the idea of a failure of market efficiency due to investor overreaction. First, we find no evidence of systematic opportunities for arbitrage over long horizons. Instead, the optimal forecast appears to be the same as the equilibrium expected return. Second, the most recent estimates of predictability support the random walk hypothesis. That is, contrary to investor overreaction, but consistent with market efficiency, stock returns appear largely unpredictable given past returns.

Appendix

A.1 Estimation of the Time-Varying Mean and Fixed Coefficient Specification

For the specification presented in equations (2) and (4), a modified version of the filter discussed in Hamilton (1989) is given by the following three steps:

<u>Step 1a</u>: Calculate the joint probability of S_t, S_{t-1}, S_{t-k}, and $S_t^* \equiv \sum_{j=2}^{k-1} S_{t-j}$ and solve for $\Pr[S_t = 1 | r_{t-1}, r_{t-2}, ...]$ by summing across all possible values of S_{t-1}, S_{t-k}, and S_t^*:

$$\Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^* = m \mid r_{t-1}, r_{t-2}, ...] = \Pr[S_t = j \mid S_{t-1} = i] \cdot \Pr[S_{t-1} = i, S_{t-k} = h, S_t^* = m \mid r_{t-1}, r_{t-2}, ...], \quad \text{(A.1)}$$

$$\Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...] = \sum_{i=0}^{1} \sum_{h=0}^{1} \sum_{m=0}^{k-2} \Pr[S_t = 1, S_{t-1} = i, S_{t-k} = h, S_t^* = m \mid r_{t-1}, r_{t-2}, ...]. \quad \text{(A.2)}$$

<u>Step 1b</u>: Calculate the conditional density of r_t:

$$f(r_t \mid S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^* = m, r_{t-1}, r_{t-2}, ...) = \frac{1}{\sqrt{2p s_{S_t}^2}} \exp\left\{ -\frac{1}{2 s_{S_t}^2} (r_{S_t}^* - b(k) \sum_{j=1}^{k} r_{S_{t-j}}^*)^2 \right\}, \quad \text{(A.3)}$$

where $r_{S_t}^* \equiv r_t - m_0 - m_1 \Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...] - d(S_t - \Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...])$. Note that given data up to time t, S_t, S_{t-1}, S_{t-k}, S_t^*, and particular values for the parameters, we observe all the elements of equation (A.3) since

$$\sum_{j=1}^{k} r^{*}_{S_{t-j}} = \sum_{j=1}^{k} r_{t-j} - m_0 \cdot k - (m_1 - d) \cdot \sum_{j=1}^{k} \Pr[S_{t-j} = 1 \mid r_{t-j-1}, r_{t-j-2}, ...] - d \sum_{j=1}^{k} S_{t-j}$$

and

$$\sum_{j=1}^{k} S_{t-j} = S_{t-1} + S_t^{*} + S_{t-k} \ .$$

<u>Step 2</u>: Calculate the joint density of r_t, S_t, S_{t-1}, S_{t-k}, and S_t^{*} and collapse across all possible states to find the marginal density of r_t:

$$f(r_t, S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m \mid r_{t-1}, r_{t-2}, ...) =$$

$$f(r_t \mid S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m, r_{t-1}, r_{t-2}, ...) \times$$

$$\Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m \mid r_{t-1}, r_{t-2}, ...], \qquad \text{(A.4)}$$

$$f(r_t \mid r_{t-1}, r_{t-2}, ...) = \sum_{j=0}^{1} \sum_{i=0}^{1} \sum_{h=0}^{1} \sum_{m=0}^{k-2} f(r_t, S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m \mid r_{t-1}, r_{t-2}, ...) \ . \qquad \text{(A.5)}$$

<u>Step 3</u>: Update the joint probability of S_t, S_{t-1}, S_{t-k}, and S_t^{*} given r_t and solve for the joint probability of S_t, S_{t-k+1}, and S_{t+1}^{*}:

$$\Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m \mid r_t, r_{t-1}, ...] =$$

$$\frac{f(r_t, S_t = j, S_{t-1} = i, S_{t-k} = h, S_t^{*} = m \mid r_{t-1}, r_{t-2}, ...)}{f(r_t \mid r_{t-1}, r_{t-2}, ...)} \ , \qquad \text{(A.6)}$$

$$\Pr[S_t = j, S_{t-k+1} = g, S^*_{t+1} = n \mid r_t, r_{t-1}, ...] =$$

$$\sum_{i=0}^{1}\sum_{h=0}^{1} \Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_{t-k+1} = g, S^*_{t+1} = n \mid r_t, r_{t-1}, ...], \quad \text{(A.7)}$$

where

$$\Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_{t-k+1} = g, S^*_t = m \mid r_t, r_{t-1}, ...] =$$

$$\Pr[S_{t-k} = h | S_{t-k+1} = g] \times \Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S^*_t = m \mid r_t, r_{t-1}, ...]$$

$$\Rightarrow \Pr[S_t = j, S_{t-1} = i, S_{t-k} = h, S_{t-k+1} = g, S^*_{t+1} = n \mid r_t, r_{t-1}, ...],$$

since $S^*_{t+1} = S_{t-1} + S^*_t - S_{t-k+1}$.

Then, given $\Pr[S_{-i^*} = 1]$ for $i^* = 0, \ldots, k-1$ and $\Pr[S_0 = i, S_{-k+1} = h, S^*_1 = m]$ for $i = 0,1$, $h = 0,1$, and $m = 0, \ldots, k-1$, we iterate through equations (A.1)-(A.7) for $t = 1, \ldots, T$ to obtain the filtered probability $\Pr[S_t = 1 \mid r_{t-1}, r_{t-2}, ...]$. We use the unconditional probabilities for $\Pr[S_{-i^*} = 1]$:

$$\Pr[S_{-i^*} = 0] = \frac{1-p}{2-p-q}, \quad \text{(A.8)}$$

$$\Pr[S_{-i^*} = 1] = \frac{1-q}{2-p-q}. \quad \text{(A.9)}$$

As for $\Pr[S_0 = i, S_{-k+1} = h, S^*_1 = m]$, deriving unconditional joint probabilities in terms of q and p is impractical for large values of k. Instead, we treat these initial probabilities, denoted $\Pi = (p_1, \ldots, p_{4(k-1)})$, as $4 \times (k-1)$ additional parameters to be estimated.

We use the marginal density of r_t given in equation (A.5) to find maximum likelihood estimates of the parameters as follows:

$$\max_q l(q) = \sum_{t=1}^{T} \ln(\, f(r_t \mid r_{t-1}, r_{t-2}, ...)) \,, \tag{A.10}$$

where $q = (b, s_{e0}, s_{e1}, q, p, m_0, m_1, d, \Pi)$.[9]

In addition to making the above inferences, we also obtain the smoothed probability $\Pr[S_t = 1 \mid r_T, r_{T-1}, ...]$ by employing Kim's (1994) smoothing algorithm. Specifically, given the filtered probability $\Pr[S_t = j \mid r_t, r_{t-1}, ...]$, which can be found by collapsing across states for (A.7), and the conditional probability $\Pr[S_t = j \mid r_{t-1}, r_{t-2}, ...]$, given in equation (A.2), we iterate backwards through the following two equations (conditional on $S_t = j$ and $S_{t+1} = l$, where $j = 0,1$ and $l = 0,1$):

$$\Pr[S_{t+1} = l, S_{t+1} = j \mid r_T, r_{T-1}, ...] =$$

$$\frac{\Pr[S_{t+1} = l \mid r_T, r_{T-1}, ...] \cdot \Pr[S_t = j \mid r_t, r_{t-1}, ...] \cdot \Pr[S_{t+1} = l \mid S_t = j]}{\Pr[S_{t+1} = l \mid r_t, r_{t-1}, ...]} \tag{A.11}$$

$$\Pr[S_{t+1} = j \mid r_T, r_{T-1}, ...] = \sum_{l=0}^{1} \Pr[S_{t+1} = l, S_{t+1} = j \mid r_T, r_{T-1}, ...] \,. \tag{A.12}$$

[9] Since we are not particularly interested in making inferences about the startup probabilities, we do not report their estimates. Also, for practical reasons, we treat their values as known for calculation of asymptotic standard errors based upon second derivatives. We consider this approach reasonable since inferences about the other parameters are virtually identical for other startup probabilities such as an equal probability for each initial state.

A.2 Estimation of the Constant Mean and Time-Varying Coefficient Specification

For specification presented in equations (4) and (5), the Kalman filter is given by the following six equations (let $b_t \equiv b_t(k)$, $y_t \equiv r_t - m$ and $x_t \equiv \sum_{j=1}^{k} y_{t-j}$):

<u>Prediction</u>

$$b_{t|t-1} = b_{t-1|t-1}, \tag{A.13}$$

$$P_{t|t-1} = P_{t-1|t-1} + s_v^2, \tag{A.14}$$

$$h_{t|t-1} \equiv y_t - y_{t|t-1} = y_t - b_{t|t-1} x_t, \tag{A.15}$$

$$f_{t|t-1} = x_t P_{t|t-1} x_t' + s_e^2, \tag{A.16}$$

<u>Updating</u>

$$b_{t|t} = b_{t|t-1} + K_t h_{t|t-1}, \tag{A.17}$$

$$P_{t|t} = P_{t|t-1} - K_t x_t P_{t|t-1}, \tag{A.18}$$

where $b_{t|t-1} \equiv E[b_t \mid r_{t-1}, r_{t-2}, ...]$, for example, is the conditional expectation of b_t; $P_{t|t-1}$ is the variance of $b_{t|t-1}$; $f_{t|t-1}$ is the variance of $h_{t|t-1}$; and $K_t \equiv P_{t|t-1} x_t' f_{t|t-1}^{-1}$ is the Kalman gain.[10]

[10] For a more general discussion of the Kalman filter and time-varying parameter models, as well as details on the derivation of the Kalman gain, refer to Hamilton (1994a,b) and Kim and Nelson (1999).

Given some initial values $b_{0|0}$ and $P_{0|0}$, we iterate through equations (A.13)-(A.18) for $t = 1,\ldots,T$ to obtain filtered inferences about b_t conditional on information up to time *t*. Also, as a by-product of this procedure, we obtain $h_{t|t-1}$ and $f_{t|t-1}$, which based on the prediction error decomposition (Harvey, 1990) can be used to find maximum likelihood estimates of the hyper-parameters as follows:

$$\max_q l(q) = -\frac{1}{2}\sum_{t=t+1}^{T}\ln(2pf_{t|t-1}) - \frac{1}{2}\sum_{t=t+1}^{T} h_{t|t-1} f_{t|t-1}^{-1} h_{t|t-1}\,, \tag{A.19}$$

where $q = (m, s_e, s_v)$.

Note that we ignore the first t observations in calculating the likelihood function. Since we do not observe b_0, and it has no unconditional expectation under the random walk specification given in equation (5), we must make an arbitrary guess as to its value and assign our guess an extremely large variance (e.g., $b_{0|0} = 0$ and $P_{0|0} >> 0$). We then use the first t observations to determine $b_{t|t}$ and $P_{t|t}$, which we treat as the initial values in the Kalman filter for the purposes of maximum likelihood estimation.[11] In practice, there is no exact rule as to what value of t to use. Roughly speaking, we choose t such that the effects of our arbitrary initial guess are minimized subject to including as

[11] Alternatively, we could treat the initial value as a hyper-parameter to be estimated by maximum likelihood estimation. Inferences are very similar in both cases. However, since the hyper-parameters are treated as known, the standard error bands surrounding the inferences in this alternative case would dramatically understate the true degree of uncertainty during the early part of the sample. This is precisely when our uncertainty should be greatest.

much data in estimation as possible. The adjusted samples given in Table 3 reflect our choices for t. The reported estimates appear to be robust to larger values of t.

Finally, given $b_{T|T}$ and $P_{T|T}$ from the last iteration of the Kalman filter, we iterate backwards through the following two equations in order to obtain smoothed inferences about b_t conditional on information up to time T:

<u>Smoothing</u>

$$b_{t|T} = b_{t|t} + P_{t|t} P_{t+1|t}^{-1} (b_{t+1|T} - b_{t+1|t}), \tag{A.20}$$

$$P_{t|T} = P_{t|t} + P_{t|t} P_{t+1|t}^{-1} (P_{t+1|T} - P_{t+1|t}) P_{t+1|t}'^{-1} P_{t|t}'. \tag{A.21}$$

A.3 Calculation of Reversion Following a Price Shock

We measure the economic significance of parameter estimates by calculating the implied reversion of a price shock over a four-year horizon. That is, we calculate the cumulative effect of a shock on j-period-ahead return forecasts, where $j = 1,\ldots,48$ months. Construction of a given j-period-ahead forecast is somewhat complicated for the Jegadeesh regression equation. Specifically, following Doan, Litterman, and Sims (1984), we need to employ an iterative procedure to calculate multi-period forecasts given a one-month dependent variable. For the first specification, the law of iterated expectations implies that the resulting forecast represents $E[\sum_{i=1}^{j} r_{t+i} \mid r_t, r_{t-1},...]$. However, it should be noted that the law of iterated expectations does not apply for the extensions since multi-period forecasts are nonlinear functions of past information.

The iterative approach to calculating economic significance works as follows. First, at time t, there is a one-unit shock. Then, for $k \geq 48$ and $j \leq 48$, the j-period-ahead expected demeaned return is calculated recursively for $j = 1,\ldots,48$ months by $r_{t+j|t} - \hat{m} = \hat{b}(k) R_{t+j-1|t}$, where $R_t = 1$ and, more generally, $R_{t+j-1} \equiv 1 + \sum_{i=1}^{j-1} (r_{t+i|t} - \hat{m})$ is the cumulative effect of a one-unit shock over a $j-1$ period horizon, with $\hat{m}$ and $\hat{b}(k)$ representing point estimates of the parameters. Finally, the four-year reversion following the initial shock is given by $R_{t+48|t} - 1$.

References

Andrews, D.W.K., 1993, "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica* **61**, 821-856.

Campbell, J.Y. and L. Hentschel, 1992, "No News Is Good News: An Asymmetric Model of Changing Volatility in Stock Returns," *Journal of Financial Economics* **31**, 281-318.

Campbell, J.Y. and R. Shiller, 1988, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies* **1**, 195-227.

Doan, T., R.B. Litterman, and C.A. Sims, 1984, "Forecasting and Conditional Projection Using Realistic Prior Distributions," *Econometric Reviews* **3**, 1-100.

Engel, C. and J.D. Hamilton, 1990, "Long Swings in the Dollar: Are They in the Data and Do Markets Know It?" *American Economic Review* **80**, 689-713.

Engle, R.F. and M. Watson, 1987, "Kalman Filter: Applications to Forecasting and Rational-Expectation Models," in *Advances in Econometrics*, Fifth World Congress, Vol. 1, 245-281.

Fama, E.F. and K.R. French, 1988, "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy* **96**, 246-273.

French, K.R., G.W. Schwert, and R.F. Stambaugh, 1987, "Expected Stock Returns and Volatility," *Journal of Financial Economics* **19**, 3-29.

Garbade, K., 1977, "Two Methods for Examining the Stability of Regression Coefficients," *Journal of the American Statistical Association* **72**, 54-63.

Garcia, R., 1995, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models," Manuscript, University of Montreal.

Hamilton, J.D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* **57**, 357-384.

Hamilton, J.D., 1994a, *Time Series Analysis* (Princeton University Press).

Hamilton, J.D., 1994b, "State-Space Models," in: R.F. Engle and D.L. McFadden, eds., *Handbook of Econometrics*, Vol. 4 (North-Holland, Amsterdam) 3014-3077.

Hamilton, J.D. and R. Susmel, 1994, "Autoregressive Conditional Heteroskedasticity and Changes in Regime," *Journal of Econometrics* **64**, 307-333.

Hansen, B.E., 1992, "The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov Switching Model of GNP," *Journal of Applied Econometrics* **7**, S61-S82.

Harvey, A.C., 1990, *The Econometric Analysis of Time Series*, 2nd ed. (MIT Press, Cambridge).

Jegadeesh, N., 1991, "Seasonality in Stock Price Mean Reversion: Evidence form the U.S. and the U.K.," *Journal of Finance* **46**, 1427-1444.

Kendall, M. and A. Stuart, 1973, *The Advanced Theory of Statistics*, Vol. 2, 3rd ed. (Hafner Publishing Co., New York).

Kim, C.-J., 1994, "Dynamic Linear Models with Markov-Switching," *Journal of Econometrics* **60**, 1-22.

Kim, C.-J., J.C. Morley, and C.R. Nelson, 1999, "Is There a Positive Intertemporal Tradeoff between Risk and Return After All?" Manuscript, Washington University.

Kim, C.-J. and C.R. Nelson, 1998, "Testing for Mean Reversion in Heteroskedastic Data II: Autoregression Tests Based on Gibbs-Sampling-Augmented Randomization," *Journal of Empirical Finance* **5**, 385-396.

Kim, C.-J. and C.R. Nelson, 1999, *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications* (MIT Press, Cambridge).

Kim, C.-J., C.R. Nelson, and R. Startz, 1998, "Testing for Mean Reversion in Heteroskedastic Data Based on Gibbs-Sampling-Augmented Randomization," *Journal of Empirical Finance* **5**, 131-154.

Kim, M.-J., C.R. Nelson, and R. Startz, 1991, "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," *Review of Economic Studies* **48**, 515-528.

McQueen, G., 1992, "Long-Horizon Mean-Reverting Stock Prices Revisited," *Journal of Financial and Quantitative Analysis* **27**, 1-18.

Morley, J.C., 1999, "Essays in Empirical Finance," Ph.D. dissertation, University of Washington.

Nyblom, J., 1989, "Testing for the Constancy of Parameters Over Time," *Journal of the American Statistical Association* **84**, 223-230.

Officer, R.R., 1973, "The Variability of the Market Factor of the New York Stock Exchange," *Journal of Business* **46**, 434-453.

Poterba, J.M. and L.H. Summers, 1988, "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics* **22**, 27-59.

Richardson, M., 1993, "Temporary Components of Stock Prices: A Skeptic's View," *Journal of Business and Economic Statistics* **11**, 199-207.

Richardson, M. and T. Smith, 1994, "A Unified Approach to Testing for Serial Correlation in Stock Returns," *Journal of Business* **67**, 371-399.

Richardson, M. and J.H. Stock, 1989, "Drawing Inferences from Statistics Based on Multiyear Asset Returns," *Journal of Financial Economics* **25**, 323-348.

Schaller, H. and S. van Norden, 1997, "Regime Switching in Stock Market Returns," *Applied Financial Economics* **7**, 177-191.

Schwert, G.W., 1989a, "Business Cycles, Financial Crises, and Stock Volatility," *Carnegie-Rochester Conference Series on Public Policy* **31**, 83-126.

Schwert, G.W., 1989b, "Why Does Stock Market Volatility Change Over Time?" *Journal of Finance* **44**, 1115-1153.

Stock, J.H. and M.W. Watson, 1998, "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of the American Statistical Association* **93**, 349-358.

Summers, L.H., 1986, "Does the Stock Market Rationally Reflect Fundamental Values?" *Journal of Finance* **41**, 591-601.

Turner, C.M., R. Startz, and C.R. Nelson, 1989, "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market," *Journal of Financial Economics* **25**, 3-22.

White, H., 1980, "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica* **48**, 817-838.

Zivot, E. and D.W.K. Andrews, 1992, "Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis," *Journal of Business and Economic Statistics* **10**, 251-270.

TABLE 1

Constant Mean and Fixed Coefficient Specification:
OLS Estimates and Chow Breakpoint Test, 1926-96

Value-Weighted Portfolio

k	$b(k)$ 1926-96	$b(k)$ 1926-46	$b(k)$ 1947-96	Chow Breakpoint F-Statistic	p-value
48	-0.0076	-0.0150	-0.0015	1.4743	0.2296
	(0.0094)	(0.0152)	(0.0074)		
72	-0.0064	-0.0338**	0.0030	5.3642	0.0049
	(0.0064)	(0.0175)	(0.0060)		
96	0.0000	-0.0269	0.0031	3.0156	0.0496
	(0.0046)	(0.0221)	(0.0044)		
120	0.0025	-0.0013	0.0023	0.2705	0.7631
	(0.0038)	(0.0199)	(0.0037)		

Equal-Weighted Portfolio

k	$b(k)$ 1926-96	$b(k)$ 1926-46	$b(k)$ 1947-96	Chow Breakpoint F-Statistic	p-value
48	-0.0096	-0.0102	-0.0086	0.0980	0.9066
	(0.0098)	(0.0142)	(0.0093)		
72	-0.0172**	-0.0324**	-0.0043	4.5636	0.0107
	(0.0089)	(0.0168)	(0.0079)		
96	-0.0102*	-0.0373**	-0.0034	4.2398	0.0148
	(0.0062)	(0.0181)	(0.0060)		
120	-0.0023	-0.0021	-0.0026	0.0363	0.9644
	(0.0057)	(0.0250)	(0.0056)		

Estimates are calculated using the continuously compounded total monthly real return on the value-weighted portfolio and the equal-weighted portfolio of all NYSE-listed stocks. Data are available for the period of January 1926 to December 1996, with sample periods adjusted to account for lagged variables. White's (1980) heteroskedasticity-consistent standard errors are reported in parentheses. Chow statistics are calculated using a breakpoint in January 1947.

* t-statistic for $H_0: b(k)=0$ is significant at 10 percent level.

** t-statistic for $H_0: b(k)=0$ is significant at 5 percent level.

TABLE 2

Time-Varying Mean and Fixed Coefficient Specification:

Maximum Likelihood Estimates, 1926-96

Value-Weighted Portfolio

k	$b(k)$ 1926-96	s_{e0}	s_{e1}	q	p	m_0	m_1	d	log-likelihood value
48	-0.0004	0.0372	0.1110	0.9931	0.9487	0.0067	-0.0096	-0.0366***	1340.69
	(0.0024)	(0.0013)	(0.0086)	(0.0034)	(0.0185)	(0.0023)	(0.0129)	(0.0135)	
72	-0.0008	0.0369	0.1106	0.9918	0.9282	0.0069	-0.0008	-0.0299**	1324.17
	(0.0019)	(0.0014)	(0.0104)	(0.0040)	(0.0252)	(0.0021)	(0.0025)	(0.0117)	
96	0.0001	0.0340	0.0752	0.9794	0.8941	0.0057	0.0005	-0.0348***	1324.39
	(0.0013)	(0.0016)	(0.0071)	(0.0082)	(0.0299)	(0.0026)	(0.0015)	(0.0090)	
120	0.0007	0.0335	0.0752	0.9783	0.8878	0.0038	0.0052	-0.0338***	1289.43
	(0.0014)	(0.0015)	(0.0071)	(0.0085)	(0.0327)	(0.0038)	(0.0107)	(0.0097)	

Equal-Weighted Portfolio

k	$b(k)$ 1926-96	s_{e0}	s_{e1}	q	p	m_0	m_1	d	log-likelihood value
48	-0.0009	0.0441	0.1478	0.9926	0.9536	0.0088	-0.0044	-0.0297*	1167.82
	(0.0030)	(0.0016)	(0.0111)	(0.0033)	(0.0166)	(0.0024)	(0.0176)	(0.0163)	
72	-0.0049	0.0441	0.1511	0.9922	0.9386	0.0091	0.0105	-0.0146	1157.86
	(0.0040)	(0.0017)	(0.0136)	(0.0036)	(0.0219)	(0.0024)	(0.0153)	(0.0171)	
96	-0.0026	0.0378	0.0950	0.9771	0.9296	0.0083	0.0026	-0.0296***	1164.86
	(0.0036)	(0.0029)	(0.0090)	(0.0095)	(0.0233)	(0.0032)	(0.0092)	(0.0112)	
120	-0.0005	0.0370	0.0921	0.9756	0.9234	0.0075	0.0022	-0.0347***	1143.97
	(0.0012)	(0.0021)	(0.0071)	(0.0089)	(0.0240)	(0.0028)	(0.0161)	(0.0114)	

Estimates are calculated using the continuously compounded total monthly real return on the value-weighted portfolio and the equal-weighted portfolio of all NYSE-listed stocks. Data are available for the period of January 1926 to December 1996, with sample periods adjusted to account for lagged variables. Asymptotic standard errors based upon second derivatives are reported in parentheses.

*t-statistics for $H_0: d=0$ is significant at 10 percent level.

**t-statistics for $H_0: d=0$ is significant at 5 percent level.

***t-statistics for $H_0: d=0$ is significant at 1 percent level.

TABLE 3

Constant Mean and Time-Varying Coefficient Specification: Maximum Likelihood Estimates, 1926-96

Value-Weighted Portfolio

k	Adjusted Sample Period	s_v	s_e	m	log-likelihood value
48	1934-96	0.0000	0.0456	0.0064	1261.47
		(0.0004)	(0.0012)	(0.0012)	
72	1935-96	0.0012**	0.0451	0.0064	1246.38
		(0.0007)	(0.0012)	(0.0013)	
96	1936-96	0.0003	0.0453	0.0070	1224.65
		(0.0007)	(0.0012)	(0.0016)	
120	1937-96	0.0000	0.0453	0.0043	1203.76
		(0.0001)	(0.0012)	(0.0021)	

Equal-Weighted Portfolio

k	Adjusted Sample Period	s_v	s_e	m	log-likelihood value
48	1934-96	0.0000	0.0591	0.0090	1064.53
		(0.0002)	(0.0015)	(0.0014)	
72	1935-96	0.0014***	0.0575	0.0097	1066.00
		(0.0008)	(0.0015)	(0.0011)	
96	1936-96	0.0015***	0.0572	0.0095	1051.18
		(0.0009)	(0.0015)	(0.0008)	
120	1937-96	0.0000	0.0574	0.0072	1034.08
		(0.0001)	(0.0015)	(0.0016)	

Estimates are calculated using the continuously compounded total monthly real return on the value-weighted portfolio and the equal-weighted portfolio of all NYSE-listed stocks. Data are available for the period of January 1926 to December 1996, with sample periods adjusted to account for lagged variables and starting up the Kalman filter. Asymptotic standard errors based upon second derivatives are reported in parentheses.

** Likelihood ratio statistic for $H_0: s_v = 0$ is significant at 5 percent level.

*** Likelihood ratio statistic for $H_0: s_v = 0$ is significant at 1 percent level.

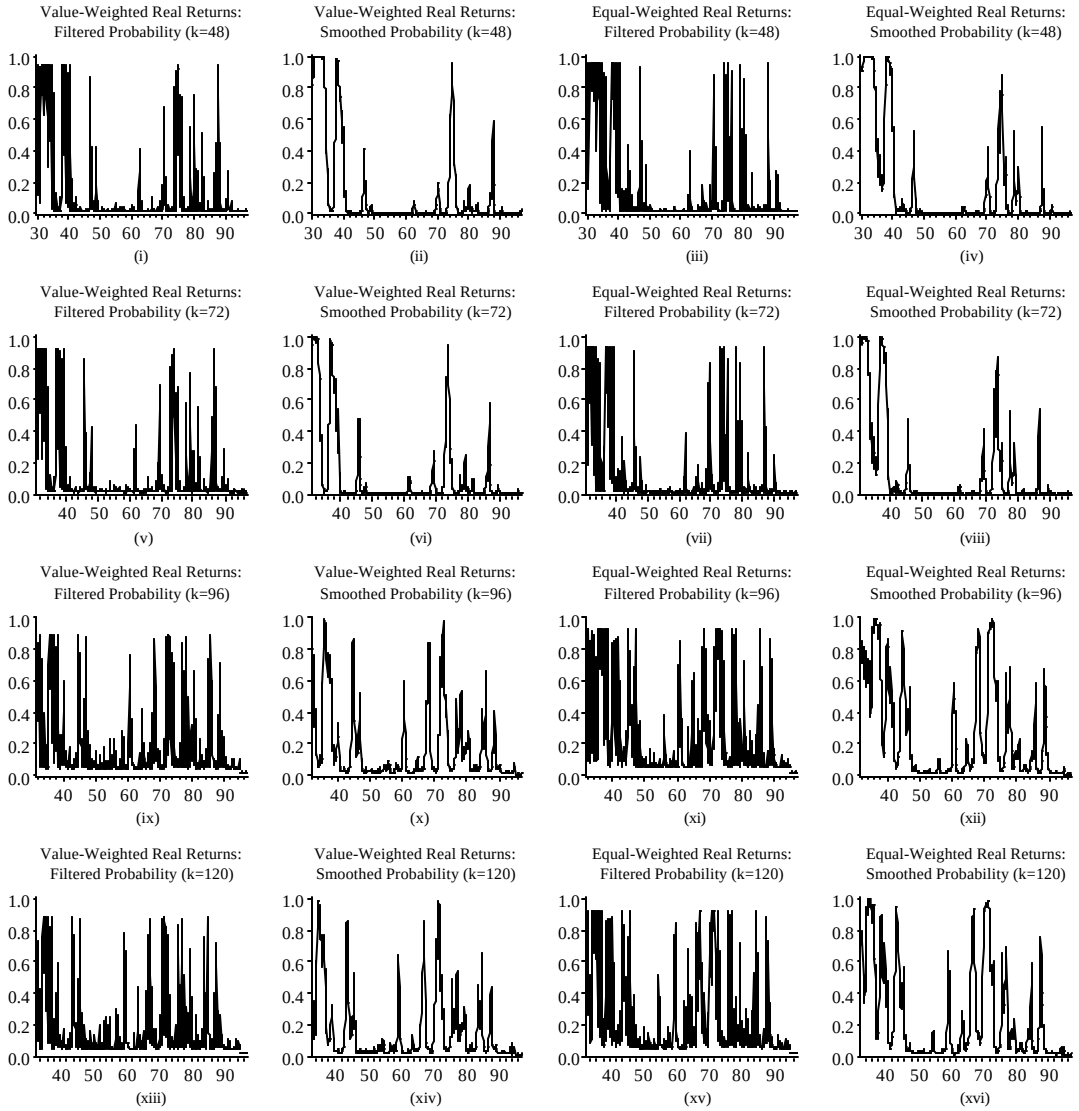

FIGURE 1. Time-Varying Mean and Fixed Coefficient Specification: Filtered and Smoothed Probabilities of the High Volatility Regime, 1926-96.

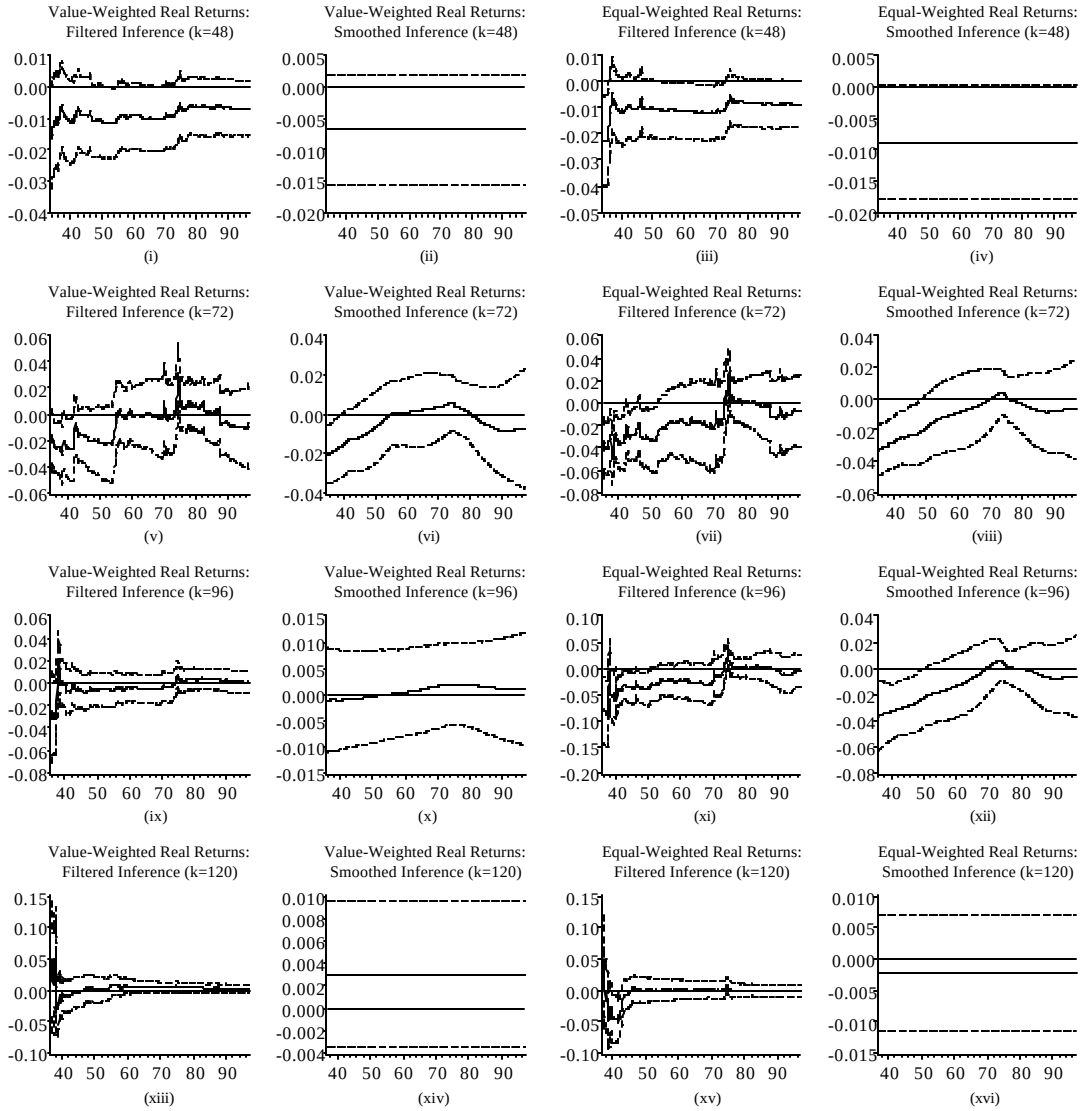

FIGURE 2. Constant Mean and Time-Varying Coefficient Specification: Filtered and Smoothed Inferences about the Mean Reversion Coefficient, 1926-96 (Dashed lines represent 95 percent confidence bands).