Markov-Switching Models with Endogenous Explanatory Variables

by

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Abstract

The maximum likelihood estimation of a Markov-switching regression model based on the Hamilton filter is not valid in the presence of endogenous explanatory variables. However, we show that there exists an appropriate transformation of the model that allows us to directly employ the Hamilton filter. The transformed model explicitly allows for a vector of bias correction terms as additional regressors, and the new disturbance term is uncorrelated with all the regressors in the transformed model. Within this framework, a Quasi maximum likelihood estimation (QMLE) procedure is presented. A procedure to test for endogeneity based on the Wald statistic or the likelihood ratio statistic is also presented.

Key Words: Bias Correction, Endogeneity, Hausman-Wu Test, Markov Switching, Forward-Looking Monetary Policy Rule.

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1. Model Specification

This paper deals with two important issues associated with a class of Markov-switching regression models in which the regressors are correlated with the disturbance term. Consistent estimation of the parameters of the model is one issue and testing for endogeneity is the other.

We consider the following Markov-switching regression model in which the explanatory variables are correlated with the disturbance term:

$$y_{1t} = x_t'\alpha_{S_t} + y_{2t}'\beta_{S_t} + e_t, \quad e_t \sim i.i.d.N(0, \sigma^2_{e,S_t}), \tag{1}$$

$$y_{2t} = (I_k \otimes z_t')\delta_{2,S_t} + v_{2,t}, \quad v_{2,t} \sim i.i.d.N(0, \Sigma_{2,S_t}), \tag{2}$$

$$Cov(v_{2,t}, e_t) = C_{2e,S_t} = (1 - S_t)C_{2e,0} + S_t C_{2e,1}, \tag{3}$$

$$Cov(z_t, e_t) = 0, \quad \forall \ \ t, \tag{4}$$

$$\alpha_{S_t} = (1 - S_t)\alpha_0 + S_t\alpha_1 \tag{5}$$

$$\beta_{S_t} = (1 - S_t)\beta_0 + S_t\beta_1 \tag{6}$$

$$\delta_{2,S_t} = (1 - S_t)\delta_{2,0} + S_t\delta_{2,1} \tag{7}$$

$$\sigma^2_{e,S_t} = (1 - S_t)\sigma^2_{e,0} + S_t\sigma^2_{e,1} \tag{8}$$

$$\Sigma_{2,S_t} = (1 - S_t)\Sigma_{2,0} + S_t\Sigma_{2,1} \tag{9}$$

where y_{1t} is 1×1; y_{2t} is a $k \times 1$ vector of explanatory variables correlated with e_t; x_t is an $l \times 1$ vector of explanatory variables uncorrelated with e_t; z_t is an $L \times 1$ vector of instrumental variables uncorrelated with e_t but correlated with y_{2t}. In a simultaneous equations model, x_t is a subset of z_t $(L > l)$. For identification of equation (1), we assume that $L - l \geq k$. The subscript S_t denotes that the corresponding parameter depends on a two-state, first-order Markov-switching variable S_t that evolves according to the following transition probabilities as in Hamilton (1989):

$$Pr[S_t = 0|S_{t-1} = 0] = q, \quad Pr[S_t = 1|S_{t-1} = 1] = p. \tag{10}$$

The maximum likelihood estimation of a Markov-switching regression model based on the Hamilton filter (Hamilton (1989)) is not valid in the presence of endogenous explanatory variables. However, we show that there exists an appropriate transformation of the model that allows us to directly employ the Hamilton filter. The transformed model explicitly allows for a vector of bias correction terms as additional regressors, and the new disturbance term is uncorrelated with all the regressors in the transformed model. A procedure to test for endogeneity based on the Wald statistic or the likelihood ratio (LR) statistic is also presented. The proposed procedures are applied to modeling a structural break in the forward-looking U.S. monetary policy rule (1960.I -1996.IV) with an unknown break point.

2. Inference Based on a Transformed Model with Bias Correction

The Hamilton filter used in the maximum likelihood estimation of Markov-switching regression models is valid only when the explanatory variables and the disturbance term in the regression equation are uncorrelated. Thus, in order to employ the Hamilton filter in our maximum likelihood estimation of the model presented in Section 1, we need to transform the model so that the explanatory variables and the disturbance terms are uncorrelated. We consider the Cholesky decomposition of the covariance matrix of $[\,v_{2,t}' \quad e_t\,]'$ in order to rewrite $[\,v_{2,t}' \quad e_t\,]'$ as a function of two independent shocks:

$$\begin{bmatrix} v_{2,t}' \\ e_t \end{bmatrix} = \begin{bmatrix} b_{11,s_t} & 0 \\ b_{21,s_t} & b_{22,s_t} \end{bmatrix} \begin{bmatrix} \omega_{1t} \\ \omega_{2t} \end{bmatrix}, \quad \begin{bmatrix} \omega_{1t} \\ \omega_{2t} \end{bmatrix} \sim i.i.d.N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} I_k & 0 \\ 0 & 1 \end{pmatrix} \right), \tag{11}$$

where $b_{11,S_t} = \Sigma_{2,S_t}$. Equation (11) allows us to rewrite equations (1) and (2) as follows:

$$y_{1t} = x_t'\alpha_{S_t} + y_{2t}'\beta_{S_t} + b_{21,s_t}\omega_{1t} + b_{22,s_t}\omega_{2t} \tag{16}$$

$$y_{2t} = (I_k \otimes z_t')\delta_{2,S_t} + \Sigma_{2,S_t}^{\frac{1}{2}}\omega_{1t}, \tag{17}$$

where ω_{it}, $i = 1, 2$, are independent standard Normal random variables; $b_{21,s_t}\omega_{1t} + b_{22,s_t}\omega_{2t} = e_t$; and $\Sigma_{2,S_t}^{\frac{1}{2}}\omega_{1t} = v_{2,t}$. As before, solving equation (17) for ω_{1t} and substituting this in equation (16) results in the following transformation of equation (1):

$$y_{1t} = x_t'\alpha_{S_t} + y_{2t}'\beta_{S_t} + (y_{2t} - (I_k \otimes z_t')\delta_{2,S_t})'\gamma_{S_t} + b_{22,s_t}\omega_{2t} \tag{18}$$

with

$$\gamma'_{S_t} = b_{21,s_t} \Sigma_{2,S_t}^{-\frac{1}{2}}. \tag{19}$$

Note that in equation (18), the explanatory variables and the disturbance term are no longer uncorrelated. This allows us to apply the Hamilton filter to consistently estimate the unknown parameters of the transformed model, with $(y_{2t} - (I_k \otimes z'_t)\delta_{2,S_t})'\gamma_{S_t}$ working as a bias correction term.

Given the above representation of the model, derivation of the log likelihood function is straightforward, and a joint estimation of the model can be performed by maximizing the following log likelihood estimation:

$$\begin{aligned}
&lnL(\alpha_0, \alpha_1, \beta_0, \beta_1, \sigma^2_{e,0}, \sigma^2_{e,1}, \delta_{2,0}, \delta_{2,1}, \Sigma_{2,0}, \Sigma_{2,1}, b_{22,0}, b_{22,1}, p, q) \\
&\quad = \sum_{t=1}^{T} ln(f(y_{1t}, y_{2t}|I_{t-1})) \\
&\quad = \sum_{t=1}^{T} ln(\sum_{s_t=0}^{1} f(y_{1t}, y_{2t}|I_{t-1}, S_t = s_t)Pr[S_t = s_t|I_{t-1}]) \\
&\quad = \sum_{t=1}^{T} ln(\sum_{s_t=0}^{1} f(y_{1t}|y_{2t}, I_{t-1}, S_t = s_t)f(y_{2t}|I_{t-1}, S_t = s_t)Pr[S_t = s_t|I_{t-1}]),
\end{aligned} \tag{20}$$

where I_{t-1} refers to information up to time $t-1$, including the instrumental variables z_t. $Pr(S_t = s_t|I_{t-1})$ is obtained from the Hamilton filter. The individual densities in equation (20) are given by:

$$\begin{aligned}
&f(y_{1t}|y_{2t}, I_{t-1}, S_t = s_t) \\
&\quad = (2\pi)^{-\frac{1}{2}}(b_{22,s_t})^{-1} exp\{-\frac{(y_{1t} - x'_t\alpha_{s_t} - y'_{2t}\beta_{s_t} - (y_{2t} - (I_k \otimes z'_t)\delta_{2,s_t})'\gamma_{S_t})^2}{2b^2_{22,s_t}}\}
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
&f(y_{2t}|I_{t-1}, S_t = s_t) \\
&\quad = (2\pi)^{-\frac{k}{2}}|\Sigma_{2,s_t}|^{-\frac{1}{2}} exp\{-0.5(y_{2t} - (I_k \otimes z'_t)\delta_{2,s_t})'\Sigma^{-1}_{2,s_t}(y_{2t} - (I_k \otimes z'_t)\delta_{2,s_t})\}.
\end{aligned} \tag{22}$$

An additional advantage of the transformed model given in equations (17) and (18) is that, when the parameters in equation (2) or (17) are time-invariant, the estimation procedure is drastically simplified. In particular, as the log likelihood function is now given by:

$$
\begin{aligned}
& lnL(\alpha_0, \alpha_1, \beta_0, \beta_1, \sigma^2_{e,0}, \sigma^2_{e,1}, \delta_2, \Sigma_2, b_{22,0}, b_{22,1}, p, q) \\
& \quad = \sum_{t=1}^{T} ln[\sum_{s_t=0}^{1} f(y_{1t}|y_{2t}, I_{t-1}, S_t = s_t) Pr(S_t = s_t|I_{t-1})] + \sum_{t=1}^{T} ln[f(y_{2t}|I_{t-1})] \qquad (23)
\end{aligned}
$$

,

where $\delta_2 = \delta_{2,0} = \delta_{2,1}$ and $\Sigma_2 = \Sigma_{2,0} = \Sigma_{2,1}$, the following two-step procedure is justified without any loss of efficiency: [2]

Step 1': Estimate equation (17) using an OLS and get $\hat{\delta}$.

Step 2': By employing the Hamilton filter, estimate equation (18) conditional on $\hat{\delta}$ obtained from the first step.

3. Testing for endogeneity

Testing for endogeneity is straightforward from equations (16) and (17). The term b_{21,s_t} in equation (16) determines the correlation between y_{2t} and e_t. For example, when $b_{21,s_t} = 0$, there exists no correlation between y_{2t} and e_t and $\gamma'_{S_t} = b_{21,s_t} b^{-1}_{11,s_t} = 0$ in equation (18). Thus, the test of endogeneity can be based on the usual Wald statistic or the likelihood ratio statistic with the following Null hypothesis:

$$H_0: \quad \gamma_{S_t} = 0, \quad S_t = 0, 1. \qquad (24)$$

Defining lnL_U and lnL_R to be the log likelihood values from unrestricted and restricted maximum likelihood estimations, respectively, and defining $\hat{\gamma} = [\,\hat{\gamma}'_0 \quad \hat{\gamma}'_1\,]'$, the appropriate test statistics and their distribution are given by:

$$LR = -2(lnL_R - lnL_U) \sim \chi^2(J), \qquad (25)$$

$$Wald = \hat{\gamma}' Cov(\hat{\gamma})^{-1} \hat{\gamma} \sim \chi^2(J), \qquad (26)$$

where J is the dimension of $\hat{\gamma}$.

[2] This is because the information matrix is block diagonal.

It is easy to show that the proposed test is an extended version of the Hausman-Wu test (Hausman (1978), and Wu (1973)) in the presence of Markov switching. For example, equation (17) can be rewritten as:

$$\begin{aligned} y_{1t} &= x_t'\alpha_{S_t} + y_{2t}'\beta_{S_t} + (y_{2t} - (I_k \otimes z_t')\delta_{2,S_t})'\gamma_{S_t} + b_{22,s_t}\omega_{2t} \\ &= x_t'\alpha_{S_t} + y_{2t}'\beta_{S_t} + y_{2t}'\gamma_{S_t} - \hat{y}_{2t}'\gamma_{S_t} + b_{22,s_t}\omega_{2t} \\ &= x_t'\alpha_{S_t} + y_{2t}'\xi_{S_t} + \hat{y}_{2t}'\gamma^*_{S_t} + b_{22,s_t}\omega_{2t}, \end{aligned} \tag{18$'$}$$

where $\hat{y}_{2t} = (I_k \otimes z_t')\delta_{2,S_t}$, $\xi_{S_t} = \beta_{S_t} + \gamma_{S_t}$, and $\gamma^*_{S_t} = -\gamma_{S_t}$. Note that, in the absence of Markov-switching regression coefficients, Wu's (1973) approach is to test for the significance of γ^* in the above augmented regression, and the result is equivalent to the Hausman test. [3] Here, under the null specified in equation (24), we have $\xi_{S_t} = \beta_{S_t}$ and $\gamma^*_{S_t} = 0$.

4. Monte Carlo Experiment

In this section, we empirically investigate the performance of the bias correction method and the hypothesis tests introduced in Section 2. For this purpose, we generate 10,000 sets of data by a simplified version of the system given by equations (1)-(9). Each sample is of length $T = 200$. For each sample, we estimate the model twice: i) without bias correction and ii) with bias correction. When estimating the model with bias correction, a two-step procedure is employed. The specific data generating process and the parameter values we assign are summarized below:

Data Generating Process (DGP)

$$y_{1t} = \beta_{S_t} y_{2t} + e_t \tag{27}$$

$$y_{2t} = \delta z_t + v_{2,t} \tag{28}$$

$$z_t \sim i.i.d.N(0,1) \tag{29}$$

$$\begin{bmatrix} v_{2,t} \\ e_t \end{bmatrix} = \begin{bmatrix} b_{11} & 0 \\ b_{21,s_t} & b_{22} \end{bmatrix} \begin{bmatrix} \omega_{1t} \\ \omega_{2t} \end{bmatrix}, \quad \begin{bmatrix} \omega_{1t} \\ \omega_{2t} \end{bmatrix} \sim i.i.d.N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \tag{30}$$

[3] For algebraic derivation of this result, readers are referred to Davidson and MacKinnon (1993).

$$p = 0.95,\ q = 0.95$$

$$\beta_0 = -1,\ \beta_1 = 1; \quad \delta = 1$$

$$b_{11} = 1; \quad b_{22} = 1; \quad b_{21,0} = 0.7,\ b_{21,1} = 0.35$$

The first two moments of the empirical distributions for the parameter estimates are summarized in Table 1. Bias in the estimation of β_0 or β_1 is clear when endogeneity in the regressors is ignored. As the DGP is such that the endogeneity problem is more serious in State 0 ($S_t = 0$) than in State 1 ($S_t = 1$), we expect the bias would be larger for β_0 than for β_1. The Monte Carlo experiment results confirm this. When the model is estimated with a bias correction term, however, the bias in the estimates of β_0 or β_1 is almost negligible. The means of the other parameter estimates are close to the true parameter values assigned for the DGP. In particular, the true values for all the parameters are well within their one standard-error confidence bands obtained from the empirical distributions of the parameter estimates.

In order to gauge the finite-sample properties of the proposed endogeneity tests, we also perform a Monte Carlo experiment based on the DGP above. We consider the empirical size of the Wald and LR tests under the Null hypothesis. We thus generate 10,000 sets of data under the Null hypothesis of no correlation between the explanatory variable and the disturbance term. This is done by assigning $b_{21,0} = 0,\ b_{21,1} = 0$. For each generated data set, we perform both Wald and likelihood ratio tests, using the critical values obtained from an appropriate χ^2 distribution. At the 1% significance level, the percentage rejections of the Null hypothesis were 1.61% for the Wald test and 1.14

5. An Application

By estimating a forward-looking monetary policy reaction function for the postwar U.S. economy, Clarida, Gali, and Gertler (2000) show that the Fed's interest rate policy since 1979 has been much more sensitive to changes in expected future inflation than before 1979. Their approach is based on the assumption that a structural break in the monetary policy reaction function occurred in 1979. In this section, however, we investigate the nature of the structural break in the post-war U.S. monetary reaction function of the Fed by assuming that the structural break date is unknown.

In the presence of an unknown structural break date, a version of the Fed's target interest rate and the interest smoothing equation specified by Clarida, Gali, and Gertler (2000) can be represented as:

Target Interest Rate

$$r_t^* = r_{S_t}^* + \beta_{1,S_t} E(g_{t,J}|I_t) + \beta_{2,S_t}(E(\pi_{t,J}|I_t) - \pi_{S_t}^*), \tag{31}$$

Interest Rate Smoothing

$$r_t = (1 - \rho_{S_t}) r_t^* + \rho_{S_t} r_{t-1} + \epsilon_t, \quad 0 < \rho_{S_t} < 1, \tag{32}$$

where r_t is the federal funds rate; $g_{t,J}$ is a measure of average output gap between time t and $t + J$; $\pi_{t,J}$ is the percent change in the price level between time t and $t + J$; $\pi_{S_t}^*$ is the target rate for inflation before ($S_t = 0$) and after ($S_t = 1$) the structural break; $r_{S_t}^*$ ($S_t = 0, 1$) is the desired nominal rate when both inflation and output are at their target levels; ϵ_t is a random disturbance term with mean zero which is uncorrelated with r_t^*; I_t is the information set at the time the interest rate is set by the Fed; and the subscript S_t determines the unknown structural break date τ such that

$$S_t = 0, \quad for\ t \leq \tau; \quad S_t = 1, \quad for\ t > \tau \tag{33}$$

In order to account for the unknown structural break in the parameters of the model, we follow Chib (1998) in treating S_t as a latent Markov-switching variable with the following transition probabilities:

$$Pr[S_t = 0|S_{t-1} = 0] = q, \quad Pr[S_t = 1|S_{t-1} = 1] = 1 \tag{34}$$

By combining equations (31) and (32) and rearranging terms, we have:

$$r_t = (1 - \rho_{S_t})(\beta_{0,S_t} + \beta_{1,S_t} g_{t,J} + \beta_{2,S_t} \pi_{t,J}) + \rho_{S_t} r_{t-1} + e_t, \tag{35}$$

$$\beta_{0,S_t} = r_{S_t}^* - \beta_{2,S_t} \pi_{S_t}^* \tag{36}$$

$$e_t = \epsilon_t + (1 - \rho_{S_t})((g_{t,J} - E(g_{t,J}|I_t)) + (\pi_{t,J} - E(\pi_{t,J}|I_t)), \tag{37}$$

where the regressors $g_{t,J}$ and $\pi_{t,J}$ are correlated with the disturbance term e_t. With a known structural break date, a GMM (Generalized Method of Moment) can be applied to each subsample divided by the break date. However, in the present case of an unknown structural break date, a GMM cannot be applied.

An empirical equation with the proposed bias correction terms is given by:

$$\begin{aligned} r_t = (1 - \rho_{S_t})[\beta_{0,S_t} + \beta_{1,S_t} g_{t,J} + \beta_{2,S_t} \pi_{t,J} + \gamma_{1,S_t}(g_{t,J} - z_t' \delta_{1,S_t}) \\ + \gamma_{2,S_t}(\pi_{t,J} - z_t' \delta_{2,S_t})] + \rho_{S_t} r_{t-1} + \omega_t + \eta_t, \end{aligned} \tag{35$'$}$$

$$\omega_t \sim (0, \sigma^2_{\omega,S_t}), \tag{38}$$

where z_t is a vector of instrumental variables; η_t is independent of e_t in equation (35) or ω_t in equation (35'); and the additional disturbance term η_t is added to account for high volatility in the federal funds rate for the period of 1979:III - 1983:IV, during which time the Fed moved temporarily away from interest rate targeting. Thus, η_t is specified as:

$$\eta_t \sim (0, \sigma^2_{\eta,t}), \quad \sigma^2_{\eta,t} = \begin{cases} \sigma^2_\eta, & \text{for } \ 1979 : III \leq t \leq 1983.IV \\ 0, & \text{otherwise} \end{cases} \tag{39}$$

Finally, as the bias correction terms in equation (35') imply, we assume that the relationships between the regressors, g_{t+J} and π_{t+J}, and the vector of instrumental variables z_t are given by:

$$g_{t+J} = z_t' \delta_{1,S_t} + v_{1t} \tag{40}$$

$$\pi_{t+J} = z_t' \delta_{2,S_t} + v_{2t} \tag{41}$$

$$[\, v_{1t} \quad v_{2t} \,]' \sim i.i.d.(0, \Sigma_{v,S_t}) \tag{42}$$

An MLE procedure based on the Hamilton filter is employed to estimate the empirical model given by equations (35') and (38)-(44). We consider the case in which $J = 1$. The data we employ are quarterly data covering the period 1960.I - 1996:IV. As in Clarida, Gali, and Gertler (2000), the interest rate is the average Federal Funds rate in the first-month of each quarter; inflation is measured by the % change of the GDP deflator; the output gap is the series constructed by the Congressional Budget Office. The instrumental variables include 4 lags of each of the following variables: the Federal funds rate, output gap, inflation, commodity price inflation, and M2 growth.

Table 2 reports the estimation results, with standard errors in parentheses. The estimate of the transition probability q provides an estimate of the break date, as the expected duration of a regime before a structural break is given by $1/(1-q)$. Thus, that $\hat{q} = 0.988$ in Table 2 implies that the estimated break date is 1981:III, while Clarida, Gali, and Gertler (2000) assume the break date is 1979:III. As in Clarida, Gali, and Gertler (2000), the response of the federal funds rate to expected future inflation increased after a structural break. The same is true of the Fed's response to the expected future GDP gap.

While most of the parameter estimates of interest are not qualitatively very different from those in Clarida, Gali, and Gertler (2000), our model provides additional information concerning the degree of endogeneity for variables of interest before and after a structural break. For example, the coefficient on the bias correction term for the inflation variable is small and not statistically significant before a structural break ($\gamma_{2,0} = -0.398$). However, it increases substantially and statically significant after a structural break ($\gamma_{2,1} = -2.934$). This result may be indicate that the Fed's response to inflation was not forward-looking before a structural break. Another possibility would be that a structural break in the dynamics of the inflation series itself might have resulted in such statistical differences.

6. Conclusion

In the presence of endogeneity in the regressors, the instrumental variables estimation technique has been applied only to regression models with stable coefficients. In this paper, we have shown how the problem of endogeneity may be solved within a class of Hamilton's (1989) Markov-switching regression models. The key is to find an appropriate transformation of the model in which the new regressors and the new disturbance term are uncorrelated, and then to apply a Quasi Maximum likelihood estimation (QMLE) using the Hamilton filter. In addition, the procedure to test for endogeneity proposed in this paper may be considered as a straightforward extension of the Hausman-Wu test to the case of Markov state-dependent regression coefficients.

We apply the proposed QMLE procedure to deal with the endogeneity problem in the estimation of a forward-looking monetary reaction function of the Fed with an unknown break date. While the results we get are not qualitatively very different from those in the literature which assume a known structural break date, our model provides additional information on the changing nature of endogeneity in one of the regressors of interest. We find that the inflation variable in the forward-looking interest rate rule does not create an endogeneity problem before a structural break. While we tentatively conclude that the Fed's response to inflation might not have been forward-looking in the 1970's, this issue warrants further studies.

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Table 1. Monte Carlo Experiment on the Performance of the Proposed MLE Procedure [T=200]

		Estimation without Bias Correction		Estimation with Bias Correction	
Parameters	True Values	Mean	SE	Mean	SE
β_0	-1	-0.645	0.091	-1.003	0.129
β_1	1	1.182	0.082	1.003	0.119
$b^2_{22,0}$	1	–	–	0.970	0.168
$b^2_{22,1}$	1	–	–	0.970	0.165
σ^2_0	1.49	1.234	0.207	–	–
σ^2_1	1.12	1.041	0.178	–	–
p	0.95	0.937	0.036	0.936	0.036
q	0.95	0.941	0.035	0.942	0.034
Implied Bias Correction Term					
γ_0	0.7	–	–	0.704	0.175
γ_1	0.35	–	–	0.349	0.168

Table 2. Estimation of a Forward-Looking Monetary Policy Rule with an Endogenous Structural Break [1960:1 - 1996:IV]

$$r_t = (1-\rho_{S_t})[\beta_{0,S_t} + \beta_{1,S_t} g_{t,J} + \beta_{2,S_t}\pi_{t,J} + \gamma_{1,S_t}(g_{t,J} - z_t'\delta_{1,S_t}) \\ + \gamma_{2,S_t}(\pi_{t,J} - z_t'\delta_{2,S_t})] + \rho_{S_t} r_{t-1} + e_t + \eta_t,$$

$$e_t \sim (0, \sigma^2_{e,S_t})$$

$$\eta_t \sim (0, \sigma^2_{\eta,t}), \quad \sigma^2_{\eta,t} = \begin{cases} \sigma^2_\eta, & \text{for } 1979: III \leq t \leq 1983.IV \\ 0, & \text{otherwise} \end{cases}$$

$$Pr[S_t = 0 | S_{t-1} = 0] = q, \quad Pr[S_t = 1 | S_{t-1} = 1] = 1$$

Parameters	**Estimates (SE)**	
σ_η	3.134	(0.514)
q	0.988	(0.012)
Before Structural Break		
$\beta_{0,0}$	0.158	(0.863)
$\beta_{1,0}$	0.642	(0.219)
$\beta_{2,0}$	1.099	(0.152)
ρ_0	0.696	(0.080)
$\gamma_{1,0}$	-1.167	(0.432)
$\gamma_{2,0}$	-0.398	(0.433)
$\sigma_{\omega,0}$	0.674	(0.063)
After Structural Break		
$\beta_{0,1}$	1.919	(1.438)
$\beta_{1,1}$	1.246	(0.320)
$\beta_{2,1}$	1.945	(0.469)
ρ_1	0.727	(0.050)
$\gamma_{1,1}$	-1.560	(0.464)
$\gamma_{2,1}$	-2.934	(0.529)
$\sigma_{\omega,1}$	0.224	(0.025)