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ORIGINAL PAPER

Bilateral trading with contingent contracts

Kiho Yoon[1]

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Abstract

We study the bilateral trading problem under private information. We characterize the range of possible mechanisms which satisfy ex-post efficiency, incentive compatibility, individual rationality, and budget balance. In particular, we show that the famous Myerson–Satterthwaite impossibility result no longer holds when contingent contracts are allowed.

Keywords Bilateral trading · Contingent contracts · Ex-post efficiency · Linear contracts · Royalty contracts

JEL Classification C72 · D47 · D82

1 Introduction

In many practical settings where the value of an asset/project is observable ex-post, the payments often depend on realized payoffs, i.e., the payments are contingent on future outcomes. In oil and gas lease auctions, for instance, buyers pay a fixed percentage of revenues in royalties in addition to upfront cash payments. Other examples include intercorporate asset sales, licensing agreements for intellectual property, and build-operate-transfer highway construction contracts in procurements.[1]

Among contingent contracts, linear contracts are popular. Schmalensee (1989) finds that "most incentive schemes observed in practice are linear". Bhattacharyya and Lafontaine (1995) note that "linear pricing rules have been found in a number of diverse areas such as, but not limited to, sales force compensation, sharecropping,

[1] See Skrzypacz (2013) for a more detailed discussion.

✉ Kiho Yoon
kiho@korea.ac.kr
http://econ.korea.ac.kr/~kiho

[1] Department of Economics, Korea University, 145 Anam-ro, Seongbuk-gu, Seoul 02841, South Korea

leasing arrangements, author's fees, legal fees, licensing agreements, commercial real estate rental fees, and franchising".[2]

In recent years, research on contingent contracts has emerged. See, for example, Hansen (1985), Saumelson (1987), Crémer (1987), Riley (1988), DeMarzo et al. (2005), Che and Kim (2010), and in particular the survey of Skrzypacz (2013).[3] See also Ekmekci et al. (2016) and the references therein for the corporate finance literature which considers contingent contracts. These papers study the (one-sided incomplete information) auction problem and show that contingent contracts can increase the seller's revenue.

In comparison, we study the (two-sided incomplete information) bilateral trading problem and show that contingent contracts can achieve ex-post efficiency. Specifically, given the prominence of linear contracts, we consider linear contracts in bilateral trading environments and characterize the range of possible mechanisms which satisfy ex-post efficiency, incentive compatibility, individual rationality, and budget balance. We show that contingent contracts reduce the information rents and ameliorate the escalation of adding-up incentive constraints, thus making ex-post efficiency possible. As far as we know, this is the first paper that examines contingent contracts for two-sided incomplete information setting.

Myerson and Satterthwaite (1983) considered a model in which the seller owns an indivisible asset, and the seller's and the buyer's valuations are statistically independent and depend only on their respective private information. They established that there does not exist an ex-post efficient, incentive compatible, individually rational, and budget balancing bilateral trading mechanism when only upfront cash payments are allowed. That is, there is no incentive compatible, individually rational, and budget balancing mechanism such that the trade occurs when and only when the buyer's valuation exceeds the seller's valuation. Subsequently, Cramton et al. (1987) showed that ex-post efficiency is possible with upfront cash payments when the ownership of the asset is distributed. The insight is that, by adjusting the 'status quo' allocation, we can satisfy the desirable properties listed above. See Segal and Whinston (2011) and the references therein on this line of research. In a different approach, McAfee and Reny (1992) showed that ex-post efficiency is possible when the players' valuations are statistically correlated and drawn from the same interval.[4] Observe that these papers alter the environment to obtain efficiency. In contrast, we maintain the same environment as Myerson and Satterthwaite's (1983) but alter the contract structure to obtain efficiency. Note in particular that, while in Cramton et al. the ownership is initially distributed but the whole ownership is given to one party after transaction, in the present paper the whole ownership initially resides with one party as in Myerson and Satterthwaite but the contracts create

[2] There have been efforts to explain the popularity of linear contracts. See the discussion section of Carroll (2015) for an excellent literature review.

[3] Contingent payments are also referred to as securities or security bids.

[4] There have been other notable attempts. Saran (2011) showed that ex-post efficiency is possible if the proportion of naive traders is greater than a lower bound (which is less than 50%). Garratt and Pycia (2016) showed that ex-post efficiency is generically possible when utilities are not quasi-linear and not too responsive to private information.

some form of shared ownership. In this sense, the present paper is orthogonal to that paper.

2 Main results

2.1 The setup

There is one seller (player 1) who owns an indivisible asset, and one buyer (player 2) who wants to buy. Let t_i denote the value of the asset to player $i = 1, 2$. Thus, t_i is player i's type, and we assume that this is private information. Player i's type is drawn independently according to the distribution F_i on the interval $T_i = [\underline{t}_i, \bar{t}_i] \subseteq I\!R_+$, with the density f_i that is continuous and positive. Let $T = T_1 \times T_2$ be the set of type profiles.

As mentioned in the introduction, we assume that the ex-post value of the asset is observable and contractible. We consider linear contracts. Specifically, we consider direct mechanisms with royalty and cash payment, (p, α, x), where $p : T \to [0, 1]$ is the probability of trade, i.e., the probability that the asset is transferred to the buyer, $\alpha : T \to [0, 1]$ is the royalty rate, and $x : T \to I\!R$ is the cash payment from the buyer to the seller.[5] Thus, when the type profile is realized as (t_1, t_2), the seller's payoff is $p(t_1, t_2)\big(\alpha(t_1, t_2)t_2 - t_1\big) + x(t_1, t_2)$ and the buyer's payoff is $p(t_1, t_2)\big(1 - \alpha(t_1, t_2)\big)t_2 - x(t_1, t_2)$ in the mechanism (p, α, x).[6,7] Note that the royalty is dependent on the realized value t_2 of the asset to the buyer. Note also that the royalty is contingent upon the transfer of the asset whereas the cash payment is not. When $\alpha(t_1, t_2) = 0$ for all $(t_1, t_2) \in T$, this setup is identical to that of Myerson and Satterthwaite (1983).

2.2 Incentive compatibility and individual rationality

We define

$$y_1(s_1) \stackrel{\text{def}}{=} \int_{\underline{t}_2}^{\bar{t}_2} x(s_1, t_2) f_2(t_2) dt_2, \quad y_2(s_2) \stackrel{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} x(t_1, s_2) f_1(t_1) dt_1,$$

$$q_1(s_1) \stackrel{\text{def}}{=} \int_{\underline{t}_2}^{\bar{t}_2} p(s_1, t_2) f_2(t_2) dt_2, \quad q_2(s_2) \stackrel{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2) f_1(t_1) dt_1.$$

Thus, $y_i(s_i)$ is the conditional expected cash payment when player i reports s_i, and $q_i(s_i)$ is the conditional expected probability of trade when player i reports s_i. In addition, define

5 By the revelation principle, it is with no loss of generality to restrict our attention to direct mechanisms.

6 We may consider more general value functions $\pi_i(t_i)$ instead of t_i for $i = 1, 2$. But, by defining $t_i' = \pi(t_i)$ and changing the distributions appropriately, we return to the original formulation.

7 Observe that the seller's payoff is non-positive, i.e., $-t_1 \leq 0$, when $p(t_1, t_2) = 1$, $\alpha(t_1, t_2) = 0$, and $x(t_1, t_2) = 0$. The bilateral trading is an environment with positive externality and consequently budget balance problem is nontrivial. This is in contrast with the auction problems.

$$r_1(s_1) \stackrel{\text{def}}{=} \int_{\underline{t}_2}^{\bar{t}_2} p(s_1, t_2)\alpha(s_1, t_2)t_2 f_2(t_2)dt_2,$$

$$r_2(s_2, t_2) \stackrel{\text{def}}{=} t_2\hat{r}_2(s_2) \stackrel{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\alpha(t_1, s_2)t_2 f_1(t_1)dt_1.$$

Note that (i) $r_1(s_1)$ is the conditional expected royalty when the seller reports s_1, (ii) $r_2(s_2, t_2)$ is the conditional expected royalty when the buyer with t_2 reports s_2, and (iii) $\hat{r}_2(s_2) \stackrel{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\alpha(t_1, s_2) f_1(t_1)dt_1$ is the conditional expected royalty rate when the buyer reports s_2. Let us define

$$\hat{q}_2(s_2) \stackrel{\text{def}}{=} q_2(s_2) - \hat{r}_2(s_2) = \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\big(1 - \alpha(t_1, s_2)\big) f_1(t_1)dt_1.$$

Observe that, when $\alpha(t_1, t_2) = 0$ for all $(t_1, t_2) \in T$ as in Myerson and Satterthwaite (1983), we have $r_1(s_1) = r_2(s_2, t_2) = \hat{r}_2(s_2) = 0$ and $\hat{q}_2(s_2) = q_2(s_2)$.

If the seller believes that the buyer will report truthfully, and he reports s_1 when his true type is t_1, then his expected payoff is

$$U_1(s_1, t_1) \stackrel{\text{def}}{=} r_1(s_1) - q_1(s_1)t_1 + y_1(s_1). \tag{1}$$

Likewise, if the buyer believes that the seller will report truthfully, and she reports s_2 when her true type is t_2, then her expected payoff is

$$U_2(s_2, t_2) \stackrel{\text{def}}{=} q_2(s_2)t_2 - r_2(s_2, t_2) - y_2(s_2) = \hat{q}_2(s_2)t_2 - y_2(s_2). \tag{2}$$

Let us define, with a slight abuse of notation, $U_i(t_i) \stackrel{\text{def}}{=} U_i(t_i, t_i)$ for $i = 1, 2$. Thus, $U_i(t_i)$ is the expected payoff when player i truthfully reports his/her type. The mechanism (p, α, x) is *incentive compatible* if

$$U_i(t_i) \geq U_i(s_i, t_i), \quad \forall i \in \{1, 2\}, \forall s_i, t_i \in T_i, \tag{IC}$$

and *individually rational* if

$$U_i(t_i) \geq 0, \quad \forall i \in \{1, 2\}, \forall t_i \in T_i. \tag{IR}$$

We first have the following proposition on incentive compatibility.

Proposition 1 *The mechanism (p, α, x) is incentive compatible if and only if*[8]

(i) $q_1(t_1)$ is decreasing,
(ii) $\hat{q}_2(t_2)$ is increasing,
(iii) $U_1(t_1) = U_1(\bar{t}_1) + \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau$,

[8] We use the terms decreasing/increasing in the weak sense.

(iv) $U_2(t_2) = U_2(\underline{t}_2) + \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau$.

Proof ($\Rightarrow$ part) Observe that $U_1(s_1, t_1) - U_1(s_1) = (s_1 - t_1)q_1(s_1)$. Therefore, the *IC* condition is equivalent to

$$U_1(t_1) \geq U_1(s_1) - (t_1 - s_1)q_1(s_1).$$

By interchanging the roles of s_1 and t_1, we get

$$U_1(s_1) \geq U_1(t_1) - (s_1 - t_1)q_1(t_1).$$

Combining these inequalities,

$$(s_1 - t_1)q_1(s_1) \leq U_1(t_1) - U_1(s_1) \leq (s_1 - t_1)q_1(t_1).$$

If $t_1 < s_1$, then $q_1(s_1) \leq q_1(t_1)$. Thus, $q_1(\cdot)$ is decreasing. Furthermore, $q_1(\cdot)$ is Riemann integrable, and $dU_1(t_1)/dt_1 = -q_1(t_1)$. Therefore,

$$U_1(t_1) = U_1(\bar{t}_1) + \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau.$$

For the buyer, we have $U_2(s_2, t_2) - U_2(s_2) = (t_2 - s_2)\hat{q}_2(s_2)$. Therefore, the *IC* condition is equivalent to

$$U_2(t_2) \geq U_2(s_2) + (t_2 - s_2)\hat{q}_2(s_2).$$

By interchanging the roles of s_2 and t_2, we get

$$U_2(s_2) \geq U_2(t_2) + (s_2 - t_2)\hat{q}_2(t_2).$$

Combining these inequalities,

$$(t_2 - s_2)\hat{q}_2(s_2) \leq U_2(t_2) - U_2(s_2) \leq (t_2 - s_2)\hat{q}_2(t_2).$$

If $s_2 < t_2$, then $\hat{q}_2(s_2) \leq \hat{q}_2(t_2)$. Thus, $\hat{q}_2(\cdot)$ is increasing. Furthermore, $\hat{q}_2(\cdot)$ is Riemann integrable, and $dU_2(t_2)/dt_2 = \hat{q}_2(t_2)$. Therefore,

$$U_2(t_2) = U_2(\underline{t}_2) + \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau.$$

($\Leftarrow$ part) We have

$$\begin{aligned} U_1(t_1) - U_1(s_1, t_1) &= U_1(t_1) - U_1(s_1) - (s_1 - t_1)q_1(s_1) \\ &= \int_{t_1}^{s_1} q_1(\tau)d\tau - (s_1 - t_1)q_1(s_1) \geq 0, \end{aligned}$$

where the first equality follows from the definition of $U_1(\cdot)$, the second equality follows from (iii), and the inequality follows from (i). Likewise,

$$\begin{aligned} U_2(t_2) - U_2(s_2, t_2) &= U_2(t_2) - U_2(s_2) - (t_2 - s_2)\hat{q}_2(s_2) \\ &= \int_{s_2}^{t_2} \hat{q}_2(\tau)d\tau - (t_2 - s_2)\hat{q}_2(s_2) \geq 0, \end{aligned}$$

where the first equality follows from the definition of $U_2(\cdot)$, the second equality follows from (iv), and the inequality follows from (ii). Hence, the mechanism (p, α, x) is incentive compatible. □

We note that players' payoffs depend on p and α but not on the cash payment x, thus a modified version of the *payoff equivalence* result holds. That is, players' payoffs depend only on the allocation rule and the royalty rate that is contingent on the allocation but not on the monetary transfer. We can in fact find an explicit cash payment given p and α.

Proposition 2 *Given any probability of trade* $p : T \to [0, 1]$ *and royalty rate* $\alpha : T \to [0, 1]$, *we can find a cash payment* $x : T \to \mathbb{R}$ *such that* (p, α, x) *is incentive compatible as long as* $q_1(t_1)$ *is decreasing and* $\hat{q}_2(t_2)$ *is increasing.*

Proof Define

$$\begin{aligned} x(t_1, t_2) = &\int_{\underline{t}_2}^{t_2} \tau d\hat{q}_2(\tau) + \hat{q}_2(\underline{t}_2)\underline{t}_2 - \int_{t_1}^{\bar{t}_1} \tau dq_1(\tau) \\ &- \int_{\underline{t}_2}^{\bar{t}_2} p(t_1, t_2)\alpha(t_1, t_2)t_2 f_2(t_2)dt_2 \\ &+ \int_{\underline{t}_1}^{\bar{t}_1} \tau F_1(\tau)dq_1(\tau) + \int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\bar{t}_1} p(t_1, t_2)\alpha(t_1, t_2)t_2 f_1(t_1) f_2(t_2)dt_1 dt_2. \end{aligned}$$

We have $y_2(t_2) = \int_{\underline{t}_2}^{t_2} \tau d\hat{q}_2(\tau) + \hat{q}_2(\underline{t}_2)\underline{t}_2$. Thus, $U_2(t_2) - U_2(\underline{t}_2) = \hat{q}_2(t_2)t_2 - y_2(t_2) - \hat{q}_2(\underline{t}_2)\underline{t}_2 + y_2(\underline{t}_2) = \hat{q}_2(t_2)t_2 - \hat{q}_2(\underline{t}_2)\underline{t}_2 - \int_{\underline{t}_2}^{t_2} \tau d\hat{q}_2(\tau) = \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau$, where the first equality follows from Eq. (2) and the last equality follows from integration by parts. Thus, $U_2(t_2) = U_2(\underline{t}_2) + \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau$. Next, we have

$$\begin{aligned} y_1(t_1) = &\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_2}^{t_2} \tau d\hat{q}_2(\tau) f_2(t_2)dt_2 + \hat{q}_2(\underline{t}_2)\underline{t}_2 \\ &- \int_{t_1}^{\bar{t}_1} \tau dq_1(\tau) - \int_{\underline{t}_2}^{\bar{t}_2} p(t_1, t_2)\alpha(t_1, t_2)t_2 f_2(t_2)dt_2 \\ &+ \int_{\underline{t}_1}^{\bar{t}_1} \tau F_1(\tau)dq_1(\tau) + \int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\bar{t}_1} p(t_1, t_2)\alpha(t_1, t_2)t_2 f_1(t_1) f_2(t_2)dt_1 dt_2. \end{aligned}$$

Thus, $U_1(t_1) - U_1(\bar{t}_1) = r_1(t_1) - q_1(t_1)t_1 + y_1(t_1) - r_1(\bar{t}_1) + q_1(\bar{t}_1)\bar{t}_1 - y_1(\bar{t}_1) = -\int_{t_1}^{\bar{t}_1} \tau dq_1(\tau) - q_1(t_1)t_1 + q_1(\bar{t}_1)\bar{t}_1 = \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau$, where the first two equalities follow from Eq. (1) and the definition of $r_1(s_1)$ and the last equality follows from integration by parts. Thus, $U_1(t_1) = U_1(\bar{t}_1) + \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau$. Therefore, (i)–(iv) of Proposition 1 is satisfied and so (p, α, x) is incentive compatible. □

We next have the following proposition on individual rationality.

Proposition 3 *An incentive compatible mechanism (p, α, x) is individually rational if and only if*

$$0 \leq \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left\{ t_2 - \frac{1 - F_2(t_2)}{f_2(t_2)} (1 - \alpha(t_1, t_2)) - \left[t_1 + \frac{F_1(t_1)}{f_1(t_1)} \right] \right\} \times p(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2.$$

Proof Observe that $U_1(\bar{t}_1) = U_1(t_1) - \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau = y_1(t_1) - q_1(t_1)t_1 + r_1(t_1) - \int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau$ by Proposition 1(iii) and Eq. (1). Since $U_1(\bar{t}_1)$ is a constant, we can integrate with respect to t_1 to get

$$\begin{aligned} U_1(\bar{t}_1) = &\int_{\underline{t}_1}^{\bar{t}_1} \int_{\underline{t}_2}^{\bar{t}_2} x(t_1, t_2) f_1(t_1) f_2(t_2) dt_2 dt_1 - \int_{\underline{t}_1}^{\bar{t}_1} \int_{\underline{t}_2}^{\bar{t}_2} t_1 p(t_1, t_2) f_1(t_1) f_2(t_2) dt_2 dt_1 \\ &+ \int_{\underline{t}_1}^{\bar{t}_1} \int_{\underline{t}_2}^{\bar{t}_2} p(t_1, t_2)\alpha(t_1, t_2) t_2 f_1(t_1) f_2(t_2) dt_2 dt_1 - \int_{\underline{t}_1}^{\bar{t}_1} \int_{t_1}^{\bar{t}_1} q_1(\tau) d\tau f_1(t_1) dt_1. \end{aligned}$$

Since

$$\begin{aligned} \int_{\underline{t}_1}^{\bar{t}_1} \int_{t_1}^{\bar{t}_1} q_1(\tau) d\tau f_1(t_1) dt_1 &= \int_{\underline{t}_1}^{\bar{t}_1} q_1(t_1) F_1(t_1) dt_1 \\ &= \int_{\underline{t}_1}^{\bar{t}_1} \int_{\underline{t}_2}^{\bar{t}_2} p(t_1, t_2) F_1(t_1) f_2(t_2) dt_2 dt_1, \end{aligned}$$

we have

$$\begin{aligned} U_1(\bar{t}_1) = &\int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} x(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2 \\ &- \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left[t_1 - \alpha(t_1, t_2) t_2 + \frac{F_1(t_1)}{f_1(t_1)} \right] p(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2. \end{aligned}$$

As for the buyer, we have $U_2(\underline{t}_2) = U_2(t_2) - \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau = \hat{q}_2(t_2)t_2 - y_2(t_2) - \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau$ by Proposition 1(iv) and Eq. (2). Since $U_2(\underline{t}_2)$ is a constant, we can integrate with respect to t_2 to get

$$U_2(\underline{t}_2) = \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} t_2 p(t_1, t_2)\big(1 - \alpha(t_2, t_2)\big) f_1(t_1) f_2(t_2) dt_1 dt_2$$
$$- \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} x(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2 - \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau) d\tau f_2(t_2) dt_2.$$

Since

$$\int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau) d\tau f_2(t_2) dt_2 = \int_{\underline{t}_2}^{\bar{t}_2} \hat{q}_2(t_2)\big(1 - F_2(t_2)\big) dt_2$$
$$= \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, t_2)\big(1 - \alpha(t_1, t_2)\big) f_1(t_1)\big(1 - F_2(t_2)\big) dt_1 dt_2,$$

we have

$$U_2(\underline{t}_2) = \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left[t_2 - \frac{1 - F_2(t_2)}{f_2(t_2)}\right]\big(1 - \alpha(t_1, t_2)\big) p(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2$$
$$- \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} x(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2.$$

Observe that $U_1(t_1)$ is decreasing in t_1 since $dU_1(t_1)/dt_1 = -q_1(t_1) \leq 0$. Observe also that $U_2(t_2)$ is increasing in t_2 since $dU_2(t_2)/dt_2 = \hat{q}_2(t_2) \geq 0$. Therefore, a necessary and sufficient condition for individual rationality is $0 \leq U_1(\bar{t}_1) + U_2(\underline{t}_2)$, which reduces to the inequality of the proposition. □

Note that x does not appear in the inequality of the proposition. Hence, if we are given any (p, α) with $q_1(t_1)$ decreasing and $\hat{q}_2(t_2)$ increasing, then we can directly check the inequality for the existence of an incentive compatible and individually rational mechanism. This is so since we can always find a cash payment $x : T \to \mathbb{R}$ such that (p, α, x) is an incentive compatible mechanism by Proposition 2.

Propositions 1–3 together correspond to Theorem 1 of Myerson and Satterthwaite (1983). In particular, the inequality in Proposition 3 is equal to inequality (2) of that paper when we set $\alpha(t_1, t_2) = 0$. Note that, with the royalty rate $\alpha(t_1, t_2)$, the buyer's expected payoff $U_2(t_2)$ is lower as reflected in part (iv) of Proposition 1 as well as the buyer's virtual valuation $t_2 - (1 - \alpha(t_1, t_2))(1 - F_2(t_2))/f_2(t_2)$ is higher as reflected in the expression in Proposition 3. We also want to note that finding an appropriate cash payment $x(t_1, t_2)$ in Proposition 2 is one of the key steps in arriving at Proposition 3.

2.3 Ex-post efficiency

The mechanism is *ex-post efficient* if the trade occurs if and only if the value of the asset to the buyer exceeds the value of the asset to the seller. In an efficient mechanism, with the probability of trade denoted by $p^0(t_1, t_2)$, we have

$$p^0(t_1, t_2) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } t_1 < t_2, \\ 0 & \text{otherwise.} \end{cases}$$

Observe that $q_1^0(t_1) = \int_{\underline{t}_2}^{\bar{t}_2} p^0(t_1, t_2) f_2(t_2) dt_2 = \int_{t_1}^{\bar{t}_2} f_2(t_2) dt_2 = 1 - F_2(t_1)$ is decreasing in t_1. On the other hand, $\hat{q}_2^0(t_2) = \int_{\underline{t}_1}^{\bar{t}_1} p^0(t_1, t_2)\big(1 - \alpha(t_1, t_2)\big) f_1(t_1) dt_1 = \int_{\underline{t}_1}^{t_2} \big(1 - \alpha(t_1, t_2)\big) f_1(t_1) dt_1$ may not be increasing in t_2. However, a sufficient condition for $\hat{q}_2^0(t_2)$ to be increasing is:[9]

$$\alpha(t_1, t_2') \le \alpha(t_1, t_2) \quad \text{for } t_2 < t_2'.$$

That is, the royalty rate is decreasing in player 2's type. In particular, this condition is satisfied when $\alpha(t_1, t_2)$ is a constant function, i.e., $\alpha(t_1, t_2) = \alpha$ for all $(t_1, t_2) \in T$. Let us choose any royalty rate $\alpha : T \to [0, 1]$ that makes $\hat{q}_2^0(t_2)$ increasing in t_2. Then, there exists an incentive compatible mechanism (p^0, α, x) by Proposition 2.

Define

$$\Delta = \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left\{ t_2 - \frac{1 - F_2(t_2)}{f_2(t_2)} (1 - \alpha(t_1, t_2)) - \left[t_1 + \frac{F_1(t_1)}{f_1(t_1)} \right] \right\} \\ \times p^0(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2.$$

Then,[10]

$$\begin{aligned} \Delta &= \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left\{ \left[t_2 - \frac{1 - F_2(t_2)}{f_2(t_2)} \right] - \left[t_1 + \frac{F_1(t_1)}{f_1(t_1)} \right] + \alpha(t_1, t_2) \frac{1 - F_2(t_2)}{f_2(t_2)} \right\} \\ &\quad \times p^0(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2 \\ &= \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \big(t_2 f_2(t_2) + F_2(t_2) - 1 \big) f_1(t_1) dt_1 dt_2 \\ &\quad - \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \big(t_1 f_1(t_1) + F_1(t_1) \big) f_2(t_2) dt_1 dt_2 \\ &\quad + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \alpha(t_1, t_2) \big(1 - F_2(t_2) \big) f_1(t_1) dt_1 dt_2 \\ &= \int_{\underline{t}_2}^{\bar{t}_2} \big(t_2 f_2(t_2) + F_2(t_2) - 1 \big) F_1(t_2) dt_2 - \int_{\underline{t}_2}^{\bar{t}_2} \min\{t_2 F_1(t_2), \bar{t}_1\} f_2(t_2) dt_2 \\ &\quad + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \alpha(t_1, t_2) \big(1 - F_2(t_2) \big) f_1(t_1) dt_1 dt_2 \end{aligned}$$

9 Recall that $\alpha(t_1, t_2) \in [0, 1]$, so the integrand is always non-negative.

10 This derivation is similar to the one in Myerson and Satterthwaite (1983).

$$
\begin{aligned}
&= -\int_{\underline{t}_2}^{\bar{t}_2} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 + \int_{\bar{t}_1}^{\bar{t}_2} (t_2 - \bar{t}_1) f_2(t_2) dt_2 \\
&\quad + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \alpha(t_1, t_2) \big(1 - F_2(t_2)\big) f_1(t_1) dt_1 dt_2 \\
&= -\int_{\underline{t}_2}^{\bar{t}_2} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 - \int_{\bar{t}_1}^{\bar{t}_2} \big(F_2(t_2) - 1\big) dt_2 \\
&\quad + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \alpha(t_1, t_2) \big(1 - F_2(t_2)\big) f_1(t_1) dt_1 dt_2 \\
&= -\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 \\
&\quad + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} \alpha(t_1, t_2) \big(1 - F_2(t_2)\big) f_1(t_1) dt_1 dt_2.
\end{aligned}
$$

The term $-\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2$ is negative as long as $\underline{t}_2 < \bar{t}_1$ and $\underline{t}_1 < \bar{t}_2$, or equivalently, *interior*$(T_1 \cap T_2) \neq \emptyset$ so that the two type-intervals properly intersect. Hence, if $\alpha(t_1, t_2) = 0$ for all $(t_1, t_2) \in T$ then $\Delta < 0$, and Proposition 3 implies the famous Myerson–Satterthwaite result that there does not exist a bilateral trading mechanism with only cash payment which satisfies ex-post efficiency, incentive compatibility, individual rationality, and budget balance. We also have the following result as a corollary of Proposition 3.

Corollary 1 *If $\bar{t}_2 \leq \bar{t}_1$ in addition to interior$(T_1 \cap T_2) \neq \emptyset$, then ex-post efficiency is impossible unless $\alpha(t_1, t_2) = 1$ for all $(t_1, t_2) \in T$.*

Proof Observe that Δ becomes

$$
-\int_{\underline{t}_2}^{\bar{t}_2} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 + \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{t_2} \alpha(t_1, t_2) \big(1 - F_2(t_2)\big) f_1(t_1) dt_1 dt_2
$$

when $\bar{t}_2 \leq \bar{t}_1$. Observe next that $\Delta = 0$ when $\alpha(t_1, t_2) = 1$ for all $(t_1, t_2) \in T$. Hence, $\Delta < 0$ for any other royalty rate $\alpha : T \to [0, 1]$ when *interior*$(T_1 \cap T_2) \neq \emptyset$. □

Now, assume that $\alpha(t_1, t_2) = \alpha(t_2)$, that is, $\alpha(\cdot)$ does not depend on t_1. That is, the royalty rate depends only on the value of the asset to the buyer. Then, Δ becomes

$$
-\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 + \int_{\underline{t}_2}^{\bar{t}_2} \alpha(t_2) \big(1 - F_2(t_2)\big) F_1(t_2) dt_2.
$$

If $\bar{t}_2 \leq \bar{t}_1$ in addition to *interior*$(T_1 \cap T_2) \neq \emptyset$, then $\Delta = -\int_{\underline{t}_2}^{\bar{t}_2} \big(1 - \alpha(t_2)\big)\big(1 - F_2(t_2)\big) F_1(t_2)\, dt_2 < 0$ unless $\alpha(t_2) = 1$ for all t_2. Thus, ex-post efficiency is impossible, as Corollary 1 has shown more generally. On the other hand, if $\bar{t}_1 < \bar{t}_2$, then $\Delta = -\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - \alpha(t_2)\big)\big(1 - F_2(t_2)\big) F_1(t_2) dt_2 + \int_{\bar{t}_1}^{\bar{t}_2} \alpha(t_2)\big(1 - F_2(t_2)\big) dt_2$ so that there may exist a bilateral trading mechanism with royalty and cash payment which satisfies ex-post efficiency, incentive compatibility, individual rationality, and budget balance.

Example 1 Let $T_1 = [0, 1]$, $T_2 = [0, 2]$, $F_1(t_1) = t_1$, and $F_2(t_2) = t_2/2$. Also, let $\alpha(t_1, t_2) = \alpha$, a constant function. Then, $\Delta = -\int_0^1 (1-\alpha)(1-t_2/2)t_2 dt_2 + \int_1^2 \alpha(1-t_2/2)dt_2 = (7\alpha - 4)/12$. Hence, ex-post efficiency is possible for $4/7 \leq \alpha \leq 1$.[11]

We note that the assumption of $\alpha(t_1, t_2) = \alpha(t_2)$ is not restrictive in the sense that any allocation rule that can be implemented with $\alpha(t_1, t_2)$ can be implemented with $\alpha(t_2)$: Given a general royalty rate $\alpha(t_1, t_2)$, define $\alpha(t_2) = \sup_{t_1 \in T_1} \alpha(t_1, t_2)$ for each $t_2 \in T_2$. Then, the inequality in Proposition 3 holds with $\alpha(t_2)$ if it holds with $\alpha(t_1, t_2)$. Moreover, $\alpha(t_2)$ is decreasing in t_2 if $\alpha(t_1, t_2)$ is decreasing in t_2, guaranteeing an increasing $\hat{q}_2^0(t_2)$. Therefore, given the sufficient condition that $\alpha(t_1, t_2)$ is decreasing in t_2 for $\hat{q}_2^0(t_2)$ to be increasing, if ex-post efficiency is impossible with any royalty rate that depends only on t_2 then it is impossible with any royalty rate that depends on both t_1 and t_2.

Summarizing the discussion, we have:

Proposition 4 *There exists a bilateral trading mechanism with royalty and cash payment which satisfies ex-post efficiency, incentive compatibility, individual rationality, and budget balance. A sufficient condition is that (i) $\alpha(t_1, t_2) = \alpha(t_2)$ is a decreasing function of t_2, (ii) $\bar{t}_1 < \bar{t}_2$, and (iii) $\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - \alpha(t_2)\big)\big(1 - F_2(t_2)\big)F_1(t_2)dt_2 \leq \int_{\bar{t}_1}^{\bar{t}_2} \alpha(t_2)\big(1 - F_2(t_2)\big)dt_2$.*

This proposition shows that contingent contracts (specifically, linear contracts with royalty and cash payment) can implement an efficient trading rule as satisfying individual rationality and budget balance in environments where standard mechanisms with simple cash payment cannot. The reason is that, by linking payments to players' private information, contingent contracts can reduce the information rents which work against the attainment of efficiency. Observe, however, that we cannot overcome the negative effect of private information unless there exists a mass of the buyer's types that are higher than any realization of the seller's type, i.e., unless $\bar{t}_1 < \bar{t}_2$ holds.

The intuition for why a royalty rate of one is the only option for implementing the efficient allocation when $\bar{t}_2 \leq \bar{t}_1$ can be explained as follows.[12] Observe first that the ex-ante gains from trade generated in an ex-post efficient mechanism is

$$\int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\min\{t_2, \bar{t}_1\}} (t_2 - t_1) f_1(t_1) f_2(t_2) dt_1 dt_2$$
$$= \int_{\bar{t}_1}^{\bar{t}_2} \big(1 - F_2(t_2)\big)dt_2 + \int_{\underline{t}_1}^{\bar{t}_1} \big(1 - F_2(t_1)\big)F_1(t_1)dt_1$$

11 Recall that $\hat{q}_2^0(t_2)$ is increasing when $\alpha(t_1, t_2)$ is a constant function.

12 I thank an anonymous referee for providing this intuition.

and the seller's expected information rent is

$$\int_{\underline{t}_1}^{\bar{t}_1}\int_{t_1}^{\bar{t}_1} q_1(\tau)d\tau\, f_1(t_1)dt_1 = \int_{\underline{t}_1}^{\bar{t}_1}\big(1 - F_2(t_1)\big)F_1(t_1)dt_1.$$

Since the contingent payments do not depend on the ex-post value of the asset to the seller, the designer must pay this information rent to satisfy the seller's incentive constraint. Subtracting the seller's information rent from the ex-ante gains from trade, the designer is left with an expected surplus of

$$\int_{\bar{t}_1}^{\bar{t}_2}\big(1 - F_2(t_2)\big)dt_2.$$

This is zero if $\bar{t}_2 \le \bar{t}_1$. Thus, in this case, the designer cannot offer any information rent to the buyer, and so the royalty rate of one is the only one that satisfies the buyer's incentive constraint.[13,14]

When $\alpha(t_1, t_2) = \alpha$, a constant function, we can explicitly derive the lower bound of α for efficiency. Define

$$G(x) \stackrel{\text{def}}{=} \int_{\underline{t}_2}^{x}\big(1 - F_2(t_2)\big)F_1(t_2)dt_2.$$

Then, the condition that $\Delta \ge 0$ is equivalent to $\alpha \ge G(\bar{t}_1)/G(\bar{t}_2)$. Thus, the lower bound of α is

$$\underline{\alpha} \stackrel{\text{def}}{=} \frac{G(\bar{t}_1)}{G(\bar{t}_2)} = \frac{\int_{\underline{t}_2}^{\bar{t}_1}\big(1 - F_2(t_2)\big)F_1(t_2)dt_2}{\int_{\underline{t}_2}^{\bar{t}_2}\big(1 - F_2(t_2)\big)F_1(t_2)dt_2}.$$

Observe that the numerator $G(\bar{t}_1)$ cannot be zero when $interior(T_1 \cap T_2) \ne \emptyset$. Thus, the lower bound $\underline{\alpha}$ is strictly positive, implying the Myerson–Satterthwaite impossibility result. Observe also that $\underline{\alpha} = 1$ when $\bar{t}_2 \le \bar{t}_1$. Observe finally that the denominator $G(\bar{t}_2)$ is equal to $\int_{\underline{t}_2}^{\bar{t}_1}\big(1 - F_2(t_2)\big)F_1(t_2)dt_2 + \int_{\bar{t}_1}^{\bar{t}_2}\big(1 - F_2(t_2)\big)dt_2$ when $\bar{t}_1 < \bar{t}_2$. Thus, $\underline{\alpha}$ is strictly less than 1 and it gets smaller as the term $\int_{\bar{t}_1}^{\bar{t}_2}\big(1 - F_2(t_2)\big)dt_2$ becomes larger.

[13] The buyer's expected information rent is decreasing in the royalty rate and equals zero if and only if the royalty rate is equal to one. When $\bar{t}_1 < \bar{t}_2$, we have $\int_{\bar{t}_1}^{\bar{t}_2}\big(1 - F_2(t_2)\big)dt_2 > 0$ and the designer can cover the buyer's expected information rent by imposing a sufficiently high royalty rate.

[14] In the appendix, we consider the case when contingent payments depend on the value of the asset to the seller as well as to the buyer and show that ex-post efficiency is possible with royalty rates other than one even when $\bar{t}_2 \le \bar{t}_1$.

3 Discussion

We have characterized the range of possible bilateral trading mechanisms which satisfy ex-post efficiency, incentive compatibility, individual rationality, and budget balance. In particular, we have shown that ex-post efficiency together with incentive compatibility, individual rationality, and budget balance is possible in exactly the same environment of Myerson and Satterthwaite (1983) when we introduce contingent contracts.

We have assumed that payments are contingent only on the ex-post value of the asset to the buyer. This reflects the real-world practices, and the reason might be that the buyer's action (such as costly investment) after the trade is crucial in utilizing the asset. Alternatively, we may consider contingent payments that depend on the value of the asset to the seller as well as to the buyer. That is, we may allow cost sharing in addition to royalty payment. Then, the seller's payoff is $p(t_1, t_2)\big(\alpha(t_1, t_2)t_2 - (1-\beta(t_1, t_2))t_1\big) + x(t_1, t_2)$ and the buyer's payoff is $p(t_1, t_2)\big((1 - \alpha(t_1, t_2))t_2 - \beta(t_1, t_2)t_1\big) - x(t_1, t_2)$, where $\beta(t_1, t_2)$ is the cost-sharing rate. It can be shown that ex-post efficiency is easier to obtain in this alternative formulation. For instance, ex-post efficiency is possible for any specification of T_i and F_i for $i = 1, 2$ if we set $\alpha(t_1, t_2) = \beta(t_1, t_2) = 1/2$ for all $(t_1, t_2) \in T$.[15] We present the analysis in the appendix. The reason for this is that the seller's incentive constraint as well as the buyer's incentive constraint is relaxed in this alternative formulation.

We have assumed that types are observable and contractible ex-post. This deterministic setup has some drawback in the sense that misreports may be detected ex-post. We note that this is mainly for expositional purposes. Alternatively, we may let the type t_i, which is unobservable, determine the distribution of the value v_i of the asset to player i, which is observable and contractible, as follows. Let $H_i(v_i|t_i)$ denote the conditional distribution of v_i given t_i which has a continuous and positive density $h_i(v_i|t_i)$ on the interval $[\underline{v}_i, \overline{v}_i]$. If we assume that types are independently drawn and $H_i(v_i|t_i)$ is ordered by first-order stochastic dominance, then essentially the same analysis goes through.

One may argue the issue of trivial solutions. Indeed, ex-post efficiency is always possible if the seller sets the royalty rate α equal to 1. A similar concern has been raised in the literature of auctions with contingent payments. Crémer (1987) argued with respect to Hansen's (1985) results that the seller could extract the entire or almost all surplus if he could virtually 'buy' the winning bidder by setting α equal to or very close to 1. This theoretical possibility is not observed in practice, however. Several reasons can be put forward.[16] First, the seller is cash constrained so that he is not able to reimburse the costs of investment that the buyer may bear to utilize the asset.[17] Next, the moral hazard problem prevents this contractual arrangement: if the buyer's action

15 Kalai and Kalai (2013) present a general theory of cooperation in strategic form games and characterize the *cooperative-competitive value*, or coco value for short. We observe that the resulting outcome when $\alpha(t_1, t_2) = \beta(t_1, t_2) = 1/2$ corresponds to the coco value.

16 See section 4 of DeMarzo et al. (2005) for a related discussion.

17 We did not explicitly model buyer's costs of investment in this paper, but it is a trivial matter to incorporate this feature.

is not fully contractible, then she may underinvest when $\alpha = 1$ or close.[18] Finally, various legal and/or practical restrictions keep α bounded away from 1.

The same reasons apply in this setting, and we want to emphasize that we have characterized all possible mechanisms and so the lower bound of α may well be bounded away from 1. We also want to mention that we chose to present the main point lucidly with an intentionally theoretical model rather than to work with a richer model which incorporates many real-world features.

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Appendix

In this appendix, we consider contingent payments that depend on the value of the asset to the seller as well as to the buyer. Thus, the seller's payoff is $p(t_1, t_2)\big(\alpha(t_1, t_2)t_2 - (1 - \beta(t_1, t_2))t_1\big) + x(t_1, t_2)$ and the buyer's payoff is $p(t_1, t_2)\big((1 - \alpha(t_1, t_2))t_2 - \beta(t_1, t_2)t_1\big) - x(t_1, t_2)$, where $\alpha(t_1, t_2)$ is the royalty rate and $\beta(t_1, t_2)$ is the cost-sharing rate.

We defined $y_1(s_1)$, $y_2(s_2)$, $q_1(s_1)$ and $q_2(s_2)$ in the text. In addition, define

$$
\begin{aligned}
r_1^\alpha(s_1) &\overset{\text{def}}{=} \int_{\underline{t}_2}^{\bar{t}_2} p(s_1, t_2)\alpha(s_1, t_2)t_2 f_2(t_2)dt_2,\\
r_2^\alpha(s_2, t_2) &\overset{\text{def}}{=} t_2\hat{r}_2^\alpha(s_2) \overset{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\alpha(t_1, s_2)t_2 f_1(t_1)dt_1,\\
r_1^\beta(s_1, t_1) &\overset{\text{def}}{=} t_1\hat{r}_1^\beta(s_1) \overset{\text{def}}{=} \int_{\underline{t}_2}^{\bar{t}_2} p(s_1, t_2)\beta(s_1, t_2)t_1 f_2(t_2)dt_2,\\
r_2^\beta(s_2) &\overset{\text{def}}{=} \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\beta(t_1, s_2)t_1 f_1(t_1)dt_1,\\
\hat{q}_1(s_1) &\overset{\text{def}}{=} q_1(s_1) - \hat{r}_1^\beta(s_1) = \int_{\underline{t}_2}^{\bar{t}_2} p(s_1, t_2)\big(1 - \beta(s_1, t_2)\big) f_2(t_2)dt_2,\\
\hat{q}_2(s_2) &\overset{\text{def}}{=} q_2(s_2) - \hat{r}_2^\alpha(s_2) = \int_{\underline{t}_1}^{\bar{t}_1} p(t_1, s_2)\big(1 - \alpha(t_1, s_2)\big) f_1(t_1)dt_1.
\end{aligned}
$$

If the seller believes that the buyer will report truthfully, and he reports s_1 when his true type is t_1, then his expected payoff is

$$U_1(s_1, t_1) \overset{\text{def}}{=} r_1^\alpha(s_1) - \hat{q}_1(s_1)t_1 + y_1(s_1).$$

[18] Suppose the buyer can either take an unobservable action to invest or not. If the costs of investment cannot be fully reimbursed by the seller due to moral hazard, then she would not invest when $\alpha = 1$ or close. We do not believe that a full-blown model of moral hazard would require a deeper insight beyond this simple observation.

Likewise, if the buyer believes that the seller will report truthfully, and she reports s_2 when her true type is t_2, then her expected payoff is

$$U_2(s_2, t_2) \stackrel{\text{def}}{=} \hat{q}_2(s_2)t_2 - r_2^{\beta}(s_2) - y_2(s_2).$$

Define, as in the text, $U_i(t_i) \stackrel{\text{def}}{=} U_i(t_i, t_i)$ for $i = 1, 2$ as well as *IC* and *IR* conditions. We can establish the following propositions.

Proposition A1 *The mechanism (p, α, β, x) is incentive compatible if and only if*

(i) $\hat{q}_1(t_1)$ *is decreasing,*
(ii) $\hat{q}_2(t_2)$ *is increasing,*
(iii) $U_1(t_1) = U_1(\bar{t}_1) + \int_{t_1}^{\bar{t}_1} \hat{q}_1(\tau)d\tau$,
(iv) $U_2(t_2) = U_2(\underline{t}_2) + \int_{\underline{t}_2}^{t_2} \hat{q}_2(\tau)d\tau$.

Proposition A2 *Given any probability of trade $p : T \to [0, 1]$, royalty rate $\alpha : T \to [0, 1]$ and cost-sharing rate $\beta : T \to [0, 1]$, we can find a cash payment $x : T \to I\!R$ such that (p, α, β, x) is incentive compatible as long as $\hat{q}_1(t_1)$ is decreasing and $\hat{q}_2(t_2)$ is increasing.*

Proposition A3 *An incentive compatible mechanism (p, α, β, x) is individually rational if and only if*

$$0 \le \int_{\underline{t}_2}^{\bar{t}_2} \int_{\underline{t}_1}^{\bar{t}_1} \left[\left\{ t_2 - (1 - \alpha(t_1, t_2)) \frac{1 - F_2(t_2)}{f_2(t_2)} \right\} - \left\{ t_1 + (1 - \beta(t_1, t_2)) \frac{F_1(t_1)}{f_1(t_1)} \right\} \right]$$
$$\times\ p(t_1, t_2) f_1(t_1) f_2(t_2) dt_1 dt_2.$$

With the probability of trade $p^0(t_1, t_2)$ defined in the text, we have

$$\hat{q}_1^0(t_1) = \int_{\underline{t}_2}^{\bar{t}_2} p^0(t_1, t_2)\big(1 - \beta(t_1, t_2)\big) f_2(t_2)dt_2 = \int_{t_1}^{\bar{t}_2} \big(1 - \beta(t_1, t_2)\big) f_2(t_2)dt_2;$$
$$\hat{q}_2^0(t_2) = \int_{\underline{t}_1}^{\bar{t}_1} p^0(t_1, t_2)\big(1 - \alpha(t_1, t_2)\big) f_1(t_1)dt_1 = \int_{\underline{t}_1}^{t_2} \big(1 - \alpha(t_1, t_2)\big) f_1(t_1)dt_1.$$

A sufficient condition for $\hat{q}_1^0(t_1)$ to be decreasing and $\hat{q}_2^0(t_2)$ to be increasing is:

$$\alpha(t_1, t_2') \le \alpha(t_1, t_2) \quad \text{for} \quad t_2 < t_2' \quad \text{and} \quad \beta(t_1', t_2) \ge \beta(t_1, t_2) \quad \text{for}\, t_1 < t_1'.$$

That is, the royalty rate is decreasing in player 2's type and the cost-sharing rate is increasing in player 1's type. In particular, this condition is satisfied when both $\alpha(t_1, t_2)$ and $\beta(t_1, t_2)$ are constant functions, i.e., $\alpha(t_1, t_2) = \alpha$ and $\beta(t_1, t_2) = \beta$ for all $(t_1, t_2) \in T$. Let us choose any royalty rate $\alpha : T \to [0, 1]$ and cost-sharing rate $\beta : T \to [0, 1]$ that make $\hat{q}_1^0(t_1)$ decreasing in t_1 and $\hat{q}_2^0(t_2)$ increasing in t_2. Then, there exists an incentive compatible mechanism (p^0, α, β, x) by Proposition A2.

Define

$$\Delta = \int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\bar{t}_1}\left[\left\{t_2-(1-\alpha(t_1,t_2))\,\frac{1-F_2(t_2)}{f_2(t_2)}\right\}-\left\{t_1+(1-\beta(t_1,t_2))\,\frac{F_1(t_1)}{f_1(t_1)}\right\}\right]$$
$$\times\, p^0(t_1,t_2)f_1(t_1)f_2(t_2)dt_1dt_2.$$

Then,

$$\begin{aligned}
\Delta &= \int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\bar{t}_1}\left[\left\{t_2-\frac{1-F_2(t_2)}{f_2(t_2)}\right\}-\left\{t_1+\frac{F_1(t_1)}{f_1(t_1)}\right\}+\alpha(t_1,t_2)\frac{1-F_2(t_2)}{f_2(t_2)}\right.\\
&\quad\left.+\beta(t_1,t_2)\frac{F_1(t_1)}{f_1(t_1)}\right]p^0(t_1,t_2)f_1(t_1)f_2(t_2)dt_1dt_2\\
&= \int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\big(t_2f_2(t_2)+F_2(t_2)-1\big)f_1(t_1)dt_1dt_2\\
&\quad-\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\big(t_1f_1(t_1)+F_1(t_1)\big)f_2(t_2)dt_1dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\alpha(t_1,t_2)\big(1-F_2(t_2)\big)f_1(t_1)dt_1dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\beta(t_1,t_2)F_1(t_1)f_2(t_2)dt_1dt_2\\
&= \int_{\underline{t}_2}^{\bar{t}_2}\big(t_2f_2(t_2)+F_2(t_2)-1\big)F_1(t_2)dt_2-\int_{\underline{t}_2}^{\bar{t}_2}\min\{t_2F_1(t_2),\bar{t}_1\}f_2(t_2)dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\alpha(t_1,t_2)\big(1-F_2(t_2)\big)f_1(t_1)dt_1dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\beta(t_1,t_2)F_1(t_1)f_2(t_2)dt_1dt_2\\
&= -\int_{\underline{t}_2}^{\bar{t}_1}\big(1-F_2(t_2)\big)F_1(t_2)dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\alpha(t_1,t_2)\big(1-F_2(t_2)\big)f_1(t_1)dt_1dt_2\\
&\quad+\int_{\underline{t}_2}^{\bar{t}_2}\int_{\underline{t}_1}^{\min\{t_2,\bar{t}_1\}}\beta(t_1,t_2)F_1(t_1)f_2(t_2)dt_1dt_2.
\end{aligned}$$

Assume that $\alpha(t_1, t_2) = \alpha(t_2)$ and $\beta(t_1, t_2) = \beta(t_1)$. Then, Δ becomes

$$
\begin{aligned}
&-\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - F_2(t_2)\big) F_1(t_2) dt_2 + \int_{\underline{t}_2}^{\bar{t}_2} \alpha(t_2)\big(1 - F_2(t_2)\big) F_1(t_2) dt_2 \\
&\quad + \int_{\underline{t}_1}^{\bar{t}_1} \beta(t_1) F_1(t_1)\big(1 - F_2(t_1)\big) dt_1 \\
&= -\int_{\underline{t}_2}^{\bar{t}_1} \big(1 - F_2(x)\big) F_1(x) dx + \int_{\underline{t}_2}^{\bar{t}_1} \big(\alpha(x) + \beta(x)\big)\big(1 - F_2(x)\big) F_1(x) dx \\
&\quad + \int_{\bar{t}_1}^{\bar{t}_2} \alpha(x)\big(1 - F_2(x)\big) dx + \int_{\underline{t}_1}^{\underline{t}_2} \beta(x) F_1(x) dx \\
&= \int_{\underline{t}_2}^{\bar{t}_1} \big(\alpha(x) + \beta(x) - 1\big)\big(1 - F_2(x)\big) F_1(x) dx \\
&\quad + \int_{\bar{t}_1}^{\bar{t}_2} \alpha(x)\big(1 - F_2(x)\big) dx + \int_{\underline{t}_1}^{\underline{t}_2} \beta(x) F_1(x) dx.
\end{aligned}
$$

Since the last two terms are non-negative, Δ is positive as long as $\alpha(x) + \beta(x) \geq 1$.[19]

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[19] Note that $1 - F(x) = 0$ for $x \geq \bar{t}_2$ and $F_1(x) = 0$ for $x \leq \underline{t}_1$. Hence, these terms are non-negative even when $\bar{t}_2 < \bar{t}_1$ and $\underline{t}_2 < \underline{t}_1$.