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the Nash Bargaining Solution』**

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# Escalation Game with Endogenous Demands and The Nash Bargaining Solution

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The paper examines the behavior of two agents who need to make a joint decision but they have conflicting preferences about the choice of the outcome. Conventionally such problem is considered as the bargaining problem described as the situation of dividing a pie. But we introduce the model that sheds a different light on the problem in question. The problem is described as the conflict situation modelled as a two-stage game. In the first stage players propose outcomes. The settlement is made if the proposed outcomes are the same. If not, the game moves onto the second stage where they play the concession game called the escalation game. In the escalation game, each player, in turn, has the choice between either to submit by accepting the other's demand or to escalate by way of insisting his demand to be accepted. Each escalation generates a probability of an inefficient outcome.

There are two main findings: (1) it is shown that the player's decision is determined by his risk limit which measures his intensity towards winning. (2) if the escalation game allocates the demand of the player with the highest risk limit, then players propose the Nash cooperative solution.

*Key Words:* Bargaining, Risk Limit, Nash Bargaining Solution.

*Subject Classification:* C72, C78

## 1. INTRODUCTION

One of the essential problems that must be dealt is that of the situation in which two individuals have difficulty in reaching a settlement because they have conflicting preferences. The paper develops the model that analyzes such two-person conflict problem. The objectives are twofold: (1) to examine the parameters that determine the behaviors of players in conflict and (2) to characterize the extensive form game that implements the solution concept proposed in the cooperative game theory.

The conflict process is modeled as a game with two stages. In the first stage, two players simultaneously make demands in the bargaining set. If they have demanded the same alternatives, then the game ends and both players receive the demand that they have made. But if they have demanded different alternatives, then they play

the escalation game in the second stage. We first analyze the escalation game, then examine the demand stage of the game.

The escalation game captures the conflict process when the players have made incompatible demands. Its outcome is either a settlement or disagreement. The settlement occurs when one of the players accepts the other's demand. There is a risk of disagreement each time the settlement is delayed. Hence, the model deals with three alternatives. Two efficient alternatives; one preferred by player 1 and the other preferred by player 2. And a disagreement outcome which is Pareto dominated by the other two alternatives.

Our model incorporates essential notions from four non-cooperative game theory models: the Nash model, the Zeuthen/Harsanyi model, the Crawford model and the Rubinstein model.

From the Nash (1950) model, it incorporates the notion that the presence of disagreement outcome serves as a threat point to enforce players to come to a settlement. By considering three alternatives, we embody the notion of risk limit, introduced by Zeuthen (1930) and further developed by Harsanyi (1977), as the determining variable of the player's strategic choice.

Crawford (1982) argued that players do not consciously choose a disagreement outcome, but they inadvertently run into it due to the result of their actions that are both uncertain and irreversible. The escalation game embodies the similar notion of exogenously chosen disagreement probabilities due to an imperfect decision making process in a conflict situation.

Rubinstein (1982) studied alternating offer model where players make sequential bids to divide the pie of size one. We also consider a sequential game where players move in turn and thus they can observe each other's actions.

Rubinstein created a powerful model capturing the process of two players reaching an agreement. But his model mainly concerns characterizing the agreement under the assumption that players have different time preferences. The purpose of our model is to characterize the variable that determines the behavior of players in a conflict. Hence our model differs from Rubinstein's model in three respects. First, we formulate the game with a finite number of moves; hence backward induction can be applied in solving the game. Second, we assume that players do not discount the future. Instead, we introduce a risk of disagreement each time agreement is delayed. Third, in our model, the demands cannot be changed once they have been chosen, while in Rubinstein's model the proposals can be revised in each period. The absence of revision of the initial proposals permits a simple characterization of equilibria and facilitates clear analysis of optimal strategies.

The paper is organized as follows. Section 3 presents the problem that will be analyzed in the paper. Section 3 introduces the escalation game and derives the sequential equilibrium of the escalation game with complete information. It is shown that the escalation game with complete information lasts at most two periods. In Section 4, we study the escalation game with endogenous demands where players first make simultaneous demands in the bargaining set before playing the escalation game. We characterize the escalation game with endogenous demands that results in the Nash cooperative solution. Section 5 concludes the paper.

## 2. PROBLEM

The problem that will be analyzed here is the situation where two individuals need to make a joint decision, but they have conflicting preferences about which

outcome should be chosen. In the joint decision making, the outcome can be obtained only by the mutual consent. Such situations include various social contexts, from friends going to a restaurant together, to the form of a political regime, to trading of goods. The decision is obtained immediately if they prefer the same outcome. But the problem arises when their preferences are conflicting. We will provide the former description of such problem.

Let  $\Omega$  denote the set of all alternatives that can be chosen in a particular situation. For clear analysis, players' preferences over the alternatives are expressed in value terms represented by a von Neumann-Morgenstern utility function. Specifically, let there be two individuals  $i = 1, 2$ . The utility function  $u_i$  of individual  $i$  assigns values to each alternatives  $\omega \in \Omega$  according to his preference relation. That is,  $u_i : \Omega \rightarrow \mathbb{R}$ , where for all  $x, y \in \Omega$ ,  $x \succsim_i y$  if and only if  $u_i(x) \geq u_i(y)$ . It is assumed that both individuals' preference rankings over the set of alternatives  $\Omega$  are common knowledge. Let  $S$  denote the set of payoff pairs that the individuals can obtain by the mutual consent. Hence, the set  $S$  consists of utility pairs from the same alternative such that

$$S = \{(u_1(\omega), u_2(\omega)) \mid \omega \in \Omega\}.$$

Thus, any chosen outcome  $\omega_i \in \Omega$  can be expressed (in value terms) as an element in  $S \subset \mathbb{R}^2$ . Notice that the set  $S$  is equivalent to the Nash (1950) bargaining set.

Let  $P(\Omega)$  denote the set of outcomes that are not Pareto dominated, hence  $P(\Omega) \subset \Omega$  such that

$$P(\Omega) = \{x \in \Omega \mid \text{if } y \succ_1 x \text{ and } y \succ_2 x, \text{ then } y \notin \Omega\}.$$

If players have conflicting preferences, then the set  $P(\Omega)$  contains more than two alternatives. Hence the Pareto set

$$P(S) = \{(u_1(\omega), u_2(\omega)) \in S \mid \omega \in P(\Omega)\}$$

with more than two elements represents the preference relation of players with conflicting preferences in the joint decision making. Hence, the problem that we consider here is the situation where an outcome must be chosen from the Pareto set  $P(S)$ . If players opt for the same alternative in the Pareto set, then their demands are compatible where  $(u_1(\omega), u_2(\omega)) \in P(S)$ . But if each player opts for different alternatives; player 1 opts for the alternative  $\omega_1 \in P(\Omega)$  and player 2 opts for the alternative  $\omega_2 \in P(\Omega)$ , then their demands are incompatible where  $(u_1(\omega_1), u_2(\omega_2)) \notin P(S)$ .

Now we will introduce the escalation game.

### 3. ESCALATION GAME

An escalation game is a model that captures the conflict process between two players who have demanded different outcomes. Specifically, it is an  $n$ - period two player non-cooperative game. The conflict is resolved if one of the players accepts the other's demand. The players move in a sequence. The player with the move either submits or escalates. In case she submits, the game ends with the outcome proposed by her opponent. In case she escalates, Nature has the move and determines with a known probability whether or not the game ends. If Nature

chooses to end the game, then the outcome  $\omega_0$  is selected; otherwise, the game moves on to the next period. The probability with which Nature choose to end the game depends upon the period and is denoted by  $r_t$  with  $t$  referring to the period. It is assumed that  $r_n = 1$ . We refer to  $\omega_0$  as the disagreement outcome.

Formally,  $u_i(\omega_i)$  denotes the payoff that player  $i$  receives when his demand  $\omega_i$  is accepted. Similarly,  $u_i(\omega_j)$  denotes the payoff that player  $i$  receives when the opponent's demand,  $\omega_j$ , is accepted and  $u_i(\omega_0)$  denotes the payoff when the game ends in disagreement  $\omega_0$ ; with  $j \neq i$ . It is assumed that accepting the other's demand is preferred to ending in disagreement. Hence,  $u_i(\omega_i) \geq u_i(\omega_j) > u_i(\omega_0)$ . Observe that  $(u_1(\omega_0), u_2(\omega_0)) \in S$ , but  $(u_1(\omega_0), u_2(\omega_0)) \notin P(S)$ .

The triple  $(u_i(\omega_i), u_i(\omega_j), u_i(\omega_0))$  is essential information and is called the type of player  $i$ . Before the game starts each player is informed about her type. Furthermore, the payoffs do not depend upon the period in which the outcome is reached. In particular, there is no discounting.

We assume (i) that players are expected utility maximizers, and (ii) that a player submits in case he cannot strictly gain by escalating. As players are expected utility maximizers, the equilibrium is independent of a positive linear transformation of the payoffs. For a given type  $(u_i(\omega_i), u_i(\omega_j), u_i(\omega_0))$ , we consider the following normalization: subtract  $u_i(\omega_j)$  and divide by  $u_i(\omega_i) - u_i(\omega_0)$ . As such, the type  $(u_i(\omega_i), u_i(\omega_j), u_i(\omega_0))$  transforms into  $(k_i, 0, k_i - 1)$  with

$$k_i = \frac{u_i(\omega_i) - u_i(\omega_j)}{u_i(\omega_i) - u_i(\omega_0)}, 0 \leq k_i \leq 1.$$

The normalized winning payoff  $k$ , also known as the risk limit (Zeuthen 1930, Harsanyi 1977), is the loss of submitting divided by the loss of disagreeing, and is invariant under positive linear transformations. Apparently, the risk limit is sufficiently rich to summarize the essential information of a player.

Now we can define the escalation game.

**Definition 1:** *An escalation game is a quadruple  $\Gamma = (n, b, r, k)$ , with  $n \geq 2$  the number of periods,  $b$  in  $\{1, 2\}$  the player with the move in period 1,  $r = (r_1, r_2, \dots, r_n = 1)$  in  $[0, 1]^n$  the disagreement probabilities, and  $k = (k_1, k_2)$  in  $[0, 1]^2$  is a couple of risk limits of each players.*

The game tree of the escalation game is depicted in Figure 1.  $N$  refers to Nature, and  $r_t$  is the probability of ending in disagreement in period  $t$ . In order to know which player has the move in a certain period, it suffices to know the player with the move in the first period. Sometimes, we will refer to the player with the move in period  $t$  ( $> 2$ ) as 'player  $t$ '. This will cause no confusion. For example, outcome  $\omega_{n-1}$  is the outcome most preferred by player  $n - 1$ .

### 3.1. Subgame Perfect Equilibrium

An equilibrium outcome results from players' behaviors that follow optimal strategies. The optimal strategies specify the best response of a player given his expectation about his opponent's action. Lemma 1 shows that player  $i$ 's strategic choice is driven by his risk limit  $k_i$ .

**Lemma 1:** *The two-period escalation game  $(2, 1, (r_1, 1), k)$  has a unique equilibrium. The normalized equilibrium payoffs are equal to  $(0, k_2)$  if  $k_1 \leq r_1$ , and  $(k_1 - r_1, 0)$  if  $k_1 > r_1$ .*

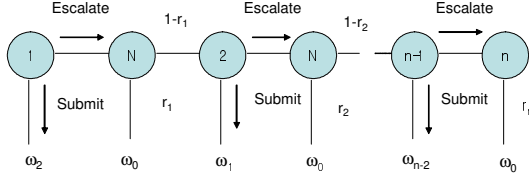


FIG. 1

**Proof.** Player 1 receives 0 if he submits. If period 2 is reached, then Nature ends the game and outcome  $\omega_1$  is selected. Thus, player 1 receives  $r_1(k_1 - 1) + (1 - r_1)k_1 = k_1 - r_1$  if he escalates. As a consequence, player 1 escalates if  $k_1 - r_1 > 0$ . Also, player 1 submits if  $k_1 - r_1 < 0$ . Finally, in case escalating and submitting generate the same payoff, then (by assumption) player 1 submits. ■

Lemma 1 implies that if player  $i$  knows the value of  $k_j$ , then player  $i$  knows whether player  $j$  will escalate or submit at her decision node. Hence players' decisions at each decision node are known to each other when both players risk limits are known.

Lemma 2 characterizes the equilibrium behavior of players in the escalation game when both players know one another's risk limits.

**Lemma 2:** *Let  $\Gamma$  be an  $n$  - period escalation game with complete information. Assume that the game has reached period  $t$ . Let player  $i$  have the move in period  $t$ . In that case, player  $i$  escalates if and only if (a)  $k_i > r_i$  and (b) player  $j$  submits.*

**Proof.** In order to allow for escalation, we assume that  $n \geq 2$ . If  $n = 2$ , then the lemma follows from Lemma 1.

Now let  $n > 2$ . Assume the lemma holds at  $t + 1$ . Thus, player  $j$  escalates if and only if player  $i$  submits and  $k_j > r_{t+1}$ .

Suppose player  $j$  escalates. Then, player  $i$  is better off submitting since he receives  $(k_i - 1)(r_t + (1 - r_t)r_{t+1}) \leq 0$  if he escalates.

Suppose player  $j$  submits, then player  $i$  receives  $r_1(k_1 - 1) + (1 - r_1)k_1 = k_1 - r_1$  if he escalates.

Hence, player  $i$  escalates if and only if player  $t + 1$  submits and  $k_i > r_t$ . Otherwise, player  $i$  submits. It follows that the lemma holds at  $t$  if the lemma holds at

$t + 1$ . By Lemma 1, the lemma holds at  $n - 1$ . By backward induction the lemma holds at  $t \geq 1$ . ■

Lemma 2 asserts that the player with the move submits and would never escalate if she knows that her opponent escalates in the next period. It follows that the escalation game with complete information lasts at most 2 periods. In the first period, player 1 will either escalate or submit. He will escalate only if he is certain that player 2 submits in the second period. Otherwise, he will submit and the game will end immediately.

**Lemma 3:** *Let  $\Gamma$  be an  $n$  - period escalation game with complete information. Then, either player 1 submits at  $t = 1$  or player 2 submits at  $t = 2$ .*

**Proof.** By backward induction and Lemma 2. At  $t = 3$ , there are only two possibilities: either player 1 submits or he escalates. If he submits at  $t = 3$ , then it is better for him to submit at  $t = 1$ . If he escalates at  $t = 3$ , then player 2 submits at  $t = 2$ . ■

The following examples provide some insights into the equilibrium behavior of the escalation game with complete information. In example 1, we consider the escalation game with a small constant risk of disagreement outcome.

EXAMPLE 1.  $n > 3$  period escalation game with complete information where  $r_t = \varepsilon > 0$  for all  $t < n$ . There are four possible cases: (i)  $k_1 > \varepsilon, k_2 > \varepsilon$ . (ii)  $k_1 \leq \varepsilon, k_2 \leq \varepsilon$ . (iii)  $k_1 \leq \varepsilon, k_2 > \varepsilon$ . (iv)  $k_1 > \varepsilon, k_2 \leq \varepsilon$ . Assume  $n$  is even. Then in case (i) player 1 escalates at  $n - 1$ . By backward induction and Lemma 2, player 1 escalates in the first period upon which player 2 submits in period 2. In cases (ii) and (iii) player 1 submits in the first period. In case (iv) player 1 escalates in the first period knowing that player 2 submits in the second period. Assume  $n$  is odd. Then in case (i) player 2 escalates at  $n - 1$ . By backward induction and Lemma 2, player 1 submits in the first period. In cases (ii) and (iii) player 1 submits in the first period. In case (iv) player 1 escalates in the first period knowing that player 2 submits in the second period.

Example 1 shows that for case (i) the outcome of the game depends on whether the game ends in odd period or in even period. For cases (ii), (iii) and (iv), the outcome is independent of the number of periods. If  $k_t \leq r_t$ , player  $t$  submits in period  $t$ . It follows that the identity of the last mover is important only if  $k_t > r_t$  for all  $t < n$ .

EXAMPLE 2.  $n > 3$  period escalation game with complete information where  $r_t = \varepsilon > 0$  for all  $t < n - 1$  and  $r_{n-1} = 1 - \varepsilon$ . Assume  $n$  is even. Let us consider the following cases: (i)  $k_1 > 1 - \varepsilon, k_1 > \varepsilon, k_2 > \varepsilon$ . (ii)  $k_1 \leq 1 - \varepsilon, k_1 > \varepsilon, k_2 > \varepsilon$ . In case (i) player 1 escalates at  $n - 1$ , and knowing this player 2 submits at  $n - 2$ . By backward induction, player 1 escalates in the first period upon which player 2 submits in period 2. In case (ii) player 1 submits at  $n - 1$ . By backward induction, player 2 escalates in period 2, and knowing this player 1 submits in the first period. Assume  $n$  is odd. Let us consider the following cases: (i)  $k_2 > 1 - \varepsilon, k_2 > \varepsilon, k_1 > \varepsilon$ . (ii)  $k_2 \leq 1 - \varepsilon, k_2 > \varepsilon, k_1 > \varepsilon$ . In case (i) player 2 escalates in period  $n - 1$  and knowing this player 1 submits at  $n - 2$ . By backward induction, player 2 escalates in period 2, and knowing this, player 1 submits in the first period. In case (ii),

player 2 submits at  $n - 1$ , and knowing this player 1 escalates in period  $n - 2$ . By backward induction, player 2 submits in period 2, and knowing this player 1 escalates in the first period.

Example 2 shows that when  $n$  is an even period (player 2 being the last mover) and  $k_1 > \varepsilon$  and  $k_2 > \varepsilon$ , then player 1 is first to submit if  $k_1 \leq 1 - \varepsilon$ . On the contrary, when  $n$  is an odd period (player 1 being the last mover), and  $k_1 > \varepsilon$  and  $k_2 > \varepsilon$ , then player 2 is first to submit if  $k_2 \leq 1 - \varepsilon$ . Hence, in the game where player 1 moves at  $n - 1$ , if  $k_1 \leq 1 - \varepsilon$ ,  $k_1 > \varepsilon$  and  $k_2 > \varepsilon$ , then the game ends in the first period. Similarly, in the game where player 2 moves at  $n - 1$ , if  $k_2 \leq 1 - \varepsilon$ ,  $k_1 > \varepsilon$  and  $k_2 > \varepsilon$ , then the game ends in the second period. It follows that if the game has reached  $t$  and the player who moves at  $t$  submits, then he submits in his first move. This is shown in example 3.

EXAMPLE 3. Assume (i)  $k_\tau > r_\tau$  where  $r_\tau > 0$ ,  $r_{\tau-1} > 0$  for all  $\tau < t$  and (ii)  $k_t \leq r_t$  for some odd  $t$ . At  $t$  player 1 submits. At  $t - 1$  player 2 escalates since by assumption  $k_{t-1} > r_{t-1}$ . By backward induction, player 1 submits in the first period. Similarly for even  $t$ , player 1 escalates in the first period, upon which player 2 submits in the second period.

Let  $t_1^* \in \arg \min_{1 \leq 2t+1 \leq n} \{r_{2t+1} \mid k_1 \leq r_{2t+1}\}$ ,  $t_2^* \in \arg \min_{1 \leq 2t \leq n} \{r_{2t} \mid k_2 \leq r_{2t}\}$ . Hence,  $t_1^*$  is the first (odd) period in which player 1's risk limit is smaller than the disagreement probability of that period, given that the game reaches the period  $t_1^*$ . Similarly,  $t_2^*$  is the first (even) period in which player 2's risk limit is smaller than the disagreement probability of that period, given that the game reaches the period  $t_2^*$ .

**Proposition 1:**  $n > 2$  period escalation game with complete information  $\Gamma$  has a unique subgame perfect equilibrium. (i) If  $t_2^* > t_1^*$ , then the normalized equilibrium payoffs are equal to  $(0, k_2)$  where player 1 submits immediately. (ii) If  $t_1^* < t_2^*$ , then the normalized equilibrium payoffs are equal to  $(k_1 - r_1, 0)$  where player 2 submits in her first move.

**Proof.** If  $t_1^* > t_2^*$ , then for some even  $t$  (a)  $k_2 \leq r_t$  and (b)  $k_1 > r_\tau$  where  $r_\tau > 0$ ,  $r_{\tau-1} > 0$  for all  $\tau \leq t - 1$ . Hence, at  $t$  player 2 submits and at  $t - 1$  player 1 escalates. By Lemma 2 and backward induction, player 2 submits in period 2 and knowing this player 1 escalates in period 1. Similarly, if  $t_1^* < t_2^*$ , then for some odd  $t$  (a)  $k_1 \leq r_t$  and (b)  $k_2 > r_\tau$  where  $r_\tau > 0$ ,  $r_{\tau-1} > 0$  for all  $\tau \leq t - 1$ . Hence, at  $t$  player 1 submits and at  $t - 1$  player 2 escalates. By Lemma 2 and backward induction, player 1 submits immediately in the first period. ■

Proposition 1 specifies the property of the disagreement probabilities  $r = (r_1, r_2, \dots, r_n = 1)$  that constitute the equilibrium outcome. Next, we will examine the disagreement probabilities that provide incentives for the player with lower risk limit to submit.

EXAMPLE 4.  $n > 3$  period escalation game with complete information  $\Gamma$  where  $r_1 = 0$ ,  $r_{t+1} = \text{Min}[r_t + \varepsilon, 1]$ ,  $\varepsilon = \frac{1}{2} |k_1 - k_2|$  for all  $t \geq 1$  and  $k_1 \neq k_2$ . As it is shown in Figure 2, when the disagreement probabilities are determined by  $k_1$  and  $k_2$  such that  $r_1 = 0, r_2 = \frac{1}{2} |k_1 - k_2|, r_3 = \frac{2}{2} |k_1 - k_2|, r_4 = \frac{3}{2} |k_1 - k_2|, \dots$ , then

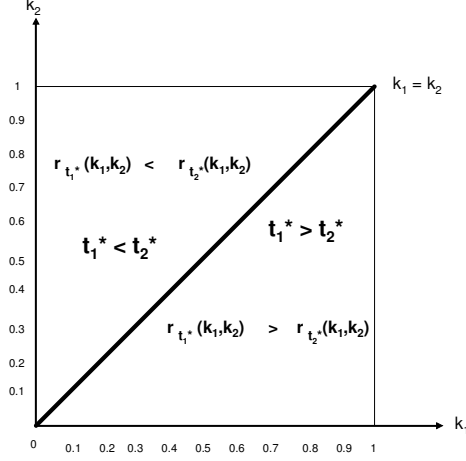


FIG. 2

$r_{t_i^*}(k_1, k_2) < r_{t_j^*}(k_1, k_2)$  if  $k_i < k_j$  where  $t_i^* < t_j^*$ . Hence by Proposition 1, (i) if  $k_1 < k_2$ , then player 1 submits immediately in the first period and (ii) if  $k_2 < k_1$ , then player 1 escalates in the first period upon which player 2 submits. ■

Example 4 shows that the player with the lower risk limit would submit to the other player if the disagreement probabilities  $r = (r_1, r_2, \dots, r_n = 1)$  increase gradually with the rate that are smaller than  $|k_1 - k_2|$ . Hence there some escalation games follow the Zeuthen (1930) principle where it argues that if  $k_i > k_j$ , then player  $i$  makes the first concession.

#### 4. THE ESCALATION GAME WITH ENDOGENOUS DEMANDS

The escalation game analyzes the conflict process after players have opted for different alternatives in the Pareto set  $P(S)$ . In this section we will examine whether there is the settlement (a point in the Pareto set) that both players would accept if they know the result of the escalation game. Hence, we introduce the model that extends the escalation game by permitting players first to demand outcomes in the Pareto set  $P(S)$ .

**Definition 2:** *The "Escalation Game with Endogenous Demands" is a two-stage game. In the first stage players simultaneously make demands in the Pareto set  $P(S)$ . In the event that demands are compatible, the game ends with the proposed demand as the outcome. But in the event that demands are incompatible, the game proceeds to the second stage, in which players play the escalation game with the two demands.*

Hence the "Escalation Game with Endogenous Demands" consists of two games; the demand game and the escalation game. The demand game is a simultaneous-move one-shot game where player  $i$  demands an alternative  $\omega_i \in P(\Omega)$  with the payoff  $(u_i(\omega_i), u_j(\omega_i)) \in P(S)$ . Thus, he determines his own risk limit  $k_i =$

$\frac{u_i(\omega_i) - u_i(\omega_j)}{u_i(\omega_i) - u_i(\omega_0)}$  given the other player's demand  $\omega_j \in P(\Omega)$ . Notice that if players make compatible demands (opts for the same point in  $P(S)$ ), then the value of both players' risk limits are equal to zero  $k_i = 0$ . The escalation game is reached only when the demand game ends with the incompatible demands. Hence if there is the settlement that is accepted by both players, then the escalation game becomes the subgame that lies off the equilibrium path.

The settlement will be accepted by both players when neither of them could gain by deviating from it. Nash (1950) characterized such settlement as the solution to his bargaining game. He considers the game  $(S, d)$  in which players have to choose the agreement in the set  $S$ . In the event that they fail to reach the agreement, then a given disagreement outcome  $d = (u_1(\omega_0), u_2(\omega_0))$  is the result. Nash proposes the solution  $F^N(S, d) \in S$  that satisfies a set of axioms. This Nash solution  $F^N$  prescribes, as the outcome of the game  $(S, d)$ , that point in  $S$  for which the product  $(x_1 - d_1)(x_2 - d_2)$  is maximal, hence

$$F^N(S, d) \in \arg \max_{x \in S} (x_1 - d_1)(x_2 - d_2).$$

There is a close relationship between the risk limit and the Nash solution  $F^N$  such that (i) if a player demands the Nash solution then he will have the highest risk limit and (ii) the demand of the player with the highest risk limit gives the largest Nash product.

Specifically, let  $\omega_N \in P(\Omega)$  denote the alternative that corresponds to the Nash solution. Suppose player  $i$  demands the Nash solution. Then, for any other demand  $\omega \in P(\Omega)$  by player  $j$

$$\begin{aligned} & (u_i(\omega_N) - u_i(\omega_0))(u_j(\omega_N) - u_j(\omega_0)) \\ > & (u_i(\omega) - u_i(\omega_0))(u_j(\omega) - u_j(\omega_0)). \end{aligned} \quad (1)$$

The equation (1) is equivalent to

$$\begin{aligned} & \frac{u_i(\omega_N) - u_i(\omega)}{u_i(\omega_N) - u_i(\omega_0)} - \frac{u_j(\omega) - u_j(\omega_N)}{u_j(\omega) - u_j(\omega_0)} \\ = & k_i - k_j > 0 \end{aligned}$$

Similarly, for any demand  $\omega_i \in P(\Omega)$  made by player  $i$  and for any demand  $\omega_j \in P(\Omega)$  made by player  $j$  observe that

$$\begin{aligned} & k_i - k_j \\ = & \frac{u_i(\omega_i) - u_i(\omega_j)}{u_i(\omega_i) - u_i(\omega_0)} - \frac{u_j(\omega_j) - u_j(\omega_i)}{u_j(\omega_j) - u_j(\omega_0)}. \end{aligned}$$

$k_i - k_j > 0$  is equivalent to

$$(u_i(\omega_i) - u_i(\omega_0))(u_j(\omega_i) - u_j(\omega_0)) - (u_i(\omega_j) - u_i(\omega_0))(u_j(\omega_j) - u_j(\omega_0)) > 0.$$

Hence if the player with lower risk limit is forced to submit in the escalation game, then it is the optimal strategy for the player to propose the Nash solution in the demand game. This is shown in the next proposition.

**Proposition 2:** *Let  $r_1 = 0$ ,  $r_{t+1} = \text{Min}[r_t + \varepsilon, 1]$ ,  $\varepsilon = \frac{1}{2}|k_1 - k_2|$  for all  $t \geq 1$ . Then, the "Escalation Game with Endogenous Demands" has a unique subgame*

perfect equilibrium such that both players propose an alternative  $\omega \in P(\Omega)$  which gives the payoff that corresponds to the Nash solution

**Proof.** Let  $z = F^N(S, d)$  denote the Nash solution. We will show that the demand  $z = (z_1, z_2)$  is a subgame perfect equilibrium.

Suppose player 1 demands the alternative  $\omega_N \in P(\Omega)$  that corresponds to the Nash solution such that  $z = (z_1, z_2) = (u_1(\omega_N), u_2(\omega_N))$ , then in equilibrium player 1 will receive the payoff  $z_1$  regardless of the demand that the opponent makes. This is because if demands are compatible, then each player receives his demand (player 1 receives  $z_1$ ). If demands are incompatible, then for any counter proposal  $y \in P(S), y \neq z$

$$\begin{aligned} & \frac{z_1 - y_1}{z_1 - d_1} - \frac{y_2 - z_2}{y_2 - d_2} \\ &= k_1 - k_2 > 0. \end{aligned}$$

The game moves on to the second stage where players play the escalation game with the disagreement probabilities  $r_1 = 0, r_{t+1} = \text{Min}[r_t + \varepsilon, 1], \varepsilon = \frac{1}{2}|k_1 - k_2|$ . By Example 4, player 2 will have to submit. Furthermore, if player 1 proposes  $z$ , then player 2 cannot receive a higher payoff than  $z_2$  for the same reason.

To see that no other demands is a subgame perfect equilibrium, suppose player 1 demands the alternative  $\omega_2 \in P(\Omega)$  such that  $(u_1(\omega_2), u_2(\omega_2)) = (x_1, x_2)$  where  $x_1 < z_1$ , then 2's best response would be to also demand  $\omega_2$ . But the demand  $\omega_2$  is not an equilibrium demand since we have seen that player 1 can assure himself  $z_1$  by demanding  $z$ , regardless of player 2's proposal. Similarly, if player 1 demands the alternative  $\omega_1 \in P(\Omega)$  such that  $(u_1(\omega_1), u_2(\omega_1)) = (x'_1, x'_2)$  where  $x'_1 > z_1$ , then any counter proposal  $y$  by player 2 such that

$$(y_1 - d_1)(y_2 - d_2) > (x'_1 - d_1)(x'_2 - d_2)$$

will yield player 2 a payoff greater than  $x'_2$ . ■

We have shown that there exists the “*Escalation Game with Endogenous Demands*” that implements the Nash solution  $F^N$ .

## 5. FINAL REMARKS

We have analyzed the joint decision making problem with conflicting preferences, generally known as the bargaining problem. We have departed from the conventional approach of analyzing the bargaining problem as the situation of dividing a cake which deals with Transferable Utility function. Instead our model permits players first to choose their type (risk limit) by demanding the outcome in the Pareto set  $P(S)$ . Then, they proceed with the concession game in the event of incompatible demands. The result of which have made two contributions:

(1) The model facilitates a simple characterization of the variable that determines players' behaviors. Hence it provides interesting tool for those who are interested in conflict management.

(2) The model characterizes the class of non-cooperative games that implements the Nash solution. Hence, it establishes the relationship between the static axiomatic theory of bargaining and the sequential strategic approach to bargaining.

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