Anti-Limit Pricing

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Abstract

Extending Milgrom and Roberts (1982) we present an infinite horizon entry model, where the incumbent(s) may use the current price to signal its strength to deter entry. We show that, due to the importance of entrants' types on the post-entry duopoly/oligopoly profits, the incumbent(s) may want to signal its weakness to invite entry of weaker firms. (*JEL* D42, D43, D82, L11)

Since the classic paper by Bain (1949) a large literature has developed on limit pricing and its impact on entry deterrence. In a two-period dynamic model Milgrom and Roberts (1982) show that the incumbent firm lowers the price in the first period below its one-period profit maximizing level in order to signal its strength and deter entry. Matthews and Mirman (1983) illustrate the possibility of limit pricing in a model where only the incumbent knows the profitability of the industry and this information is stochastically transmitted to the potential entrant through the market price. Harrington (1986) shows that an incumbent may distort its monopoly pricing upwards to deter entry[1] if the entrant does not know its own cost and it is sufficiently strongly correlated with the incumbent's cost.[2] Harrington (1987) and Bagwell and Ramey (1991) extended the analysis to allow for multiple incumbents. Extending his earlier paper Harrington (1987) shows that uncoordinated entry deterrence takes place when the entrant only observes the

[1] Anti-limit pricing refers to the same phenomenon but for the opposite reason, i.e., to promote entry.

[2] If the correlation is weak, the standard limit pricing obtains.

market price. In contrast, Bagwell and Ramey (1991) show that no distortion is the only robust equilibrium if the entrant can observe the individual choice of the incumbents rather than the aggregated outcome.

In these studies the analysis was conducted invariably in a two-period context.[3] It is implicitly left to the readers to interpret the two-period model as a reflection of more complicated dynamic competition in a longer horizon real world. One of our goals in this note is to make this implicit understanding more explicit. In all these models the incumbents want to deter entry if possible by signaling their strength or the unprofitability of the industry. Extending the two-period model of Milgrom and Roberts (1982) to an infinite horizon, we discover a completely opposite motive: The incumbents may want to invite entry of weak firms by signaling their own weakness/inefficiency via lower production than they would otherwise.

Since entry decision is not altered as long as the signaling separates the incumbents' types, this motive has negative social welfare implications due to lower consumption at a higher price. This is in contrast to the conventional limit pricing phenomenon that enhances the welfare because the incumbents want to signal their strength via higher production (hence lower price) than they would otherwise. Although informational issues make the nature of our analysis quite different from Bernheim (1984), our findings share the same spirit as his in emphasizing the caution that the incumbents may take fundamentally different deterrence decisions when faced with recurrent entry threats, rather than a single threat, potentially leading to entirely different policy implications.[4]

In the model of Milgrom and Roberts (1982) an incumbent firm and a potential entrant know their own cost but not the opponent's. In the first period the incumbent chooses a price (or equivalently, an output) as a monopolist. Since the incumbent's choice depends on its cost, the potential entrant can infer the incumbent's cost from the observed price, and make an entry decision accordingly. Anticipating this behavior the incumbent lowers the price in order to signal its strength and deter entry. If it succeeds in deterring entry, the reduction of the profit in the first period would be more than compensated by the monopoly profit in the second period. In an

[3]The only exception we know (in limit pricing literature) is Harrington (1984) who provided an infinite-horizon extension of the result reported in Harrington (1986).

[4]Specifically, he shows that policies making deterrence activity more costly, may have a perverse effect of discouraging entry by lowering the value of entry due to more entries anticipated in the future.

infinite horizon model with a new entry threat each period, the incumbent cannot resort to the monopoly price in the second period after it succeeded in deterring entry, because the second period is no different from the first.

Hence, one might think that there may be no incentive to lower the price in the first place if the incumbent has to maintain the low price, because the initial loss may never be compensated. One should realize, however, that the compensation comes not from the incumbent's profit being equal to the monopoly profit after deterring entry, but from it being higher than what would have been if there was an entry. As long as the latter is the case, a limit price plays dual roles in an infinite horizon, namely, that of signaling strength, and that of compensating for the signaling loss by delaying a lower, post-entry profit. It seems then that the two-period model can be regarded as a reduced form of an infinite-horizon model where the monopoly profit in the second period approximates the accumulated profit from continuing the limit pricing in non-initial periods. We show that this is not necessarily the case.

In our model, a new potential entrant arrives in each period with a private type. The incumbent may continue to exercise limit pricing every period to deter weak entrants from entering, but it cannot block entry in case the arrived entrant is a strong type for the first time. Hence, limit pricing delays entry but eventually there will be an entry by a strong firm, after which the incumbent will have a small duopoly profit because the competitor is strong. Alternatively, if the incumbent appears as weak by pricing high, even a weak entrant will enter, after which the incumbent's duopoly profit would be larger than that after a strong entry. This alternative behavior would mean an earlier entry by a weaker entrant on average. If i) a strong entrant arrives with a sufficiently high probability so that the expected delay of entry by limit pricing is not significant, and ii) the incumbent is sufficiently patient so that the post-entry profit is important, then it pays for the incumbent to signal its weakness by pricing high and invite weak entrants in, for the sake of enhancing post-entry profits. We refer to this phenomenon as *anti-limit pricing.*

The incumbent's motive behind anti-limit pricing is clear: Cost-effective measures will be taken that enhance the chances of having a weak competitor in the market rather than a strong one. This motive is casually observed in action. For example, Rockett (1990) reports that Du Pont licensed its polyester, cellophane and nylon patents selectively to weaker potential competitors shortly before the patents expired,[5] and Comanor (1964) cites similar motives of pharmaceutical firms.

Section I presents an infinite-horizon model and an equilibrium concept. Section II focuses on environments where no more than one entry is viable and characterizes when the anti-limit pricing arises and when the standard limit pricing does. Section III demonstrates that the phenomenon of anti-limit pricing extends to environments in which more than one entry is viable. Appendix contains some deferred technical details.

I. The Model

Our model is an extension of Milgrom and Roberts (1982) to an infinite horizon. A monopoly firm produces a certain product from period 1 onwards. At each future period $t = 2, 3, \cdots$, one potential entrant arrives at the market, observes the number of firms in the market and industry output of the last period, and decide whether to enter or not. If it does not enter, it leaves the market forever. In every period the market demand is the same. A single parameter c_i (marginal cost) describes the characteristic of firm i, $i = 1$ (incumbent) or 2_t (entrant at period t), which is private information. We assume that c_i's are drawn independently from a common distribution F on a closed interval $[\underline{c}, \bar{c}]$.[6] If entry occurs, the entrant pays a certain entry/setup cost C, and the characteristics of the firms in the market become common knowledge among them. From then on the two firms earn the Cournot duopoly profits until a third firm enters. The entry process stops when there are already enough number of firms in the market and further entry is unprofitable.[7] This description of the game is common knowledge. In this section we define separating equilibrium presuming that once the market reaches a duopoly no further entry is profitable.

[5]Rockett (1990) also justifies such a licensing behavior as optimal under certain circumstances. Her model is one of complete information, hence there is no scope of signaling which is the central issue of our paper. Similarly, Ashiya (2000) shows that if an incumbent in a differentiated market knows that a weak and a strong potential entrants arrive in that order, the incumbent may induce the weak firm in to deter entry by the second, strong firm, again in a complete information setting.

[6]It is not necessary that the incumbent's marginal cost have the same distribution as the entrants' nor that the entrants' marginal costs have identical distributions. We adopt this assumption for simplicity. In the examples, we consider two point distributions.

[7]No exit occurs in our model because there is no fixed cost other than the entry cost which is sunk.

Let $\pi(q; c_1)$ denote the monopoly profit of an incumbent with marginal cost c_1 when it produces q units of output and sell them at the market price; Let $\pi_i^D(c_1, c_2)$ denote the (one-period) Cournot equilibrium duopoly profit of an incumbent ($i = 1$) with a marginal cost c_1 and that of an entrant ($i = 2$) with a marginal cost c_2. Note that we use the subscript 2 as a shorthand for 2_t for simplicity, i.e., π_2^D for $\pi_{2_t}^D$ and c_2 for c_{2_t}. In period $t = 1$ the incumbent chooses an output level q_1 and earns a monopoly profit $\pi(q_1; c_1)$. In the next period, an entrant arrives and knowing q_1 decides whether to enter the market or not. If entry occurs the two firms earn duopoly profits $\pi_1^D(c_1, c_2)$ and $\pi_2^D(c_1, c_2)$ from that period onwards for there will be no further entry. If entry does not occur the incumbent maintains its monopoly position, and the continuation game is the same as the original game. We formalize "stationary" equilibrium in which the arrived entrants, regardless of period, make the same inference on the incumbent's type from the previous period's output, and the incumbent chooses the same output level as long as entry did not occur.[8]

Suppose that the entrants conjecture that the incumbent uses strategy $s(\cdot)$, where $s(\cdot)$ is a one-to-one function from $[\underline{c}, \bar{c}]$ to R_+. A strategy of an entrant of type c_2 is a function $e(\cdot; c_2) : \mathsf{R}_+ \to \{0, 1\}$, where $e(q; c_2) = 1$ means "entry" and $e(q; c_2) = 0$ "no entry". Observing the incumbent's output $q \in s([\underline{c}, \bar{c}])$ in the previous period, an entrant will enter only if it expects a profit stream whose present value covers the entry cost. Define $\hat{c}_2(q)$ to be the maximum of the entrant's types (marginal cost) that expect this to be the case against an incumbent whose type is inferred from q. After observing the incumbent's output q an entrant will enter if and only if $c_2 \leq \hat{c}_2(q)$. Then, the incumbent's value (i.e., the discounted sum of the expected profit stream) when its type is c_1 and it produces q, $\Pi(q; c_1)$, satisfies

$$\Pi(q; c_1) = \pi(q; c_1) + \delta \int_{c_2 \leq \hat{c}_2(q)} V_{c_1}(c_2)\, dF + \delta(1 - F(\hat{c}_2(q)))\Pi(q; c_1) \qquad (1)$$

[8] The incumbent's choice of the same output is a result of dynamic optimizing behavior, not an assumption. We believe this is the most sensible equilibrium given that the potential entrants the incumbent faces are indistinguishable across periods. If the entrants use the entire history of output levels (rather than only the last period's) in forming the belief about the incumbent, the incumbent may need to distort its output level only in the initial period for effective signaling, and produce its myopic monopoly output afterwards until an entry occurs. The examples of anti-limit pricing in Sections II and III are robust to this generalization because any signal distorted from the myopic optimum happens for only one period in those equilibria.

where $V_{c_1}(c_2) = \pi_1^D(c_1, c_2)/(1-\delta)$ is the value of a c_1-type incumbent after an entry by a c_2-type firm, and δ is the common discount factor. From (**??**) we obtain

$$\Pi(q; c_1) = \frac{\pi(q; c_1) + \delta \int_{c_2 \leq \hat{c}_2(q)} V_{c_1}(c_2)\, dF}{1 - \delta + \delta F(\hat{c}_2(q))}. \tag{2}$$

We now define a separating equilibrium when no more than one entry is viable. A more general definition will be given in the Appendix.

DEFINITION 1: A strategy profile $(s(\cdot), e(\cdot))$ is a *separating equilibrium* if

1) $s : [\underline{c}, \bar{c}] \to \mathsf{R}_+$ is a one-to-one function and satisfies $s(c_1) \in \arg\max_q \Pi(q; c_1)$,

2) $e(q; c_2) = 1$ if and only if $c_2 \leq \hat{c}_2(q)$ for $q \in s([\underline{c}, \bar{c}])$.[9]

II. When Only One Entry Is Viable

In this section we focus on environments in which no more than one entry is viable. We first construct an example of such environment in which anti-limit pricing is the only sensible outcome: Specifically, in equilibrium a strong incumbent produces its myopic monopoly output whereas a weak incumbent distorts its monopoly output below (hence, price above) its myopic optimal level to deter imitation by the strong counterpart. Then, we explain when the standard limit pricing takes place and when the anti-limit pricing arises.

We consider the environments with linear market demands, normalized as $p(q) = 1 - q$. Both the incumbent and entrants may have one of two possible types/marginal costs, $c_i = 0$ or $c > 0$. Each firm has a low cost ($c_i = 0$) with probability $m = 0.8$. The discount factor is set at $\delta = 0.95$. We also refer firms with a low cost ($c_i = 0$) as "strong" or "good", and firms with a high cost ($c_i = c$) as "weak" or "bad." The entry cost C is such that it is profitable for a strong entrant to enter against both types of incumbent, but for a weak entrant it is profitable only against a weak incumbent. This is the case for some level of entry cost if and only if $0 < c < 0.1$. For concreteness we set $c = 0.05$ in this section, but the analysis is qualitatively the same for any $0 < c < 0.1$. For $c = 0.05$, the entry conditions are satisfied if C is

[9] As is typical in signaling games, an equilibrium can be supported by various beliefs on off-equilibrium paths, i.e., for $q \notin s([\underline{c}, \bar{c}])$. We will specify sensible off-equilibrium beliefs for the equilibria we analyze later.

between $\frac{9}{5}$ and $\frac{361}{180}$.[10] Notice that once an entry occurs no further entry is profitable, since the entering firm's gross profit would be $\frac{121}{80} < \frac{9}{5}$, even when it is strong and both incumbents are weak.

We construct a separating equilibrium with the following properties. (1) A strong incumbent produces its single period monopoly output $q_s = 0.5$. (2) A weak incumbent produces q_w which is less than its single period monopoly output level, 0.475. (3) In each period t if the output level in the previous period is greater than q_w, the entrant infers that the incumbent is strong and enters only if it is also a strong type; otherwise it infers that the incumbent is weak and enters regardless of its own type. In this equilibrium the weak incumbent sacrifices some monopoly profit in order to signal that its cost is high and hence to induce even a weak firm to enter. Since its strong counterpart has the same motive (as verified below), the only way for a weak incumbent to credibly signal its type is by reducing the output (increasing the price) to the extent that a strong incumbent would find it too costly to mimic. Hence, we will observe a price higher than the myopic monopoly price, which we call an anti-limit price.

In constructing the equilibrium we select q_w at a level where the incentive constraint is binding, i.e., a strong incumbent is indifferent between (a) producing its optimal monopoly output level, q_s, and regarded as a strong type and (b) producing q_w and regarded as a weak type. This will give us a separating equilibrium with minimum sacrifice by the weak incumbent. Given the entrant's strategy (3) above, if the incumbent produces $q_s(> q_w)$, then entry occurs only if the entrant is strong and the post-entry value of the incumbent is $V_0(0) = \frac{\pi_1^D(0,0)}{1-\delta} = 2.222\ (= \frac{20}{9})$. Hence, the equilibrium value of a strong incumbent, $\Pi(q_s; 0)$, satisfies $\Pi(q_s; 0) = \pi(q_s, 0) + \delta(mV_0(0) + (1-m)\Pi(q_s; 0))$, i.e., $\Pi(q_s; 0) = (\pi(q_s, 0) + \delta m V_0(0))/(1 - \delta + \delta m)$. If this incumbent produces q_w instead, an entry occurs for sure and the value of the incumbent is $\Pi(q_w; 0) = \pi(q_w, 0) + \delta(mV_0(0) + (1-m)V_0(0.05))$. Solving the equation $\Pi(q_w; 0) = \Pi(q_s; 0)$ for q_w, we have $q_w = 0.397$ or 0.603.[11] We select

[10] The expected profit (gross of entry cost) of the weak entrant against a strong incumbent and that against a weak incumbent are

$$\frac{\pi_2^D(0, 0.05)}{1-\delta} = \frac{9}{5} \text{ and } \frac{\pi_2^D(0.05, 0.05)}{1-\delta} = \frac{361}{180}, \text{ respectively.}$$

The expected profit of a strong entrant exceeds $\frac{361}{180}$ because $\pi_2^D(0,0) > \pi_2^D(0.05, 0.05)$.

[11] All numerical values were calculated using *Mathematica* and are reported here as the three-digit approximations below the decimal point, unless more precise figures are needed

$q_w = 0.397$ because it needs to be lower than 0.475.

Then, it is obvious that it is optimal for a strong incumbent to produce q_s. It is also optimal for a weak incumbent to produce q_w and obtain an equilibrium value $\Pi(q_w; 0.05) = \pi(q_w, 0.05) + \delta(mV_{0.05}(0) + (1-m)V_{0.05}(0.05)) = 1.969$: producing less does not help because it only reduces the current profit, and the best deviation to a larger output is the myopic monopoly level of 0.475, the value of which is calculated as 1.967.[12] Thus, we have constructed an equilibrium with anti-limit pricing: (i) $s(0) = 0.5$ and $s(0.05) = 0.397$; (ii) $e(q; c_2) = 0$ if $c_2 = 0.05$ and $q > q_w$, $e(q; c_2) = 1$ otherwise; and (iii) upon arrival each entrant infers that $c_1 = 0$ if $q_{t-1} > q_w$, and $c_1 = 0.05$ otherwise. One can also check that this equilibrium is robust to the refinement criterion of Cho and Kreps (1987).[13] Furthermore, it is the unique such equilibrium in pure strategies if $\frac{1657}{900} < C < \frac{361}{180}$.[14]

Why does the anti-limit pricing occur in the considered environment? In a separating equilibrium, a strong incumbent faces entry by only a strong entrant. If it behaves as if a weak incumbent instead, it will face entry by both types of entrant. This mimicking behavior has three effects on the incumbent's profit stream: it reduces its monopoly profit; it shifts weight from the monopoly profit to post-entry, duopoly profits; and it increases the duopoly profits on average. The first two effects are clearly negative, while the third effect is positive. If duopoly profits carry sufficiently more weight than the monopoly profits in the expected stream of profits, then the positive effect dominates and a strong incumbent would imitate a weak incumbent's behavior. Then, a weak incumbent may reduce its production below the optimal monopoly level (i.e., increase its price above the optimal monopoly level) to remove the incentives for such imitation, resulting in anti-limit pricing. Note that this phenomenon does not arise in two-period settings because then the duopoly profit, which will take place at most for one period, does not carry more weight than the monopoly profit for an

for comparison. The precise solution values are $0.396565\cdots$ and $0.603435\cdots$.

[12]This last value is the value of Π that solves $\Pi = \pi(0.475, 0.05) + \delta(mV_{0.05}(0) + (1-m)\Pi)$.

[13]To be precise, it is so subject to the firms playing the Cournot equilibrium and given this, the entrants using the dominant strategy, if exists, in entry decisions.

[14]If $C > \frac{1657}{900}$, a weak firm would not enter given the prior distribution of the incumbent's type, hence no pooling may be robust to this refinement. For these values of C, all other equilibria robust to this refinement are inessential variations of this equilibrium and exhibit anti-limit pricing: A strong incumbent mixes between producing q_s and q_w, the latter with sufficiently small a probability so that a weak entrant still enters after observing q_w.

incumbent.

Several conditions are important for anti-limit pricing to arise. First, the incumbent should put a sufficient weight on post-entry profits, either because the expected duration of the market is long enough[15] or because new entry threats arise with a high frequency. In our model, this means δ is high. Second, the entrant is strong with a sufficiently high probability (i.e., m is high), so that the effect of delaying entry (of a weak firm) by the conventional limit pricing is short-lived. Third, weak and strong entrants are sufficiently differentiated so that the duopoly profit levels differ significantly depending on the entrant's type. This means that the marginal cost, c, of a weak firm is not too close to that of a strong firm, which is normalized to 0 in our model.

Given the set of possible types (in our example, $\{0, 0.05\}$) one can find the set of parameters (m, δ) for which anti-limit pricing arises in equilibrium, and the set of parameters for which the standard limit pricing arises. In Figure 1 the former set of parameters is represented by the dark area, and the latter set by the lightly shaded area. The standard limit pricing arises for a wide range of parameter values for δ not near 0 or 1 and for m not close to 1, whereas the anti-limit pricing arises when both δ and m are quite large. As c increases from 0 to 0.1, a range for which the equilibrium considered in this section is possible as mentioned earlier, the area for anti-limit pricing tends to get larger, as depicted in Figure 2. We explain how this area is determined in a proof of the next Proposition which says that anti-limit pricing arises for some parameter values so long as first entry by a weak firm can be profitable but no second entry is.

PROPOSITION: *Given a market demand $p(q) = 1 - q$, the following holds if and only if $0 < c < 0.1$: There is a nonempty open set of $(C, m, \delta) \in \mathsf{R}_+ \times (0,1) \times (0,1)$ such that if (C, m, δ) is in this set, no more than one entry is profitable and anti-limit pricing arises in an equilibrium.*

PROOF: See the Appendix.

[Figures 1 and 2 about here]

[15] Anti-limit pricing may arise in long enough, yet finite horizon models. Our search for a 3-period example was unsuccessful, though. Hence, we opted to present it in an infinite horizon model which may be simpler to analyze conceptually.

III. Extension To More Than One Viable Entries

The example in the previous section illustrates the essential forces behind anti-limit pricing in a simple setting: If entry is bound to occur eventually and the post-entry profit is important, the incumbent may prefer to have weaker firms enter because the higher post-entry profits from having a weaker competitor overcompensates the loss from allowing an entry early. Clearly, this effect is not limited to the environments that allow only one entry. In this section we demonstrate that anti-limit pricing is a phenomenon that can happen in a wider class of environments. Specifically, we provide an environment in which up to two entries are viable and anti-limit pricing arises both when there is a single incumbent in the market and when two firms operate in the market after the first entry. We are confident that anti-limit pricing can happen when more entries are viable, although it is an open question whether it is compatible with arbitrarily many viable entries. The problem becomes more complex as the number of viable entries increases because the cases to consider increases exponentially, letting alone the additional issues that arise when multiple firms operate in the market, such as joint signaling and bargaining among them.

As before, the inverse demand function is normalized as $p(q) = 1 - q$. For minimum change, we maintain that the firms may have the same two possible types, $c_i = 0$ or 0.05, and the probability of being a low cost type is $m = 0.8$. The discount factor is changed to $\delta = 0.972$, and the entry cost is set at $C = 1.89$. The separating equilibrium in this environment has the following properties. (1) A strong incumbent produces its myopic monopoly output level $q_s = 0.5$, whereas a weak incumbent produces at a level q_w below its monopoly output to deter imitation by the strong counterpart. (2) Consequently, an entrant of either type enters if the single incumbent's production last period was q_w or lower, but only a strong type enters otherwise. (3) When there are two firms in the market, they produce the Cournot (equilibrium) outputs unless both firms are weak; if both firms are weak they produce less than their Cournot outputs to deter imitation by different pairs of firms. (4) Consequently, an entrant of either type enters if the two firms' total output last period was that of two weak firms' (or lower), whereas only a strong type enters otherwise. (5) If there are three firms in the market they produce the Cournot outputs, and no additional entry takes place.

In section I we defined the separating equilibrium for the cases that no more than one entry is viable. Since the extension to general cases is intuitively straightforward (formalized in the Appendix), for smooth flow of discussion we continue with verifying the equilibrium described above.

We start with the cases that three firms operate in the market. For the case that all three are strong firms, we calculate from the standard Cournot game[16] that the market price is $\frac{1}{4}$ and each firm produces $\frac{1}{4}$ and earns a profit of $\frac{1}{16}$. Since there will be no further entry (as verified below), the value of each firm when there are three strong firms in the market, is $V_s(sss) = \frac{1}{16(1-\delta)} = 2.232$. (Here and below, the argument of V designates the types of the firms in the market and the subscript to V is the type of the firm whose value it denotes.) Likewise, if there are two strong firms and one weak firm in the market, the value of a strong firm and that of a weak firm are $V_s(ssw) = 2.461$ and $V_w(ssw) = 1.613$, respectively; If there are one strong and two weak firms, they are $V_s(sww) = 2.701$ and $V_w(sww) = 1.808$, respectively; If all three firms are weak, the value of each firm is $V_w(www) = 2.015$. To verify that no further entry will occur, note that the value of a strong firm when it enters a market with three weak firms and no further entry ensues, is calculated as 1.889, which does not cover the entry cost $C = 1.89$. Therefore, from the values calculated above, it follows that a strong entrant will always enter if two firms exist in the market regardless of their types, but a weak entrant would enter only when both existing firms are weak, because those are the cases in which the value of the entrant exceeds the entry cost.

Next, to check the incentives of the entrant when a single incumbent exists in the market, we calculate the values of firms when two firms operate in the market. If the two firms are both strong, each gets a Cournot profit of 1/9 from selling 1/3 unit at a price 1/3. Since entry will occur in the next period only if a strong entrant arrives, each firm's value is calculated as

$$V_s(ss) = \frac{1/9 + \delta m V_s(sss)}{1 - \delta + \delta m} = 2.292.$$

If one firm is strong and the other is weak, the strong firm gets a Cournot profit of 0.123 from selling 0.35 unit at a price 0.35, and the weak firm gets a profit of 0.09 from selling 0.3 unit. Since only a strong entrant will enter in next period, the respective firm's values are calculated analogously as

$$V_s(sw) = 2.527 \qquad \text{and} \qquad V_w(sw) = 1.668.$$

If the two firms are both weak, both types of entrant will enter next period in the equilibrium. If the two firms play the Cournot outcome in the

[16]See, for example, Shy (1996, p.126) for solutions of the standard Cournot game.

current period, each would get a profit of 0.1 from selling 0.317 unit at a price 0.367, and therefore, each firm's value would be

$$V_w'(ww) = 0.1 + \delta(mV_w(sww) + (1-m)V_w(www)) = 1.898.$$

However, this behavior of two weak firms cannot be sustained in equilibrium because it invites deviation when there are one strong and one weak firm in the market: If the strong firm produces 0.334 unit instead of its equilibrium level (0.35), the total output will be the same as that of two weak firms, hence both types of entrant will enter in the next period. Deviating in this way,[17] the strong firm suffers a loss in the current period, but this loss is more than compensated by inducing a weak entrant in the next period. The value of this deviation is 2.561 while the equilibrium value is 2.527. To remove incentives for such a deviation, in equilibrium the two weak firms reduce total production to 0.466[18]: Each firm produces a half of this, inducing entry by both types in the next period, thereby obtaining a value of $V_w(ww) = 1.91$. Then, the types of the two firms in the market are correctly inferred from their total output level, q_2, hence the third firm that arrives at the market enters only when $q_2 \leq 0.466$ if a weak type, while it enters regardless of q_2 if a strong type. It is straightforward to verify that the behavior by two firms of various types described above is immune to deviations.[19] From the various values of the two firms in the market calculated above, it is clear that a strong entrant will always enter if a single incumbent exist in the market regardless of the incumbent's type, but a weak entrant would enter only when the incumbent is weak, because those are the cases in which the value of the entrant exceeds the entry cost.

Finally, we determine the equilibrium output levels of the initial incumbent. If the incumbent is a strong firm, it obtains a monopoly profit of 0.25 from selling 0.5 unit at the market price 0.5. Since only a strong entrant will

[17]Here, we only consider unilateral deviation. It is conceivable that the two firms in the market may coordinate in deviation, but this approach raises an additional issue of bargaining between them, which is not central to our paper.

[18]To be precise, the maximum total output that prevents imitation is $0.4663847\cdots$.

[19]There is some ambiguity as to how the two weak firms behave subsequently if one of them deviated by increasing its production in some period and no entry ensued (because the arrived entrant was a weak firm). It is routinely calculated that this deviation is not worthwhile even under the most optimistic presumption that the deviator keeps on producing the optimal output level against the other firm who sticks to the equilibrium level (0.466/2) until a third firm enters: the value of this deviation is 1.905.

enter in the next period, the value of a strong incumbent is calculated as

$$V_s(s) = \frac{1/4 + \delta m V_s(ss)}{1 - \delta + \delta m} = 2.5231.$$

If the incumbent is weak, both types of entrant will enter next period in the equilibrium. If the incumbent produces the myopic monopoly output in the current period, it would get a profit of 0.226 from selling 0.475 unit at a price 0.525, thereby obtaining a value of

$$V_w'(w) = 0.226 + \delta\left(mV_w(sw) + (1-m)V_w(ww)\right) = 1.89433.$$

However, this behavior of weak incumbent cannot be sustained in equilibrium because it would be imitated by a strong incumbent: By producing 0.475 unit instead of 0.5, a strong incumbent would masquerade as a weak one and thereby inducing both types of entrant to enter in the next period and consequently, obtaining a value of 2.5234 that exceeds the equilibrium value of 2.5231. Anticipation of this deviation would deter a weak entrant from entering when the incumbent produces 0.475 unit, which in turn would reduce the value of the weak incumbent to 1.890. To remove incentives for such imitation, the weak incumbent reduces its production to 0.471,[20] thereby obtaining a value of $V_w(w) = 1.89431$. Then, the incumbent's type is correctly inferred from its output level, q_1, hence the entrant will enter only when $q_1 \leq 0.471$ if a weak type, while it will enter regardless of q_1 if a strong type. Given this entry strategy, neither type of incumbent has incentive to change its output level. This completes description of the desired separating equilibrium.

Appendix

A. PROOF OF PROPOSITION.

The best possible profit stream for a third firm in the market is having the Cournot profit forever when it is strong and the other two are weak. This Cournot profit is easily calculated as $\frac{1}{16}(1+2c)^2$. If C is not covered by a stream of this profit, no second entry is profitable.

[20] To be precise, the maximum output that prevents imitation is $0.470983\cdots$.

The Cournot duopoly profit of the entrant is $\pi_2^D(c_1, c_2) = \frac{1}{9}(1 + c_1 - 2c_2)^2$ where c_1 and c_2 are the types of the incumbent and entrant, respectively. If

$$\pi_2^D(0, c) < (1 - \delta)C < \pi_2^D(c, c) \quad \text{and} \quad (1 + 2c)^2/16 < (1 - \delta)C, \tag{3}$$

then it is profitable for a weak entrant to enter against a weak incumbent but not against a strong one, and no second entry is profitable. It is clear in this case that a strong entrant would enter regardless of the incumbent's type because $\pi_2^D(c, c) < \pi_2^D(0, 0) < \pi_2^D(c, 0)$. Condition (3) is rewritten as

$$\max\Big\{\Big(\frac{1+2c}{4}\Big)^2, \Big(\frac{1-2c}{3}\Big)^2\Big\} < (1 - \delta)C < \Big(\frac{1-c}{3}\Big)^2, \tag{4}$$

which holds for a nonempty interval of C if and only if $3(1 + 2c) < 4(1 - c)$, i.e., $0 < c < 0.1$.

Supposing $0 < c < 0.1$ and C satisfying (4), we now determine the area of (m, δ) for which the anti-limit pricing arises in a separating equilibrium. In this equilibrium, a weak incumbent expects entry by either type of entrant (hence with probability $\mu = 1$) and a strong incumbent expects entry only by a strong type (hence with probability $\mu = m$). With a slight abuse of notation, let $\mu = m$ and $\mu = 1$ also denote the prospects that only a strong type enters and either type of entrant enters, respectively, in each period until an entry actually occurs. Then, for each entry prospect $\mu = m, 1$, the expected post-entry value of an incumbent of type c_1 is

$$\widehat{V}_{c_1}(m) = \pi_1^D(c_1, 0)/(1-\delta) \text{ and } \widehat{V}_{c_1}(1) = (m\pi_1^D(c_1, 0) + (1-m)\pi_1^D(c_1, c))/(1-\delta).$$

We further define the expected profit of this incumbent when it produces q currently and faces an entry prospect $\mu = m, 1$, as

$$\widehat{\Pi}(q, m; c_1) = \frac{\pi(q; c_1) + \delta m \widehat{V}_{c_1}(m)}{1 - \delta + \delta m} \text{ and } \widehat{\Pi}(q, 1; c_1) = \pi(q; c_1) + \delta \widehat{V}_{c_1}(1).$$

Let $q_s^M = 0.5$ and $q_w^M = \frac{1-c}{2}$ denote the myopic monopoly outputs of a strong and a weak incumbent, respectively. An equilibrium with anti-limit pricing can be constructed if the following properties hold. (a) A strong incumbent prefers the output-entry prospect pair $(q, \mu) = (q_w^M, 1)$ to (q_s^M, m) and (q_s^M, m) to $(0, 1)$, so that there exists an output level q_w between 0 and q_w^M such that the strong incumbent is indifferent between $(q_w, 1)$ and (q_s^M, m). (b) A weak incumbent prefers $(q_w, 1)$ to (q_w^M, m), the best it can expect under deviation.

The first condition in (a) can be written as follows:

$$\begin{aligned}
[\widehat{\Pi}(q_w^M, 1; 0) - \widehat{\Pi}(q_s^M, m; 0)] &= -\frac{c^2}{4} + \frac{\delta m(1-m)}{(1-\delta+\delta m)}\frac{c(2+c)}{9(1-\delta)} \\
&\quad -\frac{\delta(1-m)}{1-\delta+\delta m}\frac{(5-4c(2+c)(1-m))}{36} \\
&> 0. \qquad (5)
\end{aligned}$$

The first and third terms of the RHS are negative (note that $c < 0.1$) and the second term is positive. The whole expression is positive when $1-\delta$ is small relative to δm. The second condition in (a) can be written as follows:

$$\begin{aligned}
[\widehat{\Pi}(q_s^M, m; 0) - \widehat{\Pi}(0, 1; 0)] &= \frac{1}{4} - \frac{\delta m(1-m)}{(1-\delta+\delta m)}\frac{c(2+c)}{9(1-\delta)} \\
&\quad +\frac{\delta(1-m)}{1-\delta+\delta m}\frac{(5-4c(2+c)(1-m))}{36} \\
&> 0. \qquad (6)
\end{aligned}$$

The first and third terms of the RHS are positive and the second term is negative. The whole expression is positive when $1-\delta$ is not too small relative to δm. The two conditions in (a) are satisfied when δ is sufficiently large but not too close to 1. Condition (b) can be written as follows:

$$\begin{aligned}
[\widehat{\Pi}(q_w, 1; c) - \widehat{\Pi}(q_w^M, m; c)] &= -\frac{(1-c-2q_w)^2}{4} + \frac{\delta m(1-m)}{(1-\delta+\delta m)}\frac{c(2-3c)}{9(1-\delta)} \\
&\quad -\frac{\delta(1-m)}{1-\delta+\delta m}\frac{5(1-c)^2+4cm(2-3c)}{36} \\
&> 0, \qquad (7)
\end{aligned}$$

where q_w is the smaller of the two solution values of q to the quadratic equation $\widehat{\Pi}(q, 1; 0) = \widehat{\Pi}(q_s^M, m; 0)$: since $\pi(q; 0) = (1-q)q$ and $\widehat{V}_0(1)$ and $\widehat{\Pi}(0.5, m; 0)$ are independent of q, we have

$$q_w = \frac{1 - \sqrt{1 + 4\delta\widehat{V}_0(1) - 4\widehat{\Pi}(0.5, m; 0)}}{2}.$$

The first and the third terms of the RHS of (7) are negative, whereas the second is positive. The whole expression is positive when $1-\delta$ is small relative to δm. This requires both δ and m to be large. The dark area of

Figure 1 is the area that satisfies all three inequalities of conditions (a) and (b) when $c = 0.05$, and Figure 2 represents it for other values of c.

Finally, we show that the three inequalities are satisfied strictly for some pair (m, δ), hence for an open set of pairs, for any $0 < c < 0.1$. Fixing such a c, note that the LHS of the inequality (5) converges to $-c^2/4 + c(2 + c)/9 = c(8 - 5c)/36 > 0$ as $m = \delta$ tends to 1 from below, and that of (6) converges to $1/4 - c(2 + c)/9 > 0$. For (7) we first note that q_w converges to $1/2 - \sqrt{2c + c^2}/3$ as $m = \delta$ tends to 1 because $\delta\widehat{V}_0(1) - \widehat{\Pi}(0.5, m; 0)$ converges to $\pi_1^D(0, c) - \pi_1^D(0, 0) - \pi(0.5; 0) = (2c + c^2)/9 - 1/4$. Hence, the LHS of (7) converges to $c^2(12\sqrt{1 + \frac{2}{c}} - 25)/36 > 0$, where the strict inequality follows because $12\sqrt{1 + \frac{2}{c}}$ decreases in c and assumes 54.99 when $c = 0.1$. For any $0 < c < 0.1$, therefore, the inequalities (5)–(7) hold when m and δ are the same value sufficiently close to 1, hence for an open set of (m, δ) by continuity. Since C can be chosen to satisfy (4) for such (m, δ), the proof is complete.

B. GENERAL DEFINITION OF SEPARATING EQUILIBRIUM

We extend the definition of separating equilibrium defined in Section I to the cases that more than one entry is viable. In any period there will be a certain number of incumbents, denoted by I. The industry in each period is characterized by a type profile of the incumbents in that period, denoted by $\mathbf{c} \in [\underline{c}, \bar{c}]^I$, an array of the marginal costs of the incumbent firms in the increasing order. We write $c_j \in \mathbf{c}$ if c_j is the j-th coordinate of $\mathbf{c}$. We also write $\mathbf{c} = (c_j, \mathbf{c}_{-j})$ and $|\mathbf{c}| = I$.

A production strategy of an incumbent in some period when there are I incumbents is a function $s(\cdot; \cdot) : [\underline{c}, \bar{c}] \times [\underline{c}, \bar{c}]^{I-1} \to \mathsf{R}_+$ where $s(c_j; \mathbf{c}_{-j})$ is the output when its type is c_j and the type profile of all other incumbents is $\mathbf{c}_{-j}$. A belief profile is a function β on the number of incumbents I and their total output q such that $\beta(I, q)$ is a measure on possible profiles of I incumbents. An entry strategy of a potential entrant of type c arriving in any period with I incumbents is a function $e(\cdot; c, I) : \mathsf{R}_+ \to \{0, 1\}$, where $e(q; c, I) = 1$ (0) means "entry" ("no entry") if q was the last period's industry output.

Observing the industry output of the previous period, an entrant makes an inference on the industry profile according to β, and will enter if and only if it expects that the entry cost will be covered by its future profit stream, given the production strategy s and the entry strategy e that will prevail. Given any inference $\beta(q, I)$, one can find the maximum type of an entrant (marginal cost) for which this is the case, which we denote by $\hat{c}(q, I, \beta)$; if for no type such is the case, let $\hat{c}(q, I, \beta) = -\infty$. Then, an entrant will enter if and only if its type is $c \leq \hat{c}(q, I, \beta)$ after observing q. Let $\pi_{c_j}(q_j; \mathbf{c}, s)$ denote the oligopoly profit of an incumbent of type c_j in an industry profile $\mathbf{c} \in [\underline{c}, \bar{c}]^I$, when it produces q_j and all other incumbents produce according to a production strategy s. Then, given s, e and β, this incumbent's value when it produces q_j (i.e., the discounted sum of the expected profit stream), $\Pi_{c_j}(q_j; \mathbf{c}, s, e, \beta)$, satisfies

$$\begin{aligned} \Pi_{c_j}(q_j; \mathbf{c}, s, e, \beta) &= \pi_{c_j}(q_j; \mathbf{c}, s) + \delta \int_{c \leq \hat{c}(q, |\mathbf{c}|, \beta))} V_{c_j}(c, \mathbf{c}, s, e, \beta)\, dF \\ &\quad + \delta(1 - F(\hat{c}(q, |\mathbf{c}|, \beta)))\Pi_{c_j}(q_j; \mathbf{c}, s, e, \beta) \end{aligned}$$

where q is the industry output and $V_{c_j}(c, \mathbf{c}, s, e, \beta)$ is the value of the c_j-type incumbent after an entry by a c-type firm. From the above equation we obtain

$$\Pi_{c_j}(q_j; \mathbf{c}, s, e, \beta) = \frac{\pi_{c_j}(q_j; \mathbf{c}, s) + \delta \int_{c \leq \hat{c}(q, |\mathbf{c}|, \beta)} V_{c_j}(c, \mathbf{c}, s, e, \beta)\, dF}{1 - \delta + \delta F(\hat{c}(q, |\mathbf{c}|, \beta)}.$$

We now define a separating equilibrium in which every industry profile is fully revealed by the industry output level.[21]

DEFINITION 2: A strategy profile (s, e) and a belief profile β constitute a *separating equilibrium* if the followings hold for each $I = 1, 2, \cdots$:

1) for each $\mathbf{c} \in [\underline{c}, \bar{c}]^I$ and $c_j \in \mathbf{c}$,
$$s(c_j; \mathbf{c}_{-j}) \in \arg\max_{q_j} \Pi_{c_j}(q_j; \mathbf{c}, s, e, \beta),$$
2) $\sum_{c_j \in \mathbf{c}} s(c_j; \mathbf{c}_{-j}) \neq \sum_{c'_j \in \mathbf{c}'} s(c'_j; \mathbf{c}'_{-j})$ if $|\mathbf{c}| = |\mathbf{c}'|$ and $\mathbf{c} \neq \mathbf{c}'$,
3) $e(q; c, I) = 1$ if and only if $c \leq \hat{c}(q, I, \beta)$ for every (q, I), and
4) β is obtained from s via Bayesian-updating whenever possible.

[21]This is needed for separation of all possible incumbent configurations that may arise in equilibrium. Note that this condition is typically not satisfied in equilibrium when the support of possible types contains a continuum. Hence, the definition in this Appendix is mainly for the cases that only finite types are possible. The definition for continuous support is not used in this paper, hence is omitted.

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Figure 1

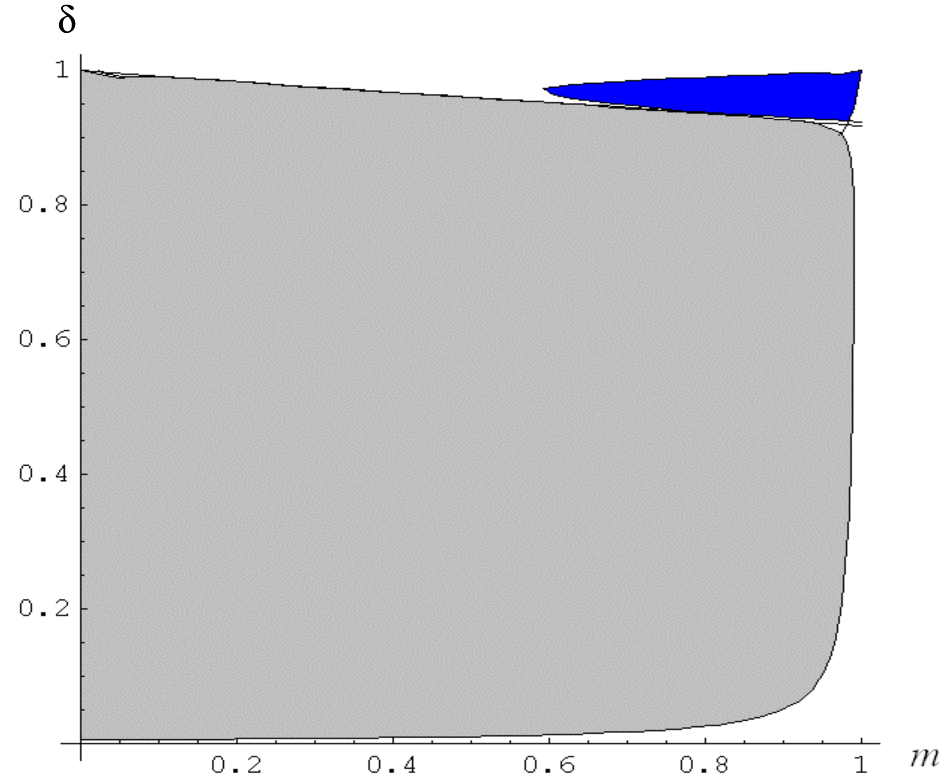

Figure 2