‘Diversity Is, Uh …, Homogeneity’: The Case of Horizontal Innovation

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The literature on ‘horizontal innovation’ claims to analyse the cases where unbounded endogenous growth comes from an increasing variety of intermediate goods. The present paper contends that a good sample of representative models in this literature share two essential assumptions regarding production technology and that these assumptions together amount to assuming the homogeneity of various intermediate goods. In these models, there is no variety of intermediate goods to increase due to the R&D activity; what increases is a mass of a single homogeneous intermediate good.

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‘[Z]ero tolerance for intellectual sloppiness’ (Paul Romer)

1. Introduction

The literature on ‘horizontal innovation’ in New Growth Theory finds the engine of unbounded endogenous growth in the increasing variety of intermediate goods which are used in the production of the final good(s). Romer (1987, 1990) is the canonical model, which has ushered in a rapidly growing number of variants. An up-to-date survey of the literature on ‘horizontal innovation’ is offered by Gancia and Zilibotti (2003). Barro and Sala-i-Martin (1995, 2004) provide a graduate textbook (and nondurable intermediate good) version of a ‘lab equipment’ variant, inspired by the model of Rivera-Batiz and Romer (1991). The ‘labour for intermediates’ variant is proposed by Young (1993) and a simplified version is found in Benassy (1998) and Heijdra and Van der Ploeg (2002). The pivotal importance of this literature in the research project of New Growth Theory as a whole is borne out by another (undergraduate) textbook: Jones (1998) takes a simplified version of the Romer (1990) model as the main model for New Growth Theory.

Against this background, the present paper calls for a critical look at this class of models. The critical look focuses on their central concept: the *variety* of intermediate goods. We shall argue that, their rhetorical emphasis notwithstanding, nothing in the models enables one to distinguish among intermediate goods: intermediate goods which, they claim, are produced in an increasing variety are in fact a mass of a homogenous good which increases in its absolute size. The variety being the crucial element of the models, the appeal of the models dries out quickly. The reader is alerted to a caveat that what is criticised is the way of modelling the variety of intermediate goods, not the idea itself that the increasing variety of intermediate goods enhances economic growth. The conclusion of the present paper indicates that this insightful idea should be presented in a different modelling framework, which is yet to come.

The popularity of the two textbooks referred to above dispenses us from presenting in detail the full structure of horizontal innovation models in the following pages. We focus instead on two features which a good sample of representative models in this class share amidst an ever-increasing variety of variants, extensions and applications. The commonality in diversity points to the key role that these two common features play. Section 2 presents these two common features. Then Section 3 discusses, on the one hand, how these two common features are crucial to the result of the models and, on the other, how they, put together, are equivalent to assuming a homogenous intermediate good. Section 4 argues that the concept of ‘differentiated’ goods featuring in the literature in question may make sense in the case of a utility function, whereas it hardly does in the case of a production function. Section 5 concludes in terms of a short remark.

2. The two common features

All the representative models have three sectors—the final output sector, the intermediate goods sector and the research and development (R&D) sector. The R&D sector produces new designs; firms in the intermediate goods sector use these designs (old and new) to produce distinct intermediate goods; the final output sector uses these distinct intermediate goods, along with other inputs, to produce the final output. That the production of designs, which is the ultimate engine of growth, is motivated by the pursuit of profits on the part of individual agents, is the basis of the claim that economic growth is endogenous. Different models are identified by different specifications of the production technologies in the three sectors. However, from amidst a diversity of models, one can extract two common features regarding the first two sectors which call for scrutiny (another common feature entertains the R&D sector, but as a result of the first two shared ones).

The first common feature is that the final output sector utilizes a technology summarized by the following 'Dixit-Stiglitz' production function (or its close variants):[1]

$$Y = L_Y^{1-\alpha}\left(\sum_{i=1}^{N} x_i^{\alpha}\right) \tag{1}$$

Final output Y is produced by means of labour L_Y and a set of N intermediate goods, x_i being the number of physical units of the ith intermediate good.[2] It is assumed that the final output is used either as the consumption good or as the investment good.

Each of the intermediate goods used in production has an 'additively separable' effect on the marginal product; the marginal product of the ith intermediate good is independent of the other remaining intermediate goods. This is taken to mean that the production of a new intermediate good is a 'basic innovation, akin to opening up a new industry' (Barro and Sala-i-Martin, 1995, p. 213), indicating that x_i 's are quantities of intermediate goods of distinct types. Note, however, that all the intermediate goods enter production function (1) in a 'symmetric' fashion. Each of the intermediate goods has the same elasticity of substitution in relation to any of the other intermediate goods.[3]

The second shared aspect is related to the production technology of the intermediate goods sector. In Romer's word, '[o]nce it owns the design, the firm can convert η units of final output into one durable unit of good i ' (1990, p. S81). The method of production for the ith intermediate good is represented by

$$\begin{aligned}&\text{(1 unit of) the ith design } \oplus \ \eta x_i \text{ units of final output}\\ &\qquad \rightarrow \ x_i \text{ units of the } i\text{th intermediate good}\end{aligned} \tag{2}$$

Note that each type of intermediate goods, though (allegedly) distinct from the others thanks to a particular design, is produced using an identical amount of the final output. Different models would have some other input(s), instead of the final output, such as labour and other intermediate goods in the production of intermediate goods. However, they invariably adopt the assumption that, besides (one unit of) a corresponding design, the production of an intermediate good requires (an) identical amount(s) of the other input(s) per unit of that intermediate good across the sector.[4] That is, whichever 'production technique' is used in the intermediate goods sector, it is composed of methods of production with equal input proportions.

No justification is given to this particular form of the method of production by

Romer. Nor by anyone else. Let us take another example, from among the abundance of such examples. Barro and Sala-i-Martin in the first edition of their textbook (1995) add almost identical footnotes regarding a uniform cost function for designs (footnote 3, p. 216) and the use of a particular production technique which is composed of methods of production similar to (2) for the intermediate goods sector (footnote 6, p. 217): ‘We are, in other words, applying the assumptions of the one-sector production model’ to the R&D sector and the intermediate goods sector, respectively. However, there is a difference. Whilst they provide a defence of the setting of the R&D sector in the main text, they are completely silent on the assumption for the intermediate goods sector.[5] They have made some changes in the second edition of the book. The footnote for the R&D sector has been graded up as part of the main text. The footnote for the intermediate goods sector has also been upgraded and transformed into a seemingly full-blown explanation: $\eta = 1$ being assumed, ‘[i]n effect, the inventor of good *j* sticks a distinctive label on the homogenous flow of final output and, thereby, converts this product into the *j*th type of intermediate good’ (2004, p. 291). This is still hardly a convincing justification of the assumption; we shall have an occasion to come back to this remark later on.

The reader is then left to presume that this assumption is made, as are some other assumptions on which Romer (1990, pp. S79–S81) spends some time to make remarks, for mathematical convenience: to make the calculation of equilibrium values simple, without affecting the essence of argument—in our current case, unbounded economic growth originating in the increasing variety of intermediate goods.

The truth is otherwise. For (*i*) the two shared features are inescapably required to obtain a sought-for result: a balanced growth equilibrium; (*ii*) however, they stir up conceptual contradiction with the very idea of the increasing variety of intermediate goods.

3. How diversity is turned into homogeneity and why such is necessary

3.1. The production technique of the intermediate goods sector

The assumption of symmetry among intermediate goods, represented by production function (1), can be taken as providing a special form of the production technology for the final output, as is the case with the Cobb-Douglas or CES production functions.[6] The nature of the matter, however, changes radically in the case of the production technology for the intermediate goods sector, represented by (2). For, in the class of models where unbounded growth is inherently related to the increasing variety of intermediate goods, a problem arises of how intermediate goods are indeed distinguished from each other within a given model. It turns out that Barro and Sala-i-Martin's passing remark that they are 'applying the assumptions of the one-sector production model' is not an innocuous one.

Let us take a short diversion for an episode of the past. Samuelson (1962) tried to demonstrate that a multiplicity of capital goods, under a special assumption, would allow for a usual well-behaved neoclassical aggregate production function. That special assumption is that 'equal proportions of inputs' are used across the production processes of the economy. This endeavour received a decisive rebuttal from Garegnani (1970). The choice of the units of the respective goods is arbitrary and, constant returns to scale being assumed, such units can always be chosen as to make the sets of 'equal proportions of inputs' identical to each other. That is, 'equal proportions of inputs' are, but for the arbitrary choice of the unit of each good, equivalent to an identical method of production. But there is no other way for different goods to be identified than by different methods of production which are applied to the production of those goods respectively.[7] Samuelson's effort was to reconfirm the result of a one-good economy. Aggregation goes through without any problem with that assumption—the result which had already been known, if implicitly, to Ricardo and Marx. This is the world where the good old labour theory of value applies. However, no more than that.

Back to the present. On the one hand, the assumption of symmetry produces an identical (and independent) demand curve for each type of intermediate goods (Romer, 1990, p. S86; Dixit and Stiglitz, 1977, p. 299; Ethier, 1982, p. 392). On the other, faced with the particular form (2) of the production technique for intermediate goods, profit-maximizing firms in the intermediate goods sector supply intermediate goods along an identical (and independent) supply curve (an identically marked-up rental rate of intermediate goods). Hence, in equilibrium, different intermediate goods are produced by an identical quantity:

$$x_i = x \quad \text{for all } i \tag{3}$$

Because one unit of each type of intermediate goods takes η units of final output for production, the value of an intermediate good is, in Romer's words, 'related to the durable goods that are actually used in production by the rule' (1990, S82)[8]

$$k_i = \eta x_i \tag{4}$$

where k_i is the amount of 'foregone output' for the production of x_i units of the *i*th intermediate good. Using the result (3), one can reduce (4) to

$$k_i = k = \eta x \quad \text{for all } i \tag{5}$$

The aggregate amount of 'foregone output', which Romer calls the 'accounting measure of total capital *K*' (1990, p. S82), is:

$$K \equiv \sum_{i=1}^{N} k_i \tag{6}$$

With the aid of (5), one gets

$$K = \sum_{i=1}^{N} k = \eta \sum_{i=1}^{N} x = \eta N x \tag{7}$$

K is the measure of 'total capital' in terms of the final good.[9] *K*, divided by η, can be expressed as a rectangle with a height of *x* and a width of *N* (Romer, 1990, S91). In equilibrium, *x* remains stationary. Thus, along the equilibrium path, the 'total capital' (*K*) increases at a common rate with the number of individual intermediate goods (*N*).

The result (7) is used to transform the Dixit-Stiglitz technology (1) into

$$Y = L_Y^{1-\alpha}(N x^{\alpha}) = L_Y^{1-\alpha} N \left(K \eta^{-1} N^{-1}\right)^{\alpha} = (L_Y N)^{1-\alpha} K^{\alpha} \eta^{-\alpha} \tag{8}$$

from which one can derive that, with the labour employment being constant, *Y* will grow at the same rate as *K* and *N*.

Finally, because the final output as the investment good is used exclusively for the increase in intermediate goods, the aggregate consumption (*C*) is obtained as

$$C = Y - \dot{K} \tag{9}$$

where $\dot{K}$ is the 'foregone output' diverted from consumption to increase the 'total capital'.[10] *C* grows at the same rate as *K*, *A* and *Y*. The economy is in a balanced growth equilibrium.

The uniformity of the productivity parameter of the material input, η, turns out to be crucial for this result of a balanced growth equilibrium. Let us suppose, to the contrary, that there is a set of intermediate goods which are produced by way of μ ($\neq \eta$) units of the final good whilst all the other types are produced by means of η

units of the final good; let us index those 'dissenting' intermediate goods by $j \in J$, where J is a proper subset of the total set M of intermediate goods. Then,

$$\begin{cases} k_j = \mu x_j \quad \text{for all } j \in J, \\ k_i = \eta x_i \quad \text{for all } i \in M - J \end{cases} \tag{10}$$

Firms in the J group of intermediate goods supply on a supply curve which is different from that entertaining the other group. However, all the firms are faced with an identical demand curve (due to the symmetry of production function (1)). Therefore, for the J group firms, the equilibrium quantity supplied and rental rate, which are stationary, will be different from those for the other group firms. Differentiate (6) with regard to time to get the rate of growth of K, with the differentiated methods of production (10) in mind, as

$$\hat{K} \equiv \frac{\dot{K}}{K} = \frac{\mu x_J \dot{N}_J + \eta x_{M-J} \dot{N}_{M-J}}{\mu x_J N_J + \eta x_{M-J} N_{M-J}} \tag{11}$$

where $\dot{N}_J$ (N_J) and $\dot{N}_{M-J}$ (N_{M-J}) are the numbers of newly-produced (pre-existing) intermediate goods of the J and the M – J groups respectively (obviously, $\dot{N}_J + \dot{N}_{M-J} = \dot{N}$ and $N_J + N_{M-J} = N$); x_J and x_{M-J} are respectively the equilibrium quantities of the J and the M – J group intermediate goods. Unless the proportion in the number of members between the J and the M – J groups of the newly produced intermediate goods are always the same as that of the pre-existing intermediate goods, the rate of growth of K changes over time even though N grows at a constant rate (this latter is the case by the construction of the model; see below). If one of the groups grows faster permanently, then the rate of growth of K (and hence that of Y and C) will asymptotically approach the rate of growth of that group. A balanced growth equilibrium will be obtained asymptotically.

In more general cases, however, this asymptotic result ceases to hold. If the faster growth of one group is not permanent, asymptotic balance growth is not guaranteed. Moreover, the general tendency of technical progress in the intermediate goods sector would be an increasing degree of fragmentation (that is, diversity) in the production technique for intermediate goods: the number of intermediate good groups which use different methods of production would increase as the total number of intermediate goods increases. This argument would get stronger support from the kernel idea of the horizontal innovation literature: when new intermediate goods are invented as distinct to the existing ones, there would be no pre-determined pattern of change in the methods of production of producing them. '[B]asic innovation, akin to opening up a new industry' (Barro and Sala-i-Martin, 1995, p. 213) would, almost by definition, preclude pre-determination. For a balanced growth equilibrium to obtain, the behaviour of the productivity parameter of the material input with the increase in N must satisfy particular (presumably very restrictive) conditions. The 'rule' (4) is one such satisfying behaviour; however, this behaviour is fatally restrictive, for the productivity parameter η of the material input, being identical across the intermediate goods, cannot constitute the criterion by which one distinguishes among intermediate goods.

3.2. The production of designs in the R&D sector

Some may object: what the literature claims is that it is different production designs used in producing intermediate goods, not the productivity parameter of the material input in the production technique of the intermediate goods sector, that makes intermediate goods differentiated from each other (recall the earlier quote from Barro and Sala-i-Martin). The increasing number of designs produced in the R&D sector is, so goes the argument, what leads to an increasing variety of intermediate goods. This claim, however, does not hold water.

Let us look at the R&D sector. The output (new designs) in the R&D sector is a function of labour employed in this sector (L_R) and the stock of previous designs (N):[11]

$$\dot{N} = \delta L_R N \tag{12}$$

The price of newly produced designs ($\dot{N}$) is uniformly P_R, however many kinds they are; by implication, the designs previously produced had the same price.[12] It is noted that this uniformity of the prices of designs is a joint result of the symmetry of production function (1) and the particular technological specification (4). The market for designs being competitive, the price of a design is adjusted until the present value of the net revenue (the gross revenue minus variable cost) of the firm producing a type of intermediate goods is equal to the interest cost of purchasing (or producing) that design. The net revenue is identical across the firms, because they are faced with an identical demand curve (from the symmetry of the production function) and an identical supply curve (from the particular production technique of the intermediate goods sector). The rate of interest being equal for every firm, the prices of designs ought to be uniform.

The ‘production function’ of the R&D sector (12) does not deviate much from the usual nature of a production function such as (1): the inputs—labour and the stock of previous designs—are both physically measured, and the output is also a physical quantity. This statement is possible because the amount of designs is measured in terms of the number of patents granted. Thus this formulation of the production of designs (a form of ‘knowledge’, broadly defined) apparently satisfies the condition of cardinal measurability of ‘knowledge’ required for one to talk of returns to scale to ‘knowledge’ or the marginal product of ‘knowledge’, and so on (Aghion and Howitt 1998, pp. 435–448; Steedman 2003). It is another matter, however, whether this formulation is informative at all for the current purpose. The fact is that it tells absolutely nothing about how designs are so different from each other that they can start up the production of distinct intermediate goods. The non-informative nature of this formulation is reported from all four fronts.

First, function (12) simply refers to the number of patents produced. As in reality patents are granted to original designs only, one can presume, but presume only, that the designs produced must be different. But this presumption is extraneous to the model; there is nothing at all in the model which would provide information about different contents of designs.

Second, let us suppose for the sake of argument that function (12) indeed provides information about the contents of designs. Now, comparative statics analysis with respect to different designs can only be carried out in terms of different values of N and δ (L_R is determined endogenously). Function (12) would then be saying that the R&D sector endowed with the same N which contributed to the increase of N by the same ratio of δ would always produce the same kind(s) of new designs. This scenario is possible but always the result of a pure fluke.

Third, as all the designs have the same price, at least the prices of designs cannot be the criterion by which to tell between different designs—between old and new designs, or among new designs currently produced.

The fourth front becomes clearly visible when one considers the production technique for the intermediate goods sector, schematized in (2). Here, as was argued in the previous section, different designs do not have anything to do with different methods of production (different input proportions). In this representation, a production design is not able to reveal itself in any material (objective) form. It serves merely as a ‘rhetorical’ device which has been devised for indicating the sparking off of the production of an allegedly distinct intermediate good. However, rhetoric alone is too feeble to bear the whole burden of running the purported engine of growth.

Production design is meant to be to ‘differentiated’ intermediate goods what ‘location’ is to ‘differentiated’ goods in the famous Hotelling model of product differentiation. But it is not up to the given task. In the Hotelling model, locations (distances) are differentiated first, and then goods, otherwise identical, are differentiated by different locations; difference in location is what contributes to different costs to consumers. In our current case, however, there is no way, from the outset, to tell between different designs; all designs, old or new, incur the same cost to their purchasers. One is left at a loss about the (material, not merely rhetorical) whereabouts of the *i*th production design which makes the *i*th intermediate good so distinct from the other intermediate goods.

A summary of our argument up until now is in order. There is no way to tell different contents of designs; an identical amount of the final output (material input) is used for the production of intermediate goods; a design and the final good exhaust the list of inputs for the production of intermediate goods; there is no other way to tell among different goods than by the list of and the proportions between the inputs used in producing them. The conclusion ought to be that there is no way to distinguish between intermediate goods produced in the way suggested in the established literature. The intermediate goods in the literature cannot but be homogeneous.[13] And this homogeneity is required to secure the sought-for result of this class of models: a balanced growth equilibrium.

4. Differentiated goods in consumption and in production

The concept of the ‘variety’ of intermediate goods in the literature in question is based on the ‘additively separable’ feature of production function (1). The initial use of this functional form (Dixit and Stiglitz, 1977) is for the consumer’s taste for variety. In this world, even though utility is obtained by consuming symmetric goods produced on a uniform cost function, there is a room in which the concept of ‘differentiated’ goods may make sense, albeit restrictively (as the authors admit explicitly, p. 298). The case is fundamentally different for production.

Let us imagine a cooking competition. Each competitor is given exactly the same ingredients, the same in kind, quality, amount and so on. The competitors must use all the ingredients, leaving nothing unused (including the total time for cooking). The only way that the competitors can produce their own distinct food (say, pizza) is, then, to use their own distinct ‘designs’, which will be expressed in different shapes, different arrangements of toppings, etc. These different designs were conceived by the toil and sweat (labour) of competitors, who have been trained in a cookery teaching institution along with the other competitors (the stock of previous designs as a public good).

Designs may even refer solely to brands or makers; so pizzas which are exactly the same physically may be differentiated in terms of who has made them.

It may be acceptable to think that to consume pizzas of a variety of shapes or topping arrangements gives greater utility to the consumer than to consume pizzas of the same shape and topping arrangement all around. To consume pizzas, though identical physically, made by a variety of cooks, sometimes including celebrities, can give greater utility than to consume pizzas made by the one and the same cook all around, even if that one cook is his/her idol celebrity.[14]

The situation radically changes for the case of production. Mere different shapes or mere different arrangements of the same components, let alone mere different makers or brands (‘labels’), would have nothing to do with the productivity of those goods which bear these differences. Productivity is a technical, objective, matter; it is not a matter of subjective taste of the producer. By ‘stick[ing] a distinctive label on the homogenous flow of final product’ (Barro and Sala-i-Martin, 2004, p. 291), one can appeal to the taste of a consumer, but not to the concern of a producer: his/her concern is the technological characteristics of a good which is to be used in production. Observe how, in this literature, the theoretical symmetry of goods produced by an identical method of production takes concrete, real-world, manifestations. New designs are ‘breakthrough innovations’ (Barro and Sala-i-Martin, 1995, p. 213); the examples of distinct goods entitled a place in production function (1) are ‘trucks’ and ‘computers’ (Romer, 1990, p. S81) or ‘hammers, trucks and computers’ (Gancia and Zilibotti, 2003, p. 3) or, for car-making, ‘tires, steel, windows, light bulbs, transistors, upholstery, crankshafts, batteries, and so on’ (Aghion and Howitt, 1998, p. 87). Real-world Avatars of intermediate goods refer unambiguously to technical diversity; their material manifestations cannot be more distinct from each other. By contrast, their theoretical existence is inescapably based on technical uniformity, effectively residing in homogeneity, just like Brahman.

Romer aspires to ‘zero tolerance of intellectual sloppiness’ (Snowdon and Vane, 2005, p. 675). Should this aspiration not imply an acceptance of the homogeneity of intermediate goods in the current models of horizontal innovation? Models in this class are not dissimilar to those in which there is only one homogeneous intermediate good. The increasing number of designs is in fact the increasing number of units of this homogeneous intermediate good. Inputs for the production of new designs are in fact inputs for the increase of this homogeneous intermediate good.[15] The only difference is that the class of models under consideration uses a particular form of production function (8). As the quantity of the homogenous intermediate good increases, represented by the increase in N while x remains constant (which implies an increase in

K at the same rate as N), the output of the final good increases linearly. There are constant returns to scale with respect to capital even if the supply of labour is fixed. A moment's look will show that production function (8) is a variant of Arrow's 'learning by doing' aggregate production function (Arrow, 1962). Function (8) is re-arranged to give

$$Y = (K^{1-\alpha} x^{\alpha-1} \eta^{-1}) L_Y^{1-\alpha} K^{\alpha} = A(K) L_Y^{1-\alpha} K^{\alpha} \tag{16}$$

where $A(K) \equiv K^{1-\alpha} x^{\alpha-1} \eta^{-1}$. Note however that (8) is an ex post production function, for a uniform x is a result of the interaction between the ex ante production function (1) and the profit maximization activity of the intermediate goods sector.

The onus of justifying this form of production function, basing that justification solely on the number of distinct intermediate goods, is on the shoulders of its users. Ethier (1982), who is the first to apply the Dixit-Stiglitz function to production, refers to the division of labour, quoting 'the hoary examples of Adam Smith's pin factory and the Swiss watch industry' (p. 391). However, our argument would no sooner prove persuasive than the charm of these 'hoary examples' for Ethier's purpose would evaporate into the thin air. For Adam Smith's pin factory is where different workers are specialized in different stages of the whole process of making pins, thus producing distinct intermediate goods at each stage; the Swiss watch industry is where different components—distinct goods—are produced by specialized workers and are assembled to produce a Swiss watch. Ethier contradicts himself soon after mentioning these examples: 'I am not interested in issues which depend upon distinctions between potential intermediate goods, so I assume that they are all producible from capital and labour via identical production functions, and that all produced components contribute in totally symmetric fashion to the finished manufactures' (p. 391). This is an excellent, if not hoary, example of self-contradiction: invoking the distinctiveness of intermediate goods (for the argument of specialization in trade), after having expelled it from the model downright by assumption (or as the result of some features of the model, as we have tried to demonstrate for our case).

5. Concluding Remarks

It is well known that most of models in New Growth Theory, though admittedly displaying a great deal of stimulating ideas, use an aggregate production function (in various forms) whose theoretical validity in an economy of multiple heterogeneous goods has been soundly put into question in the Capital Controversy of the 1960s.

As far as these models do not pretend that their results extend beyond the boundary in which an aggregate production function is valid, which is a one-good economy, they might be tolerated in the minds of the moderately-versed in capital theory and quite lenient, if only for the reason that they in the way of ‘parables’ directs us to some interesting perspectives. Yet, this tolerance expires the moment one finds a conceptual contradiction between the perspective that a model promotes and the formalization of the model that is devised to ‘demonstrate’ that very perspective.

The present paper has argued that this conceptual contradiction is the case of at least one particular class of models in New Growth Theory: models which claim that unbounded endogenous growth is possible on the basis of the ‘increasing variety of intermediate goods’ (‘horizontal innovation’). The increasing variety of intermediate goods being talked about, one naturally expects that these models incorporate a multiple number of goods and thereby go explicitly beyond the boundary of a one-good world. However, one’s expectation is betrayed. This paper is the story of this betrayal.

The literature on horizontal innovation, along with other lines of research in New Growth Theory, has had enormous impact, providing a great deal of interesting and stimulating perspectives. No one would be allowed to take lightly the idea that an increasing variety of intermediate goods contributes to economic growth. However, that idea has to be backed up by conceptual consistency in models presented, before being supported by empirical evidence. Unfortunately, this has not been the case. A good sample of representative models for horizontal innovation deal in effect with a homogeneous intermediate good whose increasing quantity contributes to economic growth, not with heterogeneous intermediate goods whose increasing variety it is that constitutes the engine of growth. Unwittingly or willingly, the existing models in the literature adopt the same strategy as Samuelson’s ‘surrogate production function’ in order to deal with heterogeneous intermediate goods—and should face the same fate as the latter. Apparently, Romer’s aspiration for ‘zero tolerance for intellectual sloppiness’ (Snowdon and Vane, 2005, p. 675) is not fulfilled in this case.

The present paper has offered criticism only. However, criticism, when proven valid as we hope ours to be, is more often than not a good starting point of constructive efforts.

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Notes

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1. Dixit and Stiglitz (1977) introduced this functional form in the context of a utility function. It is Ethier (1982) that later applied this functional form to production and interpreted it as a production function. As Dixit and Stiglitz never refer this functional form to production, to call production function (1) the 'Dixit-Stiglitz' production function/technology may be honouring them too much (or too little, as some may say after reading Section 4 below).
2. Here, following the usual practice in the literature, we proceed without questioning the sense of taking x_i as the number of physical units (contrasted to a value magnitude), in order to concentrate upon the issue we currently wish to raise. A companion paper to the present one is built on such questioning.
3. In contrast to the Dixit-Stiglitz production function, a traditional 'neoclassical' production function ('disaggregated', of the Cobb-Douglas type) can be expressed as $Y = L_Y^{1-\alpha}\left(\sum_{i=1}^{N} x_i\right)^{\alpha}$. Here each of the intermediate goods has a uniform effect on the marginal product of any other intermediate goods. In this sense, intermediate goods in this representation are not 'differentiated'.
4. In Rivera-Batiz and Romer (1991), the prototype of the 'lab equipment' model, the production functions for intermediate goods are identical to each other and

furthermore identical to the production function for the final output. The ‘labour for intermediates’ model has labour, rather than the final output, as an input for production of intermediate goods; still, different firms in this sector use an identical amount of labour per unit of their respective intermediate good.

5. Their defence regarding the R&D sector is that, as more and more new designs are produced, researchers tend to run out of new ideas but at the same time they can rely on the previous designs for further inspiration; thus, it can be assumed that ‘these effects roughly cancel so that the cost of inventing a new good does not change over time’ (1995, p. 216). They are surely standing on a sharp knife-edge. In the Problems section of that chapter, they throw to students a task to think over the possibility and implications of a declining cost of inventing with increasing N (Problem 6.2(b)). Indeed, the second edition of their textbook allocates some substantial pages for the increasing or decreasing cost of invention, reflecting the line of research pursued by Jones (1995), which has initiated a wave of literature on ‘growth with or without scale effects’. This possibility of relaxation indicates the (theoretical) non-essentiality of the shape of cost function for the R&D sector.

6. However, see Section 4 below.

7. Take the kilogram as the unit of measuring the quantity of apples. Suppose so many units of land, labour and organic fertilizer (this list exhausting the inputs for apples) produce one unit of apples. Suppose one ‘unit’ (whatever it may be) of pears requires 0.45359 times those for one unit of apples, the proportions among them being the same as for apples (the list of inputs for pears being the same as for apples). Under constant returns to scale, 0.45359 times the inputs for one kilogram of apples will produce 0.45359 kilogram of apples, or one pound (avoirdupois) of apples. As exactly the same amounts of the respective inputs are used for producing one pound of apples and one ‘unit’ of pears, one pound of apples is identical to one ‘unit’ of pears. Now, change the unit of apples to the pound, to have the true statement that one unit of apples is identical to one ‘unit’ of pears. That is, apples are pears.

8. In fact, Romer skips this ‘rule’ for individual intermediate goods and jumps to the aggregate case of the ‘rule’ (7) below.

9. Doubt can be raised as to this way of accounting the quantity of intermediate goods (individually and in aggregate), which is related to taking x_i as a physical quantity rather than as a value quantity (see footnote 2 above). The present paper leaves this doubt aside.

10. Henceforth, a dot on a variable denotes the time derivative of that variable.

11. Here we follow the specification of Jones (1998). Romer (1990) has ‘human

capital', split between the final good sector (which uses also labour) and the R&D sector. The total endowment of labour (or human capital) in the economy is split between the two sectors, and the split ratio is determined endogenously, thereby producing endogenously determined rate of balanced growth.

12. We do not take issue with the special assumption here that the output of new designs is linearly related to the stock of previous designs, other things being constant. Romer (1990, p. S84) himself stops briefly to give a reservation remark. Solow (2000) provides a forceful criticism of this aspect and we now have Jones (1995) which has sparkled off discussions on 'scale effects'. Some models make the output of new designs independent of the stock of previous designs (e.g. Barro and Sala-i-Martin, 1995, 2004). The point which is important to our argument is, however, that in whatever specification the equilibrium price of designs turns out to be uniform.

13. An extreme example of this is Rivera-Batiz and Romer (1991). In addition to assuming an identical production function for various intermediate goods and also for the final good, they assume the same production function for the production of designs, identical up to a scale factor to that of the other sectors. Thus, all three sectors of the economy effectively use an identical production function. It remains mysterious how they can still talk of a 'variety' of intermediate goods.

14. This example of pizza-making competition is not exactly appropriate, in fact. Different identities of cooks, as different kinds of labour, should be a component of different methods of production. (However, in 'labour for intermediates' models, where labour is used in the production of intermediate goods, labour is homogenous.)

15. Tirole (1988) recognizes the restrictiveness of the use of functional form (1): '[functional form (1)] treats all the differentiated products in a symmetric way. When a firm introduces a product, it does not choose its degree of differentiation relative to other products.... [I]t allows us to concentrate on the entry decision ... without complicating this by a simultaneous choice of "location"' (p. 298; see also p. 100). If the entry decision only, without consideration of difference in 'location', were not equivalent to an increase or decrease of a homogeneous good, what would it be?