DISCUSSION PAPER SERIES

高麗大學校　經濟研究所

THE INSTITUTE OF ECONOMIC RESEARCH
KOREA UNIVERSITY

『Gradual Revelation Mechanism with Two-Sided Screening』

Helena Hye-Young Kim, Frans Spinnewyn,
and Luc Lauwers

Discussion Paper No. 07-10 (May 2007)

The Institute of Economic Research Korea University
Anam-dong, Sungbuk-ku,
Seoul, 136-701, Korea
Tel: (82-2) 3290-1632 Fax: (82-2) 928-4948

Gradual Revelation Mechanism with Two-Sided Screening

Helena Hye-Young Kim
Department of Economics, Korea University,
Anam-dong, Seongbuk-Gu, Seoul, Korea
E-mail: helena@korea.ac.kr

and

Frans Spinnewyn and Luc Lauwers
Department of Economics, K.U.Leuven,
69 Naamsestraat, B-3000 Leuven, Belgium
E-mail: frans.spinnewyn@econ.kuleuven.ac.be, luc.lauwers@econ.kuleuven.ac.be

Version: May 2007

We investigate the mechanism that provides the optimal decision rule for two agents making joint decisions. It is shown that, a special rectangular mechanism with two sided screening, elicit correct information when agents' preferences are private information. Such mechanism is presented as a game of incomplete information. It is shown that if types are uniformly distributed, then a three stage sequential game with an exogenously given probability of a terminal break down cannot be improved upon within a restricted class of models.

Key Words: Mechanism Design, Efficiency, Risk Limit.

1. INTRODUCTION

This paper studies the problem of a mechanism designer who wants to resolve the dispute between two agents preferring different alternatives. Each agent has private information concerning his valuation from two alternatives. Thus, a mechanism designer does not know both agents' true preferences, and also agents themselves have incomplete information concerning their respective preferences. The objective of a mechanism designer is to elicit right information concerning agents' preferences. One way of obtaining this information is by imposing an inferior outcome that is Pareto dominated by two other alternatives that each agent has demanded.

We characterize the decision rule that a mechanism designer can employ in order to induce agents to reveal true information in Bayesian Nash equilibrium. It is shown that by assigning the appropriate probabilities to each outcomes contingent upon agents' risk limits, a mechanism designer can induce agents to reveal their true preferences.

The mechanism that we consider here has many important applications. The design of tournament procedure, the writing of contract that specifies the penalty when agreement fails, and construction of implementation procedure for resolving legal dispute between parties who will come to have private information are all examples.

There are two essential differences between our model to that of the general mechanism design problem discussed in the existing literature. First, our model deals with non-transferable utility, thus avoids a budgetary problem discussed in d'Aspremont and Gerard-Varet (1979). Second, we do not need to deal with participation constraint since the dispute ends in three possible ways; two alternatives preferred by each player and an inferior outcome.

The paper is organized as follows. In Section 2, we introduce definitions and notations. Furthermore, we derive the conditions that is necessary and sufficient for the mechanism to be Bayesian incentive compatible. Section 3 relates the escalation game in the context of the mechanism design. It is shown that a sequential escalation game is a special mechanism that is truth revelaing. Section 4 investigates the mechanism that is both truth revealing and efficient. It is shown that with uniformly distributed risk limits, a three stage escalation game cannot be improved upon within a restricted class of models.

2. TRUTH REVEALING MECHANISMS

2.1. Problem

The mechanism designer wishes to achieve a settlement between two agents who favor different alternatives. Agent i strictly prefers alternative ω_i above ω_j and his opponent j strictly prefers ω_j above ω_i. The alternatives are physical outcomes that are the result of a collective decision of two agents. For example, (i) the merger deal can only be carried out if it is agreed by both companies (ii) the child custody is given to one of the parents.

The problem that the mechanism designer faces is that he does not know agents' payoffs. We will show that by imposing the probability of a Pareto dominated alternative ω_0, the mechanism designer can elicit information concerning agents' payoffs.

The setting of the problem now involves three alternatives $(\omega_0, \omega_i, \omega_j)$ with the preference ordering $u_i(\omega_i) \geq u_i(\omega_j) > u_i(\omega_0)$. Again, we normalize utilities towards the risk limits:

$$k_i = \frac{u_i(\omega_i) - u_i(\omega_j)}{u_i(\omega_i) - u_i(\omega_0)} \qquad \text{and} \qquad k_j = \frac{u_j(\omega_j) - u_j(\omega_i)}{u_j(\omega_j) - u_j(\omega_0)}.$$

The type k_i is randomly selected from an interval K_i that is included in the closed interval $[0, 1]$. The type of an agent is private information, i.e. each agent is only informed about his own type. The interval K_i, K_j, and the distribution of the types are common knowledge.

The mechanism designer assigns the probabilites to each alternative for each report of the agents. Let D denote the set of probability distributions over the alternatives $(\omega_0, \omega_i, \omega_j)$, i.e.

$$D \;=\; \{(p_0, p_i, p_j) \,|\, 0 \leq p_0, p_i, p_j \leq 1 \text{ and } p_0 + p_i + p_j = 1\}\,.$$

A distribution $p = (p_0, p_i, p_j)$ is called an outcome and generates for each agent i an expected utility

$$\begin{aligned} E_i(p, k_i) \;&=\; p_0\, u_i(\omega_0) \;+\; p_i\, u_i(\omega_i) \;+\; p_j\, u_i(\omega_j) \\ &=\; u_i(\omega_0) \;+\; \underbrace{(1 - p_0 - p_j\, k_i)}_{U_i(p\,|\,k_i)} \times (u_i(\omega_i) - u_i(\omega_0)) \end{aligned} \tag{1}$$

We return to the term $U_i(p \,|\, k_i) = (1 - p_0 - p_j\, k_i)$ later on.

2.2. Definition of a mechanism

The mechanism designer partitions the Cartesian product $K_i \times K_j$ and defines a selection function. Let $\wp$ denote this partition, i.e. $K_i \times K_j = \cup_{B \in \wp} B$ and $B \cap B' = \varnothing$ for each pair B and B' of different elements in $\wp$. Later on, we will impose more structure upon $\wp$. In particular, we will assume that the partition $\wp$ is finite and consists out of rectangles.

The selection function is a map from the partition $\wp$ to the set D of outcomes

$$d : \wp \longrightarrow D : A \longmapsto d(A) = (d_0(A), d_i(A), d_j(A)).$$

The couple $M = (\wp, d)$ is said to be a mechanism. The interpretation is as follows. The mechanism designer announces a mechanism M and asks the agents to simultaneously report their types. If the report (λ_i, λ_j) he receives belongs to A in $\wp$, then he selects the outcome $p = d(A)$. We abuse notation and we will write

$$d(\lambda_i, \lambda_j) \qquad \text{instead of} \qquad d(A) \text{ with } (\lambda_i, \lambda_j) \in A \in \wp.$$

Note that the agents may or may not report their true types, i.e. the reports (λ_i, λ_j) may differ from the true profile (k_i, k_j).

Given such a mechanism M, each agent i tries to gain by strategically announcing a type λ_i. In other words, agent i optimizes the map

$$\lambda_i \longmapsto E_i(d(\lambda_i, \lambda_j), k_i),$$

where expectations are taken over the announcement λ_j of the opponent. Recall that the information k_i and k_j is private. Player i only knows the distribution of his opponents type k_j. Now, we return to equation (1) and we observe that the map

$$\lambda_i \longmapsto U_i\left(d(\lambda_i, \lambda_j), k_i\right) = \; 1 - d_0(\lambda_i, \lambda_j) - d_j(\lambda_i, \lambda_j)\, k_i$$

is a positive linear transformation of the expected utility. Indeed, for each agent i the value $u_i(\omega_i) - u_i(\omega_0)$ is strictly positive. Manipulating $E_i(d(\lambda_i, \lambda_j), k_i)$ corresponds to manipulating $U_i(d(\lambda_i, \lambda_j), k_i)$. From here we focus on these normalized payoff functions U_i and U_j.

2.3. Incentives under the Bayesian Principles

The process of reporting the type can be interpreted as a 2-person Bayesian game of incomplete information. In this game, the strategy λ_i of player i is going to be determined by the strategy λ_j of the opponent, since his objective function U_i depends upon the revealed type λ_j of the opponent. We want to investigate the case where it is the optimal strategy for a player to reveal his true type if his behavior is guided by Bayesian principles.

Definition 1: *Let $M = (\wp, d)$ be a mechanism and let (k_i, k_j) in $K_i \times K_j$ be a profile. The announcement $(\lambda_i^*, \lambda_j^*)$ is said to be a Bayesian Nash equilibrium if for each player i and for each λ_i in K_i, we have*

$$U_i\left(d(\lambda_i^*, \lambda_j^*) \,|\, k_i\right) \;\geq\; U_i\left(d(\lambda_i, \lambda_j^*) \,|\, k_i\right).$$

The ultimate goal of the mechanism designer is to develop a mechanism such that it is in both players' interest to reveal their true types. Formally:

Definition 2: *Let $M = (\wp, d)$ be a mechanism. Then, M is said to be truth-revealing if for each (true) profile (k_i, k_j) the announcement (k_i, k_j) is a Bayesian Nash equilibrium.*

Let us provide an example. Although we do not yet insert a truth revealing mechanism, we will learn how to improve this mechanism and detect conditions that are strong enough to guarantee truth revelation.

Example 1 (a naive mechanism)**:** Let $K_1 = K_2 = [0, 1]$. Consider a designer wanting to implement the Nash bargaining solution. He wants to select that alternative ω that maximizes the Nash product $[u_i(\omega) - u_i(\omega_0)] \times [u_j(\omega) - u_j(\omega_0)]$. One can check that this designer must select (with certainty) alternative ω_i (resp. ω_j) if $k_i > k_j$ (resp. $k_j > k_i$).

The designer partitions $K_1 \times K_2$ into three sets (illustrated in Figure 1): the diagonal A_3, the triangle A_1 below, and the triangle A_2 above the diagonal. The selection function $d = (d_0, d_1, d_2)$ is as follows:

$$d(A_1) = (0, 1, 0)\,,\ d(A_2) = (0, 0, 1)\,,\ \text{and}\ \ d(A_3) = (0, 0.5, 0.5)\,.$$

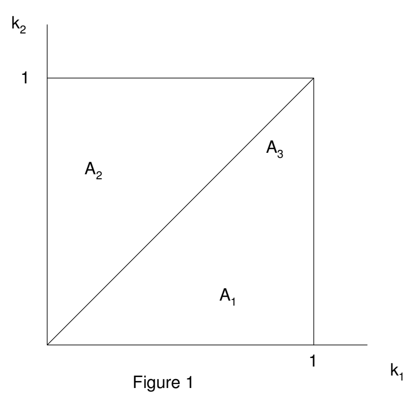

Figure 1

That is, alternative ω_1 (resp. ω_2) is selected in A_1 (resp. A_2), and on the diagonal both ω_1 and ω_2 have probability 0.5 to be selected. The probability of selecting the disagreement outcome ω_0 is everywhere zero.

This mechanism has a unique Bayesian Nash equilibrium: each player i announces $\lambda_i = 1$. Indeed, whatever the true value of k_i and whatever the opponent announces, choosing $\lambda_i = 1$ is the best option for player i. As a consequence, this naive mechanism is not truth-revealing, neither of the agents has incentive to reveal their true types. The equilibrium outcome is a random choice between ω_1 and ω_2. ■

Requiring a mechanism to be truth-revealing strongly limits the outlook of the mechanism. The next lemma summarizes the effects of being truth-revealing upon the selection function $d = (d_0, d_i, d_j)$.

Lemma 1: *Let $M = (\wp, d)$ be a mechanism and let (k_i, k_j) be a profile. If M is truth-revealing, then*

- *(i) the map $\lambda_i \mapsto d_j(\lambda_i, k_j)$ is nowhere increasing, and*
- *(ii) the map $\lambda_i \mapsto d_0(\lambda_i, k_j)$ is nowhere decreasing.*

Proof: M is truth-revealing. Hence, for each λ_i and $\tilde{\lambda}_i$ in K_i we have

$$U_i(d(\tilde{\lambda}_i, k_j) \,|\, \tilde{\lambda}_i) - U_i(d(\lambda_i, k_j) \,|\, \tilde{\lambda}_i) \geq 0 \quad \text{and} \quad U_i(d(\lambda_i, k_j) \,|\, \lambda_i) - U_i(d(\tilde{\lambda}_i, k_j) \,|\, \lambda_i) \geq 0.$$

Add up these inequalities and obtain

$$\begin{aligned} & U_i(d(\tilde{\lambda}_i, k_j) \,|\, \tilde{\lambda}_i) \; - \; U_i(d(\tilde{\lambda}_i, k_j) \,|\, \lambda_i) \; + \; U_i(d(\lambda_i, k_j) \,|\, \lambda_i) \; - \; U_i(d(\lambda_i, k_j) \,|\, \tilde{\lambda}_i), \\ = \; & [d_j(\tilde{\lambda}_i, k_j) - d_j(\lambda_i, k_j)] \times (\lambda_i - \tilde{\lambda}_i), \\ \geq \; & 0. \end{aligned}$$

It follows that $\lambda_i > \tilde{\lambda}_i$ implies $d_j(\tilde{\lambda}_i, k_j) \geq d_j(\lambda_i, k_j)$. Hence, condition (i) is checked.

Recall $U_i = 1 - d_0 - d_j\, k_i$. The combination of $U_i(d(\lambda_i, k_j) \,|\, \lambda_i) \geq \max U_i(d(\tilde{\lambda}_i, k_j) \,|\, \lambda_i)$ and the monotonicity of $\lambda_i \mapsto d_j(\lambda_i, k_j)$ implies that also (ii) is satisfied. ■

This lemma states the following. If a player announces a higher risk limit, then a truth-revealing mechanism gives more weight to the disagreement outcome and less weight to the opponent's most preferred outcome. Although this lemma is a first step in characterizing truth-revealing mechanisms, it does not provide necessary conditions. Indeed, the naive mechanism is not truth-revealing and satisfies both properties: $d_0 = 0$ and d_j is nowhere increasing in λ_i.

Let us now study the linear maps $k_i \mapsto U_i(d(\lambda_i, k_j) \,|\, k_i) = 1 - d_0(\lambda_i, k_j) - d_j(\lambda_i, k_j)\, k_i$ and do some sensitivity analysis (with λ_i as exogenous parameter). From the lemma we learn that both the vertical intercept $1 - d_0$ and the absolute value d_j of the slope are nowhere increasing in the announced risk limit λ_i. Of course, the graphs of two such linear maps (for two different values of λ_i) have at most one intersection point in the strict positive part of the (k_i, U_i)-plane. Apparently, whether two such graphs do intersect (in the relevant area) will influence the properties of the mechanism.

Example 2 (a naive mechanism, continued)**:** Let us exercise the above approach and consider linear maps $k_1 \mapsto U_1$ for different values of λ_1. The intercepts coincide: $1 - d_0 = 1$ (see Figure 2). Concerning the slope $-d_j$ there are three cases to distinguish: $-d_j \in \{-1, -0.5, 0\}$ depending upon whether λ_1 is smaller tan, equal to, or larger than k_2.

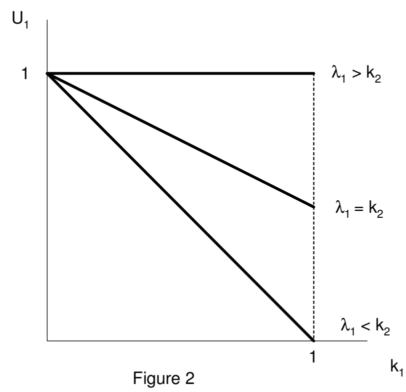

Figure 2

Obviously, the line corresponding to $\lambda_1 > k_2$ allocates the higher utilities to player 1. Whatever the (true) values of k_1, player 1 benefits from announcing a risk limit λ_1 that is higher than k_2. Given the uncertainty about k_2, player 1 should announce $\lambda_1 = 1$. As the mechanism is symmetric in both players, we arrive again at the unique equilibrium: $(\lambda_1, \lambda_2) = (1, 1)$. ■

This naive mechanism indicates that if the different utility maps $\lambda_i \mapsto U_i$ do not intersect in the strict positive part of the (k_i, U_i)-plane, then the players do not have an incentive to reveal any information about their true risk limits. They will announce strategically and choose the dominating utility map. Hence, in order to force a player to reveal some information, the mechanism designer should use utility maps that intersect. Two utility functions intersecting at the '$k_i > 0$'-area must have different vertical intercepts. As a consequence, the mechanism designer must attach a positive probability to the disagreement outcome. Therefore, imposing truth-revelation is at the cost of selecting the disagreement outcome with a strictly positive probability.

Example 3 (almost truth-revealing)**:** Consider the next mechanism. Let $0 < \varepsilon < \mu < 1$. The partition $\wp$ contains the set $A_1 = \{(\lambda_1, \lambda_2) \,|\, \lambda_1 \leq \mu\}$ and the complementary set A_2. Figure 3 presents the partition $\wp$ and Figure 4 shows the graphs of the utility map of player 1.

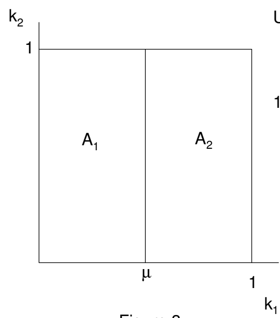

Figure 3

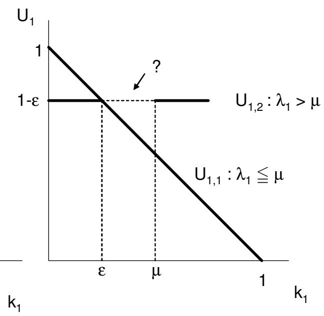

Figure 4

The selection $d = (d_0, d_1, d_2)$ is defined by $d(A_1) = (0, 0, 1)$ and $d(A_2) = (\varepsilon, 1 - \varepsilon, 0)$. If the designer receives a signal in A_1, then ω_2 is selected. If the signal is in A_2, then the outcome is a weighted random selection between ω_0 and ω_1. The probability of selecting the disagreement outcome is denoted by ε. Player 1 is the only player able to manipulate the outcome, he has to reveal the position of his risk limit with respect to μ. The figure 4 shows the graphs of the corresponding utility maps (denoted by $U_{1,1}$ and $U_{1,2}$). Player 1's choice between $U_{1,1}$ and $U_{1,2}$ depends upon his true risk limit k_1. If $k_1 \leq \varepsilon$, then he truly announces $\lambda_1 \leq \mu$. Also, if $k_1 > \mu$, he truly announces his risk limit. But, in the case $\varepsilon < k_1 \leq \mu$, truth telling is not optimal. Here, player 1 has an incentive to lie.

It is easy to avoid such a problematic part in the picture (indicated by a '?'). The graphs of $U_{1,1}$ and $U_{1,2}$ should intersect at $k_1 = \mu$ (indicated by the $\bullet$). Hence, one can

- either set $\varepsilon = \mu$,
- or add the alternative ω_2 to the support of $d(A_2)$.

In the first approach, the utility graph $U_{1,2}$ shifts downwards and intersects $U_{1,1}$ at $k_1 = \mu$. As a consequence, the risk of choosing the disagreement point increases. In the second approach, the value of ε remains fixed. Hence, the vertical intercept of the graph remains at $1-\varepsilon$. The graph $U_{1,2}$ rotates until the intersection occurs at $k_1 = \mu$. Both interventions lead to a mechanism in which player 1 has an incentive to reveal the true position of his risk limit with respect to μ. ∎

Inspired by these examples we put more structure upon a mechanism. In particular, we impose the partitioning $\wp$ to be finite and rectangular, i.e. the intervals K_i and K_j are partitioned into a finite number of intervals and the partition $\wp$ collects all the Cartesian product of two such intervals. Formally, let $K_i = [\underline{k}_i, \overline{k}_i]$ and partition K_i into the n_i intervals

$$I_{i,1} = [\underline{k}_i, \mu_{i,1}], \quad I_{i,2} =]\mu_{i,1}, \mu_{i,2}], \quad \ldots, \quad I_{i,n_i} =]\mu_{i,n_i-1}, \mu_{i,n_i}] \quad \text{with} \quad \mu_{i,n_i} = \overline{k}_i.$$

The rectangle $I_{i,\ell} \times I_{j,m}$ belongs to the partition P, for each $\ell = 1, 2, \ldots, n_i$ and for each $m = 1, 2, \ldots, n_j$. We will use such a rectangular structure to enforce each player i to truly reveal his position against the values $\mu_{i,k}$. A mechanism $(\wp, d)$ with a finite and rectangular partition $\wp$ is said to be rectangular. For a rectangular mechanism, announcing $k_i = \mu_{i,\ell}$ is synonymous to announcing $k_i \in I_{i,\ell}$.

The next proposition strengthens Lemma 1. First, the mechanism is assumed to be rectangular. Second, we impose that the utility graphs corresponding to two neighboring intervals $I_{i,\ell}$ and $I_{i,\ell+1}$ intersect at $k_i = \mu_{i,\ell}$ (see condition (iii)). As such we obtain conditions that are strong enough to generate a truth-revealing mechanism.

Proposition 1: *Let $M = (\wp, d)$ be a rectangular mechanism. Then M is truth-revealing if and only if for each player i we have*

- *(i) the map $\lambda_i \mapsto d_j(\lambda_i, k_j)$ is nowhere increasing, and*
- *(ii) the map $\lambda_i \mapsto d_0(\lambda_i, k_j)$ is nowhere decreasing,*
- *(iii) $U_i\left(d\left(\mu_{i,\ell}, k_j\right) \mid \mu_{i,\ell}\right) = U_i\left(d\left(\mu_{i,\ell+1}, k_j\right) \mid \mu_{i,\ell}\right)$ for each $\ell = 1, 2, ..., n_i - 1$.*

Proof: Let M be truth-revealing. Lemma 1 implies that (i) and (ii) are satisfied. Suppose condition (iii) is not fulfilled. Let us follow the argument in Example 3. If the two graphs do not intersect at $k_i = \mu_{i,\ell}$, then some types in K_i have an incentive to tell lies.

Now suppose that conditions $(i), (ii)$, and (iii) are satisfied. We have to check whether

$$U_i\left(d\left(\mu_{i,\ell}, k_j\right) \mid \mu_{i,\ell}\right) \geq U_i\left(d\left(\mu_{i,m}, k_j\right) \mid \mu_{i,\ell}\right),$$

holds for each ℓ and m, by studying the graph of the map

$$k_i \mapsto \max_{\lambda_i} U_i\left(d\left(\lambda_i, k_j\right) \mid k_i\right) = \max_{\lambda_i} \left\{1 - d_0\left(\lambda_i, k_j\right) - d_j\left(\lambda_i, k_j\right) k_i\right\}.$$

We start with k_i in $I_{i,1}$. As the conditions $(i), (ii)$, and (iii) are satisfied, player i has an incentive to reveal the true position of the risk limit. Indeed, the false announcement $k_i > \mu_{i,1}$ decreases the intercept $1 - d_0\left(\lambda_i, k_j\right)$ and increases the slope $-d_j\left(\lambda_i, k_j\right)$ of the graph: player i (with k_i small) will end up with a lower utility. The intersection condition (iii) makes player i with type $k_i = \mu_{i,1}$ indifferent between announcing $k_i \in I_{i,1}$ and announcing $k_i \in I_{i,2}$.

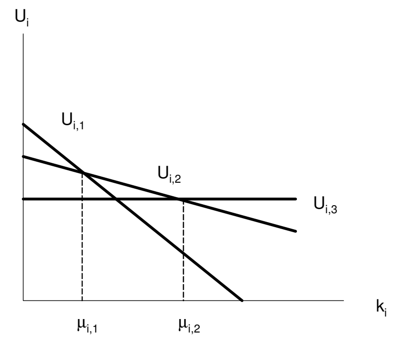

Figure 5

Next, consider k_i in $I_{i,2} =]\mu_{i,1}, \mu_{i,2}]$. Denote the utility obtained by player i when announcing $k_i \in I_{i,1}$ (resp. $I_{i,2}, I_{i,3}, ...$) by $U_{i,1}$ (resp. $U_{i,2}, U_{i,3}, ...$). Conditions (i) , (ii) ,and (iii) force the positions of the graphs of $U_{i,1}, U_{i,2}$, and $U_{i,3}$ as in the Figure 5.4 : the intercepts decrease, the slopes increase, and the intersections are at the right positions. Obviously, player i obtains the highest level of utility by revealing the true position of his risk limit. As a matter of fact, the previous picture remains valid when $U_{i,1}$, $U_{i,2}$, and $U_{i,3}$ is replaced with $U_{i,\ell}, U_{i,\ell+1}$, and $U_{i,\ell+2}$. ■

Example 4 (one-sided screening)**:** Proposition 1 indicates that there are many ways to turn the "almost-truth-revealing-mechanism" in Example 3 into a truth revealing mechanisms. Consider the following mechanism depicted in Figure 5 and 6. :

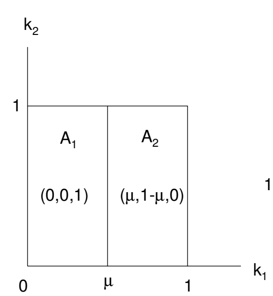

Figure 6

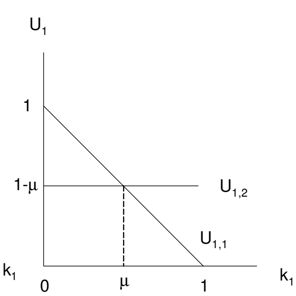

Figure 7

The selection map d is indicated in the picture of partition. E.g. $d(A_1) = (0, 0, 1)$: if the mechanism designer receives a report that is in A_1, then alternative ω_2 is selected. The outcome does not depend upon the information revealed by player 2. Whether the report (λ_1, λ_2) belongs to A_1 or A_2 only depends upon the value of λ_1. In other words, the designer only screens for player 1.

Let us investigate the effect of a refined mechanism where player 1 can select out of three rectangles (denoted by B_1, B_2, and B_3) instead of the rectangles A_1 and A_2. Figures 8 and 9 represents this new mechanism.

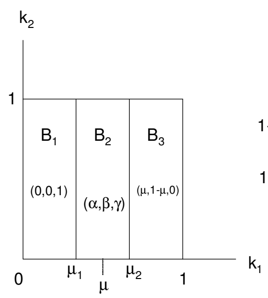

Figure 8

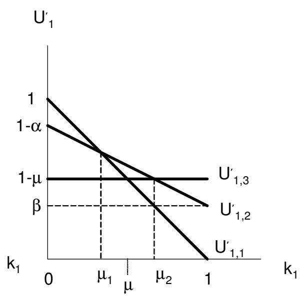

Figure 9

Again, the selection map is denoted within the rectangles. Observe that $U'_{1,1}$ coincides with the $U_{1,1}$ in the previous mechanism, similarly $U'_{1,3}$ coincides with $U_{1,2}$. The utility map $U'_{1,2}$ corresponds to the rectangle B_2. The outcome $d(B_2) = (\alpha, \beta, \gamma)$ in D is chosen in order to obtain a truth revealing mechanism: the two intersection conditions completely determine the values of α, β and γ (see below).

Let us compare the one-sided-screening mechanism (A_1, A_2) with the one-sided-screening mechanism (B_1, B_2, B_3). We start with the utility level obtained by player 1. The types $k_1 \leq \mu_1$ and $k_1 > \mu_2$ are indifferent between the two mechanisms. The corresponding utility levels coincide. On the other hand, types in between μ_1 and μ_2 do prefer the second mechanism. For these types the utility map $U'_{1,2}$ dominates the levels of $U_{1,1} = U'_{1,1}$ and $U_{1,2} = U'_{1,3}$. Conclusion: each type of player 1 weakly prefers the finer screening mechanism, moreover some types of player 1 are strictly better off in this finer mechanism.

We now focus on player 2. In case player 1 selects B_1 or B_3, the outcome remains at $d(A_1)$or $d(A_2)$, and player 2 obtains the same utility level as the mechanism (A_1, A_2). In case player 1 selects B_2, the payoff of player 2 is affected. The outcome $d(B_2) = (\alpha, \beta, \gamma)$ is selected instead of $(0, 0, 1)$ at the left of μ and $(\mu, 1-\mu, 0)$ at the right of μ. Hence, at the left of μ player 2 looses and at the right of μ player 2 gains. In case k_1 has a low probability within the interval (μ, μ_2) and a high probability within (μ_1, μ) then player 2 is better off with the first mechanism (A_1, A_2) ; in the opposite case, player 2 definately prefers the mechanism (B_1, B_2, B_3).

We will check whether the move from (A_1, A_2) to (B_1, B_2, B_3) has a positive effect in the (ex ante) expected payoff of player 2 for the special case that the type k_1 of player 1 is uniformly distributed over the interval $[0, 1]$. Alas, we need some calculations.

First, we determine the expected payoff of player 2 for the mechanism (A_1, A_2). The map $F_1 : k \mapsto k$ describes the distribution of player 1's type. The expected payoff of player 2 is described by the map

$$\begin{aligned}
k_2 &\mapsto U_2 \\
&= F_1(\mu)\times 1 + (1 - F_1(\mu)) \times (1-\mu)(1-k_2), \\
&= \mu + (1-\mu)^2 (1-k_2).
\end{aligned}$$

In order to determine the expected payoff for the mechanism (B_1, B_2, B_3), we first find out the outcome $d(B_2) = (\alpha, \beta, \gamma)$. The graphs of $U'_{1,1}$ and $U'_{1,2}$ (resp. of $U'_{1,2}$ and $U'_{1,3}$) should.intersect at $k_1 = \mu_1$ (resp. at $k_1 = \mu_2$). We obtain the linear system:

$$\begin{cases} 1 - \mu_1 = 1 - \alpha - \gamma\mu_1, \\ 1 - \mu = 1 - \alpha - \gamma\mu_2. \end{cases}$$

The solution for (α, β, γ) reads

$$\gamma = \frac{\mu - \mu_1}{\mu_2 - \mu_1}, \quad \alpha = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}\mu_1, \quad \text{and} \quad \beta = 1 - \alpha - \gamma = \frac{(\mu_2 - \mu)(1-\mu)}{\mu_2 - \mu_1}.$$

Next, we write down the expected payoff for player 2:

$$\begin{aligned}
k_2 &\mapsto U'_2 \\
&= F_1(\mu_1) + (F_1(\mu_2) - F_1(\mu_1))(1 - \alpha - \beta k_2) + (1 - F_1(\mu_1))(1-\mu)(1-k_2), \\
&= \mu_1 + (\mu_2 - \mu_1)(1 - \alpha - \beta k_2) + (1-\mu_2)(1-\mu)(1-k_2).
\end{aligned}$$

Finally, we verify whether for each type k_2 the expected utility U'_2 exceeds U_2. Observe that the expressions for U_2 and U'_2 are linear in k_2. Hence it is sufficient to check whether $U'_2 \geq U_2$ holds for the types $k_2 = 0$ and $k_2 = 1$.

If $k_2 = 1$,then

$$U'_2 - U_2 = \mu_1 + (\mu_2 - \mu_1)\gamma - \mu = 0,$$

if $k_2 = 0$, we proceed as follows:

$$\begin{aligned}
U'_2 - U_2 &= \mu_1 + (\mu_2 - \mu_1)(1-\alpha) + (1-\mu_2)(1-\mu) - \mu - (1-\mu)^2, \\
&= \mu_2 - \mu_1(\mu_2 - \mu)\mu_1 - (\mu_2 - \mu)(1-\mu) - \mu, \\
&= (\mu_2 - \mu)(1-\mu_1) - (\mu_2 - \mu)(1-\mu), \\
&= (\mu_2 - \mu)(\mu - \mu_1), \\
&> 0.
\end{aligned}$$

Hence, the finer mechanism (B_1, B_2, B_3) treats each type of player 2 at least as good as the mechanism (A_1, A_2). In addition, the weak types of player 2 are better of in the finer mechanism.

In sum, if the types are uniformly distributed over the interval $[0,1]$, then refining a truth revealing one-sided screening mechanism (by replacing the threshold μ by the combination of μ_1 and μ_2) improves the mechanism: the finer mechanism remains truth revealing and ex ante all the types of both players weakly prefer the finer mechanism and some types strictly prefer the finer mechanism. The final section of this chapter further develops this idea. ■

3. ESCALATION GAMES AND HERRINGBONE MECHANISMS

This section reinterprets a sequential escalation game in terms of mechanism design. Apparently a sequential escalation game generates a mechanism with a special rectangular mechanism, which we will coin “a herringbone mechanism". A herringbone mechanism is a two-sided screening mechanism. We close this section by investigating escalation games with a small number of stages and their corresponding herringbone mechanisms.

Let us recall that an escalation game is a quadruple $\Gamma = (n, b, r, F)$, with $n \geq 2$ the number of stages, b in $\{1, 2\}$ the player with the move at stage 1, $r = (r_1, r_2, \ldots, r_n = 1)$ in $[0, 1]^n$ the disagreement probabilities, and $F = (F_1, F_2)$ a couple of continuous distribution functions the supports of which are closed intervals (subsets of $[0, 1]$). Furthermore, we have shown that each perfect Bayesian equilibrium is described through a sequence $k_1^*, k_2^*, k_3^* \ldots, k_s^* = \overline{k}_s$ (with $s \leq n$) of critical risk limits. At stage t type k of the player with the move escalates if $k > k_t^*$ and submits if $k \leq k_t^*$.

The submission of player i in stage t should be interpreted as a report (k_i, k_j) with $k_i \leq k_t^*$ and $k_j > k_{t-1}^*$. We present such an equilibrium (i.e. such a sequence of critical risk limits) as a two-sided-screening mechanism. We assume that player 1 is at the move at stage 1. We start the discussion with a particular example. A general definition follows.

Consider an equilibrium $(k_1^*, \ldots, k_5^*)$. In the first stage player 1 reveals his risk limit against k_1^*. If the game continues to stage 2, then player 2 reveals his position against k_2^*. And so forth, if the game reaches stage 3, then player 1 reveals his position against k_3^*, ... The mechanism screens alternately players 1 and 2. It is a two-sided screening mechanism. In this example the escalation game partitions the square $K_1 \times K_2$ into six sets: $A_1, A_2, ..., A_6$ (see Figure 10).

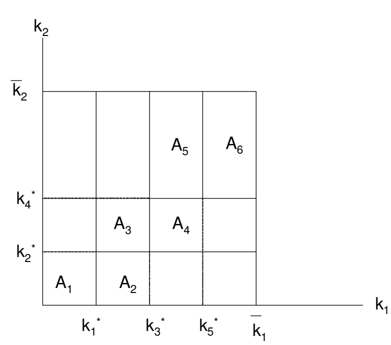

Figure 10

The selection map d is not presented in the figure. The outcome in a certain rectangle coincides with the outcome proposed by the escalation game in case the game ends in that particular rectangle. As such, we have

$$d(A_1) = (0,0,1), d(A_2) = (r_1, 1-r_1, 0), d(A_3) = (r_2, 0, 1-r_2), ..., d(A_6) = (r_5, 1-r_5, 0).$$

In order to obtain a truth revealing mechanism, the selection map should satisfy the conditions of Proposition 5.1. This imposes restrictions upon the escalation game.

Finally, observe the zigzag pattern of the solid lines within the square $K_1 \times K_2$. This zigzag pattern resembles the pattern of a herring's bones. This clarifies the term 'herringbone mechanism'.

Let us now proceed the other way around and indicate that each herringbone mechanism generates an escalation game. Suppose the herringbone mechanism $(A_1, A_2, ..., A_6)$ is given. Let player 1 first consider A_1 versus *not*-A_1. In case A_1 is selected, we say that player 1 submits and the game ends by selecting the alternative most preferred by player 2. In case ' *not*-A_1' is selected, we say that player 1 escalates. Then, the rectangle A_1 is removed from the partition and player 2 considers (within the area ' *not*-A_1') the rectangle A_2 versus *not*-A_2. In case player 2 submits (the report belongs to A_2), then the outcome is a weighted random selection between the disagreement outcome and the outcome most preferred by player 1. In case player 2 escalates, then the rectangle A_2 also is removed from the partition and player 1 has the move. And so forth.

Take notice of a typicality of herringbone mechanisms: during the subsequent removals of the rectangles $A_1, A_2, ...$, the remaining pattern continues to resemble the zigzag pattern of a herring's bones.

We now formulate the definition of a herringbone mechanism.

Definition 3: *Let $(\wp, d)$ be a rectangular mechanism based upon the following partitioning of the intervals K_i and K_j :*

$$\begin{aligned} I_{i,1} &= [\underline{k}_i, \mu_{i,1}], \quad I_{i,2} =]\mu_{i,1}, \mu_{i,2}], \quad \ldots, \quad I_{i,n_i} =]\mu_{i,n_i-1}, \mu_{i,n_i}] \quad \text{with} \quad \mu_{i,n_i} = \overline{k}_i, \\ I_{j,1} &= [\underline{k}_j, \mu_{j,1}], \quad I_{j,2} =]\mu_{j,1}, \mu_{j,2}], \quad \ldots, \quad I_{j,n_i} =]\mu_{j,n_j-1}, \mu_{j,n_j}] \quad \text{with} \quad \mu_{j,n_j} = \overline{k}_j. \end{aligned}$$

Then, $(\wp, d)$ is said to be a herringbone mechanism if the selection function d satisfies

- *$d(I_{i,1} \times I_{j,k})$ with $k \geq 1$ does not depend upon k and gives a zero weight to ω_i,*
- *$d(I_{i,k} \times I_{j,1})$ with $k \geq 2$ does not depend upon k and gives a zero weight to ω_j,*
- *$d(I_{i,2} \times I_{j,k})$ with $k \geq 2$ does not depend upon k and gives a zero weight to ω_i,*
- *$d(I_{i,k} \times I_{j,2})$ with $k \geq 3$ does not depend upon k and gives a zero weight to ω_j,*
- *and so forth.*

In the assumption that the disagreement outcome ω_0 is never selected with probability 1 ($d_0 < 1$), a herringbone mechanism can be recognized as follows.

Also assume that the k_i- axis is drawn horizontally and the k_j-axis vertically. If in a certain rectangle $R = I_{i,1} \times I_{j,k}$ the outcome ω_i is never selected $(d_i(R) = 0)$, then also in the rectangles above R we have $d_i = 0$. Similar for player j. If $d_j(R) = 0$, then also in the rectangles at the right of R we have $d_j = 0$. Note that we do not demand that a herringbone mechanism is truth revealing.

Let us recall and modify the previous figure.

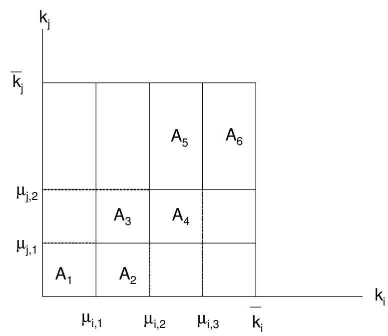

Figure 11

The interval K_i is broken up into four parts, the breaking points are $\mu_{i,1}, \mu_{i,2}$ and $\mu_{i,3}$. The interval K_j is broken up into three parts, the breaking points are $\mu_{j,1}, \mu_{j,2}$ and $\mu_{j,3}$. The three rectangles $I_{i,1} \times I_{j,k}$ with $k = 1, 2, 3$ have the same outcome and are glued together. Also the three rectangles $(I_{i,k} \times I_{j,1})$ with $k = 2, 3, 4$ have the same outcome and are glued together. And so forth. The dotted lines (...) reflect the rectangular mechanism before certain areas were glued together.

We close this section by presenting a truth revealing herringbone mechanism in which each player is screened only once. Figure 12 represents this mechanism.

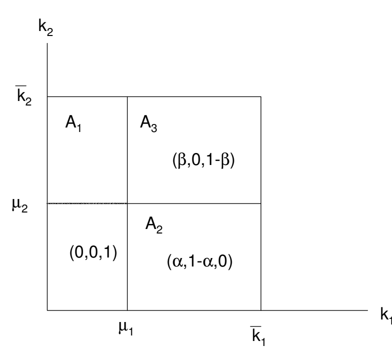

Figure 12

The thresholds are denoted by μ_1 and μ_2. In each area the column vector summarizes the selection map. As we want the herringbone mechanism to be truth revealing, the values of α and β depend upon μ_1 and μ_2. The intersection condition in Proposition 5.1 generates the linear system:

$$\begin{cases} 1-\mu_1 = E_1\left[1-d_0(\mu_1,k_2)-d_2(\mu_1,k_2)\mu_1\right], \\ 1-\beta = (1-\alpha)(1-\mu_2). \end{cases}$$

Elaborating the expectations (over the types of player 2) in the first equation leads to

$$\begin{aligned} 1-\mu_1 &= 1-F_2(\mu_2)\alpha-(1-F_2(\mu_2))\beta-(1-F_2(\mu_2))(1-\beta)\mu_1, \\ &= F_2(\mu_2)(1-\alpha)+(1-F_2(\mu_2))(1-\beta)(1-\mu_1). \end{aligned}$$

A further simplification results in:

$$\mu_1 = 1-\frac{(1-\alpha)F_2(\mu_2)}{1-(1-\beta)(1-F_2(\mu_2))}, \text{ and } \mu_2 = 1-\frac{1-\beta}{1-\alpha}=\frac{\beta-\alpha}{1-\alpha}. \tag{2}$$

Hence, for a given couple (μ_1,μ_2) the designer should solve these two equations for the disagreement probabilities α and β. Note that the final equation implies $\beta>\alpha$: the higher the announced risk limit (in a truth revealing mechanism) the higher the disagreement probability.

4. INCENTIVE EFFICIENCY

In this final section we investigate whether the designer can Pareto improve a truth revealing herringbone mechanism. Does there exist a truth revealing mechanism that Pareto dominates a truth revealing herringbone mechanism (i.e. ex ante player i and player j both prefer this dominating mechanism) ? In the context of mechanism design the term "incentive efficiency" is used to refer to this Pareto-property. We start with a formal definition.

Definition 4: *Let $M=(\wp,d)$ be a truth revealing mechanism. Then M is said to be incentive efficient if there does not exist a truth revealing mechanism $M'=(\wp',d')$ such that (i) for each player $i=1,2$ and for each type k_i in K_i we have $U_i(d'\mid k_i)\geq U_i(d\mid k_i)$; and (ii) for at least one player i and one type k_i of this player we have $U_i(d'\mid k_i)>U_i(d\mid k_i)$.*

Checking whether or not a mechanism is incentive efficient is far from trivial. Either one has to find a Pareto superior mechanism or, even worse, one has to prove that such a Pareto superior mechanism does not exist. Let us investigate the incentive efficiency of truth revealing herringbone mechanisms with a small number of reports.

The trivial herringbone mechanism ($K_1\times K_2$; $d=(0,1,0)$) consists out of the full rectangle and selects the outcome most preferred by player 1. This mechanism coincides with the dictatorship of player 1 and is incentive efficient. Each finer mechanism definitely hurts some types of player 1.

On the other hand, herringbone mechanisms with two rectangles are not incentive compatible. Indeed, such a mechanism coincides with one-sided screening (with one threshold). In Example 5.4, we already have shown that one-sided screening is not incentive efficient: a finer screening never hurts a player and some types will benefit. Notice that a one-sided screening mechanism with two or more thresholds is not a herringbone mechanism.

Finally, we consider truth revealing herringbone mechanisms in which each player is screened only once. We will show that such a mechanism satisfies a restricted version of incentive efficiency: refinements of a certain type do not Pareto improve the mechanism. As a matter of fact we will only check for refinements that involve one more threshold for one of the players. We provide illustration in Figures 13,14 and 15:

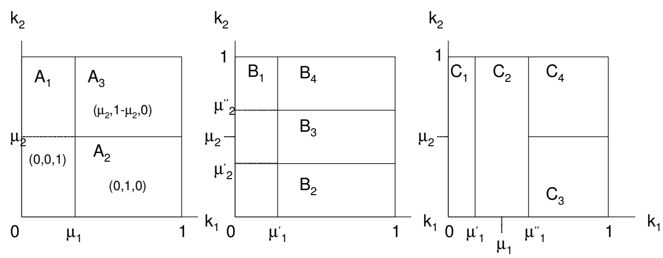

Figure 13 Figure 14 Figure 15

The move from the mechanism $A = (\{A_1, A_2, A_3\}; d)$ to $B = (\{B_1, B_2, B_3, B_4\}; d')$ involves the replacement of the threshold μ_2 by a pair of thresholds (μ_2', μ_2'') satisfying $\mu_2' < \mu_2 < \mu_2''$. In order to end up with a truth revealing mechanism, the threshold against which player 1 is screened decreases $(\mu_1' < \mu_1)$.

The move from A to the mechanism $C = (\{C_1, C_2, C_3, C_4\}; d')$ only involves the replacement of the threshold μ_1 by a pair of thresholds (μ_1', μ_1'') satisfying $\mu_1' < \mu_1 < \mu_1''$.

As already announced, we will check the existence of a mechanism B or C that is Pareto superior to A. Again, we suppose that the types of both players are uniformly distributed over the closed interval $[0, 1]$.

Consider mechanism A. The expected payoffs $U_i = E_i [1 - d_0(k_i, k_j) - d_j(k_i, k_j)k_i]$ reduce to

$$k_1 \mapsto U_1 = \begin{cases} 1 - k_1 & \text{if } k_1 \leq \mu_1, \\ \mu_2 + (1 - \mu_2)(1 - \mu_2)(1 - k_1) & \text{if } k_1 > \mu_1, \end{cases}$$

$$k_2 \mapsto U_2 = \begin{cases} \mu_1 + (1 - \mu_1)(1 - k_2) & \text{if } k_2 \leq \mu_2, \\ \mu_1 + (1 - \mu_1)(1 - \mu_2) & \text{if } k_2 > \mu_2. \end{cases}$$

The intersection property is satisfied for player 2. Type μ_2 is indifferent between reporting $k_2 \leq \mu_2$ and reporting $k_2 > \mu_2$. For player 1 we impose the type μ_1 to be indifferent between the two reports. This results in the condition

$$1-\mu_1=\mu_2+(1-\mu_2)^2(1-\mu_1), \qquad \text{or} \qquad \mu_1=\frac{1-\mu_2}{2-\mu_2}.$$

Mechanism A is truth revealing as soon this relationship between μ_1 and μ_2 holds. Note that the above condition coincides with expression (2) with $\alpha=0$ and $F_2(\mu_2)=\mu_2$.

Consider mechanism B. The selection function satisfies $d'(B_1)=d(A_1)$, $d'(B_2)=d(A_2)$, and $d'(B_4)=d(A_3)$. For the area B_3 we put $d'(B_3)=(\alpha,\beta,\gamma)$. The intersection property determines the values of α,β, and γ as following

$$\begin{cases} \mu_1+(1-\mu_1)(1-\mu_2')=1-\alpha-\beta\mu_2', \\ \mu_1+(1-\mu_1)(1-\mu_2)=1-\alpha-\beta\mu_2''. \end{cases}$$

We solve for α and β, and obtain

$$\alpha=\frac{\mu_2'(1-\mu_1)(\mu_2''-\mu_2)}{\mu_2''-\mu_2'},\beta=\frac{(1-\mu_1)(\mu_2-\mu_2')}{\mu_2''-\mu_2'}.$$

Let us investigate whether there is a Pareto improvement for player 2. The expected payoff of player 2 in mechanism B is described by the map

$$k_2\mapsto U_2'=\begin{cases} \mu_1'+(1-\mu_1')(1-k_2) & \text{if } k_2\le\mu_2', \\ \mu_1'+(1-\mu_1')(1-\alpha-\beta k_2) & \text{if } \mu_2'<k_2\le\mu_2'', \\ \mu_1'+(1-\mu_1')(1-\mu_2) & \text{if } k_2>\mu_2''. \end{cases}$$

Observe that for the types $k_2\le\mu_2'$, the utility map $U_2'=\mu_1'+(1-\mu_1')(1-k_2)$ lies below the utility map $U_2=\mu_1+(1-\mu_1)(1-k_2)$. Hence, the types $k_2\le\mu_2'$ are worse off in mechanism B in comparison to mechanism A.

Let us investigate the effect on the types $\mu_2'<k_2\le\mu_2''$. Because $\mu_2'<\mu_2<\mu_2''$, it is sufficient to check whether $U_2'\ge U_2$ holds for the type $k_2=\mu_2$.

If $k_2=\mu_2$, then

$$\begin{aligned} U_2'-U_2 &= \mu_1'+(1-\mu_1')(1-\alpha-\beta\mu_2)-\mu_1-(1-\mu_1)(1-\mu_2), \\ &= (1-\mu_1)\left(\mu_2-(1-\mu_1')\left(\frac{\mu_2'(\mu_2''-\mu_2)+\mu_2(\mu_2-\mu_2')}{\mu_2''-\mu_2'}\right)\right), \\ &= (1-\mu_1)\left(\frac{\mu_2((\mu_2''-\mu_2')-(\mu_2-\mu_2')(1-\mu_1'))-(1-\mu_1')(\mu_2''-\mu_2)\mu_2'}{\mu_2''-\mu_2'}\right), \\ &= (1-\mu_1)\left(\frac{\mu_2((\mu_2''-\mu_2)+\mu_1'(\mu_2-\mu_2'))-(1-\mu_1')(\mu_2''-\mu_2)\mu_2'}{\mu_2''-\mu_2'}\right), \\ &= \frac{(1-\mu_1)}{\mu_2''-\mu_2'}((\mu_2''-\mu_2)(\mu_2-\mu_2')+\mu_1'\mu_2'+\mu_2\mu_1'(\mu_2-\mu_2')), \\ &> 0. \end{aligned}$$

For the types $k_2>\mu_2''$.the difference between U_2' and U_2 is equal to

$$\begin{aligned} U_2'-U_2 &= \mu_1'+(1-\mu_1')(1-\mu_2)-\mu_1-(1-\mu_1)(1-\mu_2), \\ &= -(\mu_1-\mu_1')+(1-\mu_2)(\mu_1-\mu_1'), \\ &= -(\mu_1-\mu_1')\mu_2, \\ &< 0. \end{aligned}$$

Result: The types $\mu_2' < k_2 \leq \mu_2''$ gain, but the types $k_2 \leq \mu_2'$ and $k_2 > \mu_2''$ lose. Hence, the move from mechanism A to B can not benefit some types of player 2 without hurting the other types. Our objective is to examine whether the move from the mechanism A to B yields Pareto improvement. Hence, it is not necessary to investigate the effect on player 1.

Finally, consider mechanism C. The restriction of d' to the areas C_1, C_3, and C_4 coincide with the selection function d of mechanism A: $d'(C_1) = d(A_1)$, $d'(C_3) = d(A_2)$, and $d'(C_4) = d(A_3)$. For the remaining area we put $d'(C_2) = (\alpha, \beta, \gamma)$. Let us examine the effect that the move from mechanism A to C has on the utility level of player 1.

The expected payoff for player 1 from the mechanism C is described by the map

$$k_1 \mapsto U_1' = \begin{cases} 1 - k_1 & \text{if } k_1 \leq \mu_1', \\ 1 - \alpha - \gamma k_1 & \text{if } \mu_1' < k_1 \leq \mu_1'', \\ \mu_2 + (1-\mu_2)(1-\mu_2)(1-k_1) & \text{if } k_1 > \mu_1''. \end{cases}$$

As it is illustrated in Figure 16, the types $k_1 \leq \mu_1'$ and $k_1 > \mu_1''$ receive the same payoff, and the types $\mu_1' < k_1 \leq \mu_1''$ benefit.

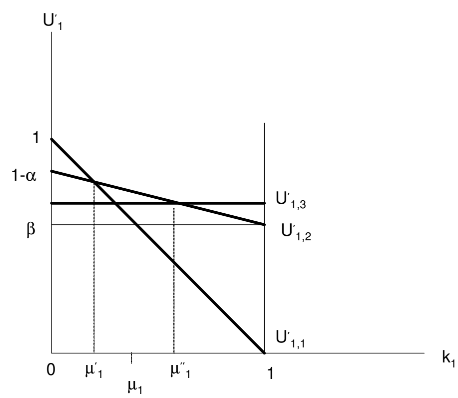

Figure 16

Now, let us examine the effect on player 2. The expected payoff of player 2 from the mechanism C is described by the map

$$k_2 \mapsto U_2' = \begin{cases} \mu_1' + (\mu_1'' - \mu_1')(1 - \alpha - \beta k_2) + (1 - \mu_1'')(1 - k_2) & \text{if } k_2 \leq \mu_2, \\ \mu_1' + (\mu_1'' - \mu_1')(1 - \alpha - \beta k_2) + (1 - \mu_1'')(1 - \mu_2) & \text{if } k_2 > \mu_2. \end{cases}$$

The intersection condition yields the values of (α, β, γ) as following:

$$\begin{aligned} \alpha &= \mu_1' \left(1 - \frac{\mu_2(\mu_1 - \mu_1')}{(\mu_1'' - \mu_1')(1 - \mu_1)}\right), \beta = \frac{\mu_2(\mu_1 - \mu_1')}{(\mu_1'' - \mu_1')(1 - \mu_1)}, \\ \gamma &= \left(1 - \frac{\mu_2(\mu_1 - \mu_1')}{(\mu_1'' - \mu_1')(1 - \mu_1)}\right)(1 - \mu_1'). \end{aligned}$$

First let us examine the effect on the types $k_2 \leq \mu_2$. Observe that the utility map U_2 intersects the utility map U_2' at $k_2 = \mu_2$. Hence, it is sufficient to examine the difference between U_2 and U_2' for the type $k_2 = 0$.

If $k_2 = 0$, then

$$\begin{aligned}
U_2 - U_2' &= \mu_1 + (1-\mu_1) - (\mu_1' + (\mu_1'' - \mu_1')(1-\alpha) + (1-\mu_1'')), \\
&= 1 - (\mu_1' + (\mu_1'' - \mu_1')(1-\alpha) + (1-\mu_1'')), \\
&= 1 - ((\mu_1'' - \mu_1')(1-\alpha-1) + 1), \\
&= \alpha(\mu_1'' - \mu_1'), \\
&= \mu_1' \left(1 - \frac{\mu_2(\mu_1 - \mu_1')}{(\mu_1'' - \mu_1')(1-\mu_1)}\right)(\mu_1'' - \mu_1'), \\
&> 0.
\end{aligned}$$

Hence, the types $k_2 \leq \mu_2$ are worse off.

Now, let us examine the effect on the types $k_2 > \mu_2$. The intersection condition yields

$$\mu_1 + (1-\mu_1)(1-\mu_2) = \mu_1' + (\mu_1'' - \mu_1')(1-\alpha-\beta\mu_2) + (1-\mu_1'')(1-\mu_2).$$

Hence, the difference between U_2 and U_2' for the types $k_2 > \mu_2$ is equal to

$$\begin{aligned}
U_2 - U_2' &= \mu_1 + (1-\mu_1)(1-\mu_2) - \mu_1' - (\mu_1'' - \mu_1')\gamma - (1-\mu_1'')(1-\mu_2), \\
&= (\mu_1'' - \mu_1')(1-\alpha-\beta\mu_2-\gamma), \\
&= (\mu_1'' - \mu_1')\beta(1-\mu_2), \\
&= \frac{\mu_2(\mu_1 - \mu_1')(1-\mu_2)}{(1-\mu_1)}, \\
&> 0.
\end{aligned}$$

Hence, the move from mechanism A to the mechanism C hurts each type of player 2.

We have shown that if types are uniformly distributed over the interval $[0,1]$, then refining a truth revealing herringbone mechanism (in which each player is screened once) does not improve the mechanism: (i) further screening of player 1 benefits some types of player 1, but hurts each type of player 2. (ii) further screening of player 2 benefits some types of player 2, but hurts other types of player 2. In conclusion, there does not exist a finer mechanism that is Pareto superior to A. ■

5. FINAL REMARKS

The main contribution of this chapter is to show that playing escalation game can possibly an efficient way to resolve the conflict. This is shown by following three steps. In first step we characterize the mechanism that is incentive compatible. In second step, we show that the escalation game is an incentive compatible mechanism with two-sided screening. In final step, we show that if types are uniformly distributed, then there is no other incentive compatible mechanism that Pareto improves the payoffs of some types by adding additional reports to each player.

REFERENCES

[1] Abreu, D. and H. Matsushima (1994) "Exact Implementation", Journal of Economic Theory, 64, 1-19.

[2] d'Aspremont, C., and L.A. Gerard-Varet. (1979). "Incentives and incomplete information.", Journal of Public Economics 11, 25-45..

[3] Jackson, M.O. (1991) "Bayesian Implementation", Econometrica, 59, 461-477.

[4] Matthews, S., M.Okuno-Fujiwara, and A. Postlewaite (1990) "Strategic Information Revelation", Review of Economic Studies, 57, 25-47.

[5] Moore, J. and R. Repullo (1988) "Subgame Perfect Implementation", Econometrica, 56, 1191-1220.

[6] Myerson, R.B. (1984) "Two-Person Bargaining Problems with Incomplete Information", Econometrica, 52, 461-487.

[7] Myerson, R. and M. Sattherthwaite (1983) "Efficient Bilateral Trading Mechanism", Journal of Economic Theory, 265-281.

[8] Myerson, R.B. (1979) "Incentive Compatibility and the Bargaining Problem", Econometrica, 47, 61-73.

[9] Myerson, R.B. (1981) "Optimal auction design", Mathematical Operation Research, 6, 58-73.

[10] Roth, A.E. (1979)," Axiomatic Models of Bargaining", Lecture Notes in Economics and Mathematical Systems No 170, Berlin: Springer-Verlag.

[11] Schelling, T.C. (1980), "The Strategy of Conflict", 2nd ed, Cambridge: Harvard University Press.

[12] Wolinsky .A. (1990), "Information Revelation in a Market with Pairwise Meetings", Econometrica, 58, 1-23.

[13] Yu. P.L. (1973) "A Class of Solutions for Group Decision Problem", Management Science, 19, 936-946.

[14] Zeuthen, F. (1930), "Problems of Monopoly and Economic Welfare", London, Routledge.atisfied. ■