DISCUSSION　PAPER　SERIES

高麗大學校　經濟研究所

THE　INSTITUTE　OF　ECONOMIC　RESEARCH
KOREA　UNIVERSITY

『Informational Disadvantage and Bargaining Power』

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Discussion Paper No. 07-11 (May 2007)

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Informational Disadvantage
and Bargaining Power

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May, 2007

Abstract

We consider an alternating offer model where the size of the total surplus is stochastic. Furthermore, the size changes during the time when the offer is being considered. As a result the responder may obtain more information than the proposer. We analyze how the asymmetry in ability to access good information affects the bargaining power, both in terms of the resulting share and in terms of the delay in agreement.

Keywords : alternating offer bargaining, stochastic surplus, informational disadvantage

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1 Introduction

Since Nash (1950) published his seminal paper there have been a lot of rigorous studies about bargaining, both axiomatic and strategic. One important research in strategic approach is the alternating offer bargaining game of Rubinstein (1982). He showed how time preference affects the bargaining power, using an infinite horizon model where players alternate in making offers and couter-offers until an agreement is reached. We consider a situation where the total size of the surplus fluctuates as in Merlo and Wilson (1995, 1998). They considered a quite general n-person bargaining model where the total size of the surplus as well as the order of offers and responses are stochastically determined and the process need not necessarily be symmetric. However, the players know at the beginning of each period what the size of the "cake" is and who moves when in that period. One of the main implications in the transferable utility case (Merlo and Wilson (1998)) is that the set of the agreement states of the unique stationary subgame perfect outcome depends only on the "cake process". Furthermore, the outcome is efficient in the sense that delay occurs when and only when the players expect a bigger cake later on.

Avery and Zemsky (1994) considered a bargaining model where the size of the surplus changes after an offer is made and before a response is determined. For example, while on a strike the firm may gain or lose the market share, the price of the product the firm sells may change. There will always be some changes in economic environment which may affect the size of the cake while the offer is being considered. Offers contingent on the realization of the size of the surplus is ruled out, which is justified when the actual surplus is not

verifiable to a third party. Avery and Zemsky (1994) assumed multiplicative i.i.d. shocks. Hence the cake which is of size x_n when an offer w is made can grow to $x_n a > x_n$ (with probability p) or shrink to $x_n b < x_n$ (with probability $1 - p$) by the time when a response is made. If the offer is accepted the proposer receives $x_n s - w$, $s = a$ or b, and the respondent gets w. If the offer is rejected then the players switch roles and start the same procedure with the new cake of size $x_{n+1} = x_n s$. The common discount factor is δ. They showed that there is a unique stationary subgame perfect equilibrium in which the proposer uses the same strategy regardless of his/her identity and the same for the respondent. They completely characterized the equilibrium strategy. For each (a, p) the strategy is one of three types. If players are sufficiently patient (regime D), the proposer tries to screen by offering w which corresponds to the reservation value of the low type ($s = b$). This offer will be accepted only by the low type and hence inefficient delay can occur, asymmetry of information being the source. If the players are sufficiently impatient (regime A), the proposer gives up and offers w which corresponds to the reservation of the high type ($s = a$). This will be accepted by both types, hence no delay. Between these regimes there is an intermeidiate case where the proposer uses a mixed strategy between high and low offers. They also showed that in regime A (low discount factor) this (pooling equilibrium) is the only subgame perfect equilibrium if the uncertainty is small (i.e., if $\delta a < 1$).

Our model is quite similar to that of Avery and Zemsky (1994). The main difference is that we introduce asymmetry between the two players which represents different abilities to obtain information. We focus on how this

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asymmetry in the access to good information affects the bargaining power. We also extend their model substantially by considering continuous distributions for the (multiplicative) shocks. We obtain the general uniqueness (not just for small δ) as long as the uncertainty is samll. Avery and Zemsky (1994) deals with the case where the proposer is a "buyer"; i.e., the respondent receives a fixed amount offered by the proposer and the proposer gets the residual. We consider both the "seller" case and the "buyer" case.

In the next section we introduce our model where offers are made in terms of the proposer's share. We introduce some basic and preliminary results, and then characterize the subgame perfect equilibrium. In the last section we introduce an alternative model where offers are made in terms of the respondent's share. We also characterize the equilibrium in terms of the resulting share and the delay possibilities.

2   Model I: Selling Game

Two players can generate a surplus and bargain over each player's share of the surplus. The size of the surplus is determined stochastically. It follows a stochastic process characterized by a multiplicative shock; $x_{n+1} = x_n t_n$, where t_n's are i.i.d. random variables with density $f(\cdot)$, and distribution funtion $F(\cdot)$. In period n both players observe x_n, and in period $n = 0, 2, 4, \cdots$ player 1 proposes $x_n a$ for her share. (We will consider an alternative model where each proposer offers the opponent's share.) When player 2 responds, he knows x_{n+1}. If he accepts 1's offer the game ends and the players divide

x_{n+1}. If player 2 rejects 1's offer, he proposes $x_{n+1}b$ for 1's share.[1] Hence, offers are made in terms of player 1's share. Player 1 must respond without observing x_{n+2}.. Hence there is informational asymmetry. If 1 accepts 2's proposal, then the game ends and they divide x_{n+1}. If 1 rejects 2' offer, then she observes $x_{n+2} = x_{n+1}t_{n+1}$, and proposes $x_{n+2}a'$, and so on.[2] $f(\cdot)$ is assumed to have support $[\tau_0, \tau_1]$. We assume that the support is sufficiently small, especially that $\tau_1 - \tau_0 \le 1 - \delta$. One implication of this assumption is that $\delta\tau_1 \le 1$, which corresponds to the small uncertainty case of Avery and Zemsky (1994). The stochastic process $\{x_n\}$ is already assumed to be "stationary". We also assume that the stochastic process $\{x_n\}$ is martingale, i.e., $E(x_{n+m} \mid x_n) = x_n$ or $E(t_n) = 1$. We assume that f is continuous and weakly single-peaked with the maximum peak not too small. More precisely, there is $\widehat{\tau} \in [\tau_0, \tau_1]$ such that $f(t)$ is weakly increasing on $[-\tau_0, \widehat{\tau}]$ and weakly decreasing on $[\widehat{\tau}, \tau_1]$ with $\tau_1 - \widehat{\tau} \le (\tau_1 - \tau_0)\widehat{\tau}$. The last inequality holds if the peak is not too smaller than the mean 1. Uniform distributions as well as any symmetric single peaked distributions satisfy these assumptions. The common discount factor is δ.

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[1]Since there is no uncertainty to be resolved between them, this is equivalent to an offer in terms of 2's share.

[2]We could have introduced a shock which occurs after a player rejected an offer and before that player makes a counter offer. This would not change our results, only making the exposition more complicated.

3   Preliminary Results

One can check that the following "stationary" strategies constitute a subgame perfect equilibrium. (i) Player 1 offers $x_n a^*$; (ii) player 2 accepts $x_n a$ if and only if $x_n a \leq x_{n+1} c^*$; (iii) player 2 offers $x_{n+1} b^*$; and (iv) player 1 accepts $x_{n+1} b$ if and only if $x_{n+1} b \geq x_{n+1} b^*$, where $c^* \equiv (1 - \delta) + \delta b^*$, and a^* and b^* are described in the following proposition.

Proposition 1. The strategy profile described by (i)-(iv) constitutes a subgame perfect equilibrium, if a^* and b^* are the solutions to the following equations

$$a^* = \arg\max_a a \left(1 - F(\frac{a}{c^*}) \right) + \delta b^* \int_{\tau_0}^{a/c^*} t dF(t), \tag{1}$$

Proof.

$$b^* = \delta \left[a^* \left(1 - F(\frac{a^*}{c^*}) \right) + \delta b^* \int_{\tau_0}^{a^*/c^*} t dF(t) \right]. \tag{2}$$

Given 2's strategy, the best 1 can do is to offer $x_n a$ that maximizes

$$
\begin{aligned}
x_n u_1(a, b^*) &\equiv \int_{a/c^*}^{\tau_1} x_n a dF(t) + \delta \int_{\tau_0}^{a/c^*} x_n t b^* dF(t) \\
&= x_n \left[a \left(1 - F(\frac{a}{c^*}) \right) + \delta b^* \int_{\tau_0}^{a/c^*} t dF(t) \right].
\end{aligned}
$$

Hence, 1's strategy is a best response to 2's strategy in any subgame. In a subgame beginning in period $n + 1$ with 2 making an offer, the best 2 can do is to offer $x_{n+1} b^*$. Thus in any subgame beginning with 1 making an offer the best 2 can do is to accept any offer $x_n a$ that satisfies

$$x_{n+1} - x_n a \leq \delta x_{n+1}(1 - b^*),$$

which leads to the strategy accepting any offer with $x_n a \leq x_{n+1} c^*$.   ∎

Next, we want to show that the equilibrium described above is the only subgame perfect equilibrium. In a stationary equilibrium, 1's payoff is

$$x_n u_1(a^*, b^*) = x_n \left[a^* \left(1 - F(\frac{a^*}{c^*}) \right) + \delta b^* \int_{\tau_0}^{a^*/c^*} t dF(t) \right].$$

Choosing a in order to maximize $u_1(a, c^*)$ is equivalent to choosing t in order to maximize

$$\widehat{u}_1(t, b^*) = c^* t [1 - F(t)] + \delta b^* \int_{\tau_0}^{t} r dF(r).$$

By definition c^* satisfies the following

$$c^* = (1 - \delta) + \delta b^*.$$

Hence, we have

$$
\begin{aligned}
\widehat{u}_1(t, b^*) &= (1 - \delta + \delta b^*)[1 - F(t)]t + \delta b^* \int_{\tau_0}^{t} r dF(r) \\
&= (1 - \delta + \delta b^*)[1 - F(t)]t + \delta b^* \left(tF(t) - \int_{\tau_0}^{t} F(r) dr \right) \\
&= (1 - \delta)(1 - F(t))t + \delta b^* t - \delta b^* \int_{\tau_0}^{t} F(r) dr.
\end{aligned}
$$

Lemma 1. Given b^*, the following problem has a unique solution t^*.

$$\max_{t} (1 - \delta)(1 - F(t))t + \delta b^* t - \delta b^* \int_{\tau_0}^{t} F(r) dr \quad s.t. \ t \in [\tau_0, \tau_1]$$

Proof. Define $h(t) = (1 - \delta)(1 - F(t))t + \delta b^* t - \delta b^* \int_{\tau_0}^{t} F(r) dr$. Then,

$$
\begin{aligned}
h'(t) &= (1 - \delta)(1 - F(t) - tf(t)) + \delta b^* - \delta b^* F(t) \\
&= (1 - \delta + \delta b^*)(1 - F(t)) - (1 - \delta) t f(t).
\end{aligned}
$$

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For $t \in [\tau_0, \widehat{\tau}]$, $h'(t)$ is decreasing since $F(t)$ and $tf(t)$ are strictly increasing. $h'(0) > 0$. Furthermore $h'(t) < 0$ for $t \in [\widehat{\tau}, \tau_1)$ since

$$
\begin{aligned}
(1 - \delta + \delta b^*)(1 - F(t)) \quad &< \quad 1 - F(t) \\
&\leq \quad (\tau_1 - t)f(t) \\
&\leq \quad (\tau_1 - \widehat{\tau})f(t) \\
&\leq \quad (\tau_1 - \tau_0)\widehat{\tau}f(t) \quad \text{by the assumption about } \widehat{\tau} \\
&\leq \quad (1 - \delta)tf(t).
\end{aligned}
$$

Hence, there exists a unique maximizer $t^* \in (\tau_0, \widehat{\tau})$ ∎

Hence, we can write as $a^* = c^*t^*$. This lemma guarantees that, given b^*, a^* and $u_1(a^*, b^*)$ are well-defined functions of b^*. Define $g(b^*) \equiv \delta u_1(a^*, b^*)$ for $b^* \in [0, 1]$. Then (a^*, b^*) is a solution to equations (1) and (2) if and only if b^* is a fixed point of $g(\cdot)$ and $a^* = c^*t^*$. The next lemma shows that g has a unique fixed point.

Lemma 2. g has a unique fixed point.

Proof. When $b^* = 0$, player 1 can guarantee a payoff of $\tau_0(1 - \delta)$ by choosing $a = \tau_0(1 - \delta)$. Hence $g(0) > 0$. When $b^* = 1$, we have $c^* = 1$ and $a^* = t^*$. Then $u_1(t^*, 1) \leq \int_{\tau_0}^{\tau_1} \min\{t, t^*\}dF(t) \leq \int_{\tau_0}^{\tau_1} tdF(t) = 1$. Hence, $g(1) \leq \delta < 1$. Furthermore, by the envelope theorem

$$
\begin{aligned}
g'(b^*) \quad &= \quad \frac{d}{db^*}\widehat{u}_1(t^*, b^*) \\
&= \quad \delta t^*[1 - F(t^*)] + \delta \int_{\tau_0}^{t^*} tdF(t) \\
&= \quad \delta \int_{\tau_0}^{\tau_1} \min\{t, t^*\}dF(t) < 1.
\end{aligned}
$$

Therefore, $g(\cdot)$ must have a unique fixed point. ∎

4   Equilibrium

We now prove the uniqueness of the subgame perfect equilibrium.

Theorem 1. Under our assumptions, there exists a unique subgame perfect equilibrium[3], which consists of a strategy porfile described by (i)-(iv), with

$$a^* = (1 - \delta + \delta b^*)t^*,$$

$$b^* = \delta \left[a^* \left(1 - F(t^*)\right) + \delta b^* \int_{\tau_0}^{t^*} t dF(t) \right].$$

Proof. Let us denote the game beginning in period n with player 1 making an offer by Game n and the subgame beginning in period $n + 1$ with player 2 making an offer by Game $n + 1$. Define

$$
\begin{aligned}
\overline{u}_n &= \sup\{u \mid x_n u \text{ is 1's subgame perfect equilibrium payoff in Game } n\}, \\
\underline{u}_n &= \inf\{u \mid x_n u \text{ is 1's subgame perfect equilibrium payoff in Game } n\}, \\
\overline{v}_{n+1} &= \sup\{v \mid x_{n+1} v \text{ is 2's subgame perfect equilibrium payoff in Game } n+1\}, \\
\underline{v}_{n+1} &= \inf\{v \mid x_{n+1} v \text{ is 2's subgame perfect equilibrium payoff in Game } n+1\}.
\end{aligned}
$$

Since Games n and n' (Games $n + 1$ and $n' + 1$) are identical except for the size of the initial surplus, we must have $\overline{u}_n = \overline{u}_{n'} \equiv \overline{u}$ *and* $\underline{u}_n = \underline{u}_{n'} \equiv \underline{u}$ $(\overline{v}_{n+1} = \overline{v}_{n'+1} \equiv \overline{v}$ and $\underline{v}_{n+1} = \underline{v}_{n'+1} \equiv \underline{v})$ for any two even numbers n and

---

[3]To be precise, there are other equilibria including player 2 rejecting 1's offer of $x_n a^*$ with positive probability. Since the probability of this event is zero and neither player's payoff is affected, we ignore this.

$n'.$[4] Then, we have

$$x_{n+1}\underline{v} = x_{n+1} - \delta E_{n+1}(x_{n+2}\overline{u}) = x_{n+1}(1 - \delta\overline{u}), \text{ or}$$

$$\underline{v} = 1 - \delta\overline{u}. \tag{3}$$

Similarly we have

$$x_{n+1}\overline{v} = x_{n+1} - \delta E_{n+1}(x_{n+2}\underline{u}) = x_{n+1}(1 - \delta\underline{u}), \text{ or}$$

$$\overline{v} = 1 - \delta\underline{u}. \tag{4}$$

According to Lemma 1, we have

$$\overline{u} = [1 - \delta + \delta(1 - \underline{v})][1 - F(t^*)]t^* + \delta(1 - \underline{v})\int_{\tau_0}^{t^*} t dF(t) \tag{5}$$

$$\underline{u} = [1 - \delta + \delta(1 - \overline{v})][1 - F(t_*)]t_* + \delta(1 - \overline{v})\int_{\tau_0}^{t_*} t dF(t), \tag{6}$$

where t^* and t_* are the maximizers corresponding to $1 - \underline{v}$ and $1 - \overline{v}$, respectively. Substituting $\delta\overline{u}$ and $\delta\underline{u}$ for $1 - \underline{v}$ and $1 - \overline{v}$, we obtain

$$\overline{u} = [1 - \delta + \delta^2\overline{u}][1 - F(t^*)]t^* + \delta^2\overline{u}\int_{\tau_0}^{t^*} t dF(t), \text{ and}$$

$$\underline{u} = [1 - \delta + \delta^2\underline{u}][1 - F(t_*)]t_* + \delta^2\underline{u}\int_{\tau_0}^{t_*} t dF(t).$$

Unless $\overline{u} = \underline{u}$, we have two fixed points of $g(\cdot)$, namely $\delta\overline{u}$ and $\delta\underline{u}$, which contradicts Lemma 1. Hence we must have $\overline{u} = \underline{u}$. Since player 1 can secure a payoff of $\delta\overline{u}$ in Game $n+1$, the sum of the payoffs of the players in Game $n+1$ is equal to x_{n+1}, which means that the game ends without delay. Hence, the unique subgame perfect equilibrium in Game $n+1$ includes player 2 offering

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[4] For any equilibrium of Game n or $n+1$, we obtain an equilibrium of Game n' or $n'+1$ by multiplying a suitable numer for each strategy.

$x_{n+1}\delta\overline{u}$ and player 1 accepting the offer. Given this, the only equilibrium response of player 2 in Game n is to accept $x_n a < x_{n+1}(1 - \delta + \delta^2\overline{u})$ and reject $x_n a > x_{n+1}(1 - \delta + \delta^2\overline{u}) = x_{n+1}(1 - \delta + \delta b^*)$. Then, $x_n a^*$ is the only equilibrium offer player 1 makes in Game n. ∎

If we write $\overline{u} = \underline{u} = u^*$ and $\overline{v} = \underline{v} = v^*$, then the equations (3) - (6) can be rewritten as

$$
u^* = (1 - \delta)[1 - F(t^*)]t^* + \delta(1 - v^*)\left([1 - F(t^*)]t^* + \int_{\tau_0}^{t^*} t\, dF(t)\right), \tag{7}
$$
$$
v^* = 1 - \delta + \delta(1 - u^*). \tag{8}
$$

Equation (8) represents the normal relationship between the two players' payoffs in Rubinstein (1982) model. The proposer (player 2) extracts all the surplus from early agreement $(1 - \delta)$, which is added to the disagreement payoff $\delta(1 - u^*)$ to yield his equilibrium payoff. Comparing this with equation (7), we can see how the informational disadvantage weakens player 1's bargaining position. If there is no disadvantage, we would have 1 instead of $[1 - F(t^*)]t^*$ or $[1 - F(t^*)]t^* + \int_{\tau_0}^{t^*} t\, dF(t)$. Notice that

$$
[1 - F(t^*)]t^* < [1 - F(t^*)]t^* + \int_{\tau_0}^{t^*} t\, dF(t) = \int_{\tau_0}^{t^*} \min\{t, t^*\}dF(t) < 1.
$$

$1 - F(t^*)$ is the probability that an agreement is reached and t^* is the minimum (normalized) size of the surplus to be harvested this period. If the potential surplus is small $(t < t^*)$ then agreement is delayed and 1's share is proportional to the realized value t. However, when the potential surplus is large $(t \geq t^*)$ 1 receives less than proportional share of the realized value.

5   Model II: Buying Game

In this section we consider a game which is identical to the selling game except that the offer is specified in terms of player 2's share. The analysis is the same as in selling game except that the uniqueness of s^* (an analogue of t^* in Model I) is guaranteed only for large enough δ. Consequently, the uniqueness of the equilibrium is guaranteed only for large enough δ. For such δ, the equilibrium is characterized as follows.

> (i') 1 offers $x_n \alpha^*$ for 2's share;
>
> (ii') 2 accepts 1's offer $x_n \alpha$ if and only if $x_n \alpha \geq x_{n+1} \delta \beta^*$;
>
> (iii') 2 offers $x_{n+1} \beta^*$ for 2's share;
>
> (iv') 1 accepts 2's offer $x_{n+1} \beta$ if and only if $x_{n+1} \beta \leq x_{n+1} \beta^*$.

α^* and β^* satisfies the following equations;

$$
\alpha^* = \arg\max_{\alpha} \int_{\tau_0}^{\alpha/\delta\beta^*} (s - \alpha) dF(s) + \delta \int_{\alpha/\delta\beta^*}^{\tau_1} s(1 - \beta^*) dF(s)
$$

$$
\beta^* = 1 - \left[\int_{\tau_0}^{\alpha^*/\delta\beta^*} (s - \alpha) dF(s) + \delta \int_{\alpha^*/\delta\beta^*}^{\tau_1} s(1 - \beta^*) dF(s) \right]
$$

The equilibrium payoffs are characterized by the following relations;

$$
u^* = (1 - \delta) \int_{\tau_0}^{s^*} s\, dF(s) + \delta \left[1 - v^* \left\{ 1 + \int_{\tau_0}^{s^*} (s^* - s) dF(s) \right\} \right]
$$

$$
v^* = (1 - \delta) + \delta(1 - u^*).
$$

Since $\int_{\tau_0}^{s^*} s\, dF(s) < 1$ and $\int_{\tau_0}^{s^*} (s^* - s) dF(s) > 0$, player 1 is at a disadvantage. In this game delay occurs when the cake is large and so the respondent expects a larger payoff next period, a property which is carried over from Avery and Zemsky (1994).

6   Further Research

In a simple framework we showed how informational disadvantage affects the bargaining power by analyzing the unique subgame perfect equilibrium. It would be nice if we can generalize the information structure so that each player obtains some but not perfect information about future periods. It is yet unknown whether the uniqueness result will be extended to the general case. Another line of extension would be to allow the proposer to choose the type of offer, buying or selling. One might also alow the possibility of proposing a menu that contains both selling and buying prices as in Ben-Ner and Jun (1996).

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