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『How to give up 'wrestling with time': the case of horizontal innovation models』

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Discussion Paper No. 08-03 (June 2008)

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A paper to appear in
Paul Samuelson, Heinz D. Kurz, Stan Metcalfe and John Vint (eds.),
Economic Theory and History of Economic Thought:
Essays in Honour of Ian Steedman

How to give up ‘wrestling with time’:
the case of horizontal innovation models

There was a period of time in Korea in which Ian’s *Marx after Sraffa* (Steedman, 1977) was categorically forbidden along with all other books that carried the name of Marx. I, a non-English speaking fledgling student near the end of the 1970s, pronounced his name ‘eye-an’ when I, cautiously and somewhat pompously, showed a Xeroxed copy of it to a small group of wannabe economists. A guy in the group, who was to become a hard-line marginalist economist, corrected me immediately, in an embarrassing way. Time has passed since then. I have now grown, hopefully, capable of correcting that guy (his economics, not his pronunciation), and it is Ian, more than anyone else, that has undoubtedly helped me grow like this.

I thought, when I first saw Sraffa’s book, that the title was poetic. Ian made the aspiring inchoate, who in the early 1980s was eager to learn Sraffa’s and Sraffian economics, appreciate the full—economic and logical as well as poetic—significance of the book. Ian advised his PhD student struggling with his thesis: ‘You’ve got to work out an *n*-sector case first and then move on to present a simpler or the simplest case to which the properties of the *n*-sector case carry over’; he is still trying to follow this advice. Ian, without any word, patted the self-conscious youngster on the shoulder when he found him going through a personal and academic crisis; it was a quiet gesture which, however, gave the recipient unbelievable encouragement; the recipient’s gratitude was inexpressible (and unexpressed) at that time. Ian, about 20 years later, showed his former student what a model teacher would be like, when he discussed with him every line of his lecture notes before delivering a series of invited lectures to a group of Korean students, checking whether the students would already know this stuff and might be perplexed if presented with that stuff.

Those times did exist and time can never be denied in reality.

The same should be true of the economy and economic theory. Ian investigated implications of time required in consumption (Steedman, 2001). He, with Martin Currie, also explored some eminent economists’ ‘wrestling with time’ and, therefore, with capital and production (Currie and Steedman, 1990). His voice was scarcely heard by mainstream economists. Some even seem to have chosen to give up the economist’s struggle with time entirely, trying to get it round by a sleight of hand, ingenious but logically flawed. One such example is horizontal innovation models.

1. Prologue

Production requires time: a positive quantity of time must elapse from the moment of applying an input to the moment of producing an output. Marginalism explains that the factor that reflects this elapse of time is *interest*; interest is the reward for the waiting involved with the passing of time between input and output.

The close association between time and interest is vividly (if with flaws) pictured by the Austrians. For such economists as Menger, Böhm-Bawerk, Wicksell and Hicks, each of the multiple stages of production, moving along a 'one-way avenue' from the original factors of production to the final good, takes time, and these stages are sequentially connected. If we call a stage of production which is carried out with a positive elapse of time a 'layer' of production,[1] the Austrian economy, with at least one intermediate stage, is a 'multi-layered' economy. The old classical economy of a 'circular process', where commodities are produced by means of commodities, is also a 'multi-layered' economy (if more than one commodity are produced). This follows from Sraffa's (1960, p. 8) assumption that any economic system contains at least one basic product.[2] In contrast, as we shall see in detail shortly, the economy of the traditional neoclassical Solow model is a 'single-layered' one, with time being felt only in one productive process.

The economy in horizontal innovation models is, at first sight, very similar to the Austrian one or, rather, to the old classical one.[3] The economy consists of three sectors: the R&D sector, the intermediate goods sector, and the final good sector. The R&D sector produces new designs; the intermediate goods sector uses these new designs to produce intermediate goods; the final good sector in turn uses intermediate goods to produce the final good; and, finally, the final good is either used for consumption or ploughed back as investment in one or more of the sectors. Production is sequentially connected and in a circle. This economy *ought* to be a 'multi-layered' one.

However, the similarity of (currently available) horizontal innovation models with the Austrian or the classical conception is a mirage. The fact is that, by a sleight of

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[1] The term 'layer' conveys two senses—positive thickness (however thin) and sequence.

[2] The number of 'layers' is at least equal to the number of basic products (and at most equal to the number of commodities). An 'indecomposable' system with $n(\geq 1)$ commodities is an 'n-layered' economy.

[3] The representative models in the horizontal innovation literature are Romer (1990), Rivera-Batiz and Romer (1991) and Bénassy (1998), each representing the three groups of models ('baseline', 'lab equipment', and 'labour for intermediates' models). See Gancia and Zilibotti (2005) for a useful survey and Barro and Sala-i-Martin (2004, Ch. 6) for a good exposition.

hand, these models reduce the essentially 'multi-layered' economy to a 'single-layered' one equivalent to the Solovian economy. The models are so manipulated that production takes time only in *one* of the sectors, whilst in the remaining sectors production is carried out in a timeless setting. The result is an economy with a single layer. This is the result of high dexterity in modelling—but at the cost of economic reality and, alas, also logic.

2. The reign of time: a 'single-layered' economy

The production process of a traditional neoclassical growth model (*e.g.* Solow, 1956) can be schematized as in Figure 1.

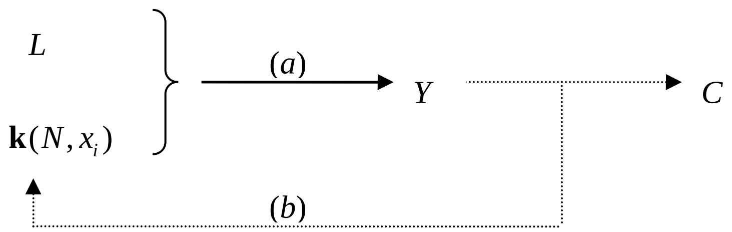

Figure 1: The production processes in the traditional model

Y = final output;
C = consumption;
L = labour;
$\mathbf{k}(N, x_i) = (x_1, x_2, ..., x_N)$ = vector of N types of capital goods.

The final good is produced by means of labour and 'capital goods' (Process *a*).[4] The process is represented by the production function:

(1) $$Y = F(L, \mathbf{k}(N, x_i))$$

In accordance with this production function, one has gross national income as the sum (Z) of the payments for the use of labour and the N types of capital goods:

[4] We are here referring, in difference from the usual understanding, to *multiple* capital goods; see footnote 6 below.

(2) $$Z = wL + \sum_{i=1}^{N} \rho_i x_i$$

where w = the wage rate; ρ_i = the gross rental rate of the ith capital good. The sum is also the aggregate *value* of the final output, measured in terms of the final output itself.

Time elapses in Process *a*; thus, interest *must* accrue on capital. If one uses the usual neoclassical formula of user cost (*e.g.* Jorgenson, 1963),[5] the ith gross rental rate is the sum of the depreciation rate (δ_i) and the rate of interest (r_i), multiplied by the price of the capital good (p_i):

(3) $$\rho_i = (\delta_i + r_i) p_i$$

The produced final good is either used for consumption or 'foregone' for investment. Process *b* describes the 'production' of capital goods by means of no input other than the final good. The 'production' of capital goods is *immediate*; there is no elapsing of time between the application of the final good as the input and the 'production' of capital goods as the output—all that is required is abstaining from, or foregoing, consumption.

Suppose the ith capital good requires ζ_i units of the final good for unit production.[6] That is, Process *b* is represented by the following methods of production:

(4) ζ_i units of the final good → 1 unit of the ith capital good

Here ζ_i stands for the *physical* quantity of the final good foregone as the input in producing one unit of a capital good. Conceptually this quantity *must* be differentiated from the unit *value* (or price) of the ith capital good. The determination of the value of a

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[5] It is well known that this formula is in general at odds with the one that is derived from the Sraffian scheme of joint production. Steedman (1994) shows how even in a 'one-commodity' (one type of fixed-capital good) model the proper (joint-production) way of calculating depreciation produces results that go against those coming from the neoclassical way.

[6] We are assuming that capital goods are 'produced' differentiated by means of nothing other than postponing the consumption of the homogeneous final good, without questioning how such differentiation is possible. The more usual construal is that capital goods are homogenous to each other and also to the final good, so that $\zeta_i = 1, \forall i$ (this construal is, of course, more congenial to the concept of 'production' as foregoing consumption). We use this setting of (pseudo-) multiple capital goods in anticipation of horizontal innovation models, in which the homogenous final good is transformed into different intermediate goods depending on different 'designs'.

good requires (*i*) the choice of the standard of value and (*ii*) the consideration of the passage of time in which inputs are locked up in the process of production. The lock-up of an input over a positive length of time is reflected in the emergence of *interest*, for interest is the reward for waiting.[7] Requirement (*i*) proves essential when there are heterogeneous goods. Requirement (*ii*) should be in effect even when there is only one homogenous good. As for Process *b*, there is only one input (the foregone final good); the standard of value is the final good itself; and the sole input is locked up in the process of production in no time. It follows that the unit value of the capital good, measured in terms of the final good, is simply equal to ζ_i, the physical amount of the final good that has been foregone for that capital good. That is,

(5) $$p_i = \zeta_i$$

Then, the *N* kinds of capital goods, expressed in value terms, are summed to yield the aggregate measure of capital:

(6) $$K \equiv \sum_{i=1}^{N} p_i x_i = \sum_{i=1}^{N} \zeta_i x_i$$

The aggregate 'quantity of capital' *K*, measured in terms of the final good, *is* the total amount of foregone final good.

If one assumes a uniform rate of depreciation ($\delta_i = \delta$), and if one considers long run equilibrium, where the rate of interest is uniform at *r*, one will have the rental rate on capital goods as $\rho_i = (\delta + r)\zeta_i$ and, consequently, the familiar accounting relationship

(7) $$Z = wL + (\delta + r)K$$

Interest on capital in (7) can be construed in two different, but equivalent, ways. If payment for the use of capital is made at the beginning of the production period (*ante factum* payment), interest on capital represent the opportunity cost for the user of the capital: the fund she had spent on capital could instead have been used for lending, thereby obtaining interest. Here, interest is the reward to the waiting on the part of the capital- good user. If payment is made at the end of the production period (*post factum* payment), interest now represents the reward to the waiting on the part of the capital-

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[7] The existence of interest with a positive elapse of time is a conceptual necessity; interest may be zero in actuality.

good provider.[8] Capital goods are provided at the beginning of the period, thus incurring current cost to their provider at the beginning of the period (this current cost is K in total); however, as he is waiting until the end of the period to get paid for renting out these capital goods, his waiting should be compensated for by interest on the current cost. Either way, the result is the same: the existence of interest in (7).

The final good, understood as the physical quantity, is either used for consumption or foregone for investment, the foregone output being in turn used for replenishing used-up capital goods (F_1) and increasing the stock of capital goods (F_2):

$$(8) \qquad Y = C + F_1 + F_2$$

The ‘transformation’ of the final good to capital goods implies

$$(9) \qquad F_1 \equiv \sum_{i=1}^{N} \delta_i \zeta_i x_i = \delta K \quad \text{and} \quad F_2 \equiv \sum_{i=1}^{N} \zeta_i \dot{x}_i = \dot{K}$$

(a dot over a variable denotes the time derivative of the variable). The physical output of the final good which is used in replenishing used-up capital goods, understood as *physical* quantities, is identically equal to the depreciation in the ‘quantity of capital’, understood as *value*. The same is true of the final good foregone for net investment and the increase in the ‘quantity of capital’. Thanks to this, the use of the final good, understood as physical quantity, is represented by the following relationship:

$$(10) \qquad Y = C + \delta K + \dot{K}$$

Z in (7) is the value of the final output measured in terms of itself; thus, it is equivalent to a certain number of the final goods. Y in (10) is the physical quantity of the final output. The national accounting requires that the two be equal. One thus has

$$(11) \qquad wL + rK = C + \dot{K}$$

This is the equation which stands for the usual national accounting constraint in a macroeconomic optimization problem.

What is important for our argument is the observation that from (9) one gets

[8] This case fits better into the understanding of interest as the reward for the foregoing of consumption.

(12) $$K(\tau) \equiv \int_0^{\tau} \dot{K}(t)dt = \int_0^{\tau} F_2(t)dt$$

That is, the 'quantity of capital' (K) *on which the rate of interest is calculated*—which is thus the theoretically proper *value* measure of capital—is the same as the accumulated foregone final good.

3. The vanishing of time: horizontal innovation models

The scene changes (or *must* change) with horizontal innovation models. We shall take the model of Barro and Sala-i-Martin (2004, Ch. 6) as our reference model.[9] The production process of this economy is schematized as Figure 2.

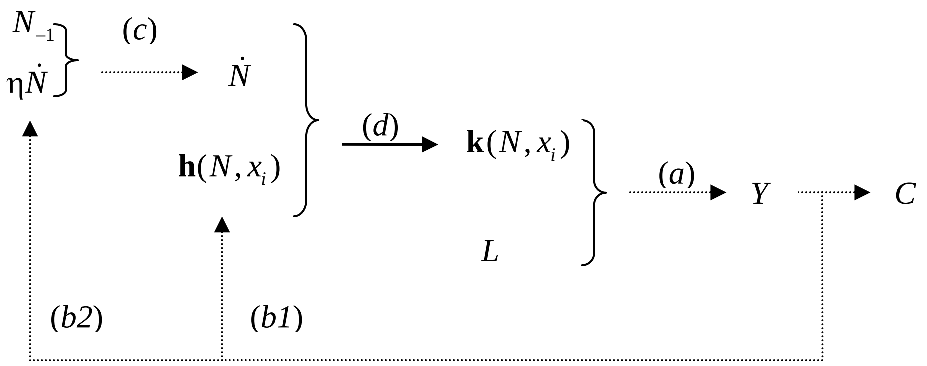

Figure 2: The production processes in an economy *à la* Barro and Sala-i-Martin (2004)

η = quantity of final output required to produce a design;
$\dot{N}$ = number of new designs;
N = total number of designs (N_{-1} = pre-existing designs);
x_i = (physical) quantity of the ith intermediate good;
$\mathbf{h}(N, x_i)$ = vector of final output required to produce N kinds of intermediate goods;
$\mathbf{k}(N, x_i) = (x_1, x_2, ..., x_N)$ = vector of N kinds of intermediate goods.

[9] The same analysis applies to other representative models of horizontal innovation, such as Romer (1990) where the 'accounting measure of capital' is defined in such a way that it is both the physical quantity of foregone final good and the 'quantity of capital' on which interest accrues (which thus must be a value term).

Process *a* is represented by the production function of the final good sector such as (1), which specifically takes the 'Dixit-Stiglitz' form:

(13) $$Y = L^{1-\alpha}\sum_{i=1}^{N} x_i^{\alpha}$$

Similarly to the 'single-layer' case, one will also have the sum *Z* of the wages of labour and gross rentals on capital goods, represented by (2), and the definition of the rental rate, represented by (3). The *numéraire* of the economy is the final good.

Barro and Sala-i-Martin *assume* $r_i = 0, \quad \forall i$ in (3). They seem to think that this follows from their explicit assumption that perfect competition prevails in the final good market. But what this latter assumption implies is no more than that long-run equilibrium will prevail so that *extra* profit is zero in this sector and also the *normal* rates of profit on the respective capital goods are uniform. For the case of $r_i = 0, \quad \forall i$, where r_i stands for the *normal* rate of profit on the *i*th capital goods, one needs an additional assumption: that the production of the final good is *immediate*—that there is *no* time elapsing from the purchase and application of the inputs to the production and selling of the output.[10]

Barro and Sala-i-Martin make another, non-essential, assumption that intermediate goods are non-durable so that they are used up in a unit period of production: $\delta_i = 1$. Thus one has, from (3),

(14) $$\rho_i = p_i$$

It is assumed that the production of one unit of the *i*th intermediate good requires, besides the *i*th design, a uniform amount θ of the final output;[11] thus,

(15) $$\mathbf{h}(N, x_i) = (\theta x_1, \theta x_2, ..., \theta x_n) = \theta \mathbf{k}(N, x_i)$$

This uniformity of the production technique yields an identical supply function of

[10] Some may argue that production time does pass positively in the final good sector so that, at least conceptually if not actually, interest exists; it is only that zero rate of interest is assumed on the input. But, as we shall discuss shortly, one will see this potentially effective defence collapse when one considers the final good sector *vis-à-vis* the other sectors of the economy.

[11] As for the unit of an intermediate good, a more precise statement is that the unit of each intermediate good is so *defined* as to require θ units of final good for the production of one unit of that intermediate good.

intermediate goods. As the Dixit-Stiglitz production function (13) implies an identical demand function for intermediate goods, one consequently has a 'symmetric equilibrium' where

(16) $x_i = x, \quad \forall i$

The same situation also implies that

(17) $p_i = p, \forall i$

Then one will have the national accounting relationship, based on (2),

(18) $Z = wL + pxN$

The value $p_i x_i$ of an intermediate good, of the amount of x_i and measured in terms of the final good, is obtained through the 'arbitrage equation' (Jones, 1998) for the intermediate goods sector. With the assumptions that labour supply is constant and that intermediate goods are nondurable, one has

(19) $rP_R = p_i x_i - \theta x_i$, or, using the results (16) and (17),

(20) $rP_R = px - \theta x, \quad \forall i$

where P_R is the price of a design. The right-hand side of (19) is the profit margin in an intermediate good firm in the same unit period; the left-hand side is the interest cost on a design accruing in a unit period. An arbitrage in the use of fund between purchasing a design and using it in production (thereby obtaining profit) on the one hand and purchasing a design and renting it (thereby obtaining interest) on the other requires that in equilibrium the return in either use of the fund be equal.[12]

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[12] Relationship (20) can be understood in various ways. Rearranging it gives

(20') $P_R = \dfrac{px - \theta x}{r}$

The left-hand side is the purchasing price of a design. The right-hand side is the present value of the flow of 'net revenue' in perpetuity. The two must be equal for equilibrium in an intermediate good sector. Still another rearrangement of (20) enables us to construe it from the perspective of costs (the 'price equation'):

Time runs, for however short an interval, in the production of intermediate goods. The proof is the existence of *interest* on a design. If the production of an intermediate good were immediate, there would be no room for an arbitrage between profit and interest. This is because charging interest in compensation for renting a design presupposes a positive length, however short, of the renting period.

However, one should not fail to notice, the positive existence of time applies *only to the design*. In (20), the cost of the material input is measured of θ. θ is originally given as an engineering constant, the physical amount of the final good required for the unit production of an intermediate good. But the quantity to appear in (20) must be the *value* of this input. Suppose that the intermediate-good-producing firm pays for the final good input at the beginning of the production period. If there is a positive passage of time in production, the firm must be subject to an additional cost, that is, the cost of waiting: interest. Thus the value of the final good input should be $(1+r)\theta$ (the depreciation rate of the final good is 1). Suppose by contrast that the payment for the use of the final good input is made at the end of the period. Then the cost of waiting is incurred to the provider of the input, and she—aware of this fact—will charge the purchaser the price which takes account of this cost of waiting. The price should be, again, $(1+r)\theta$. The formulation (20) is *internally inconsistent* in the matter of treating time: time applies asymmetrically to the inputs in one and the same sector.

This observation is now combined with our previous observation that the final good sector attracts zero rate of profit, only to reveal a further internal consistency of the model. The model is concerned with long run equilibrium—the state characterized by the uniformity of the rates of interest across different sectors (as well as across different capital goods in the final good sector). As there is a positive rate of interest on a design in the intermediate goods sector, the same rate of interest must prevail, in long run equilibrium, in the other sectors. The fact is that, in the model under consideration, the final good sector (and, as we shall see shortly, the design sector) attracts zero rate of interest whereas the intermediate goods sector attracts a positive rate of interest (and that, only on one of the two inputs while no interest accrues to the other). The internal inconsistency, which is double, is conspicuous.

Profit maximization in the R&D sector yields the following relationship:

(20'') $px = rP_R + \theta x$

The right-hand side stands for the total costs of producing a type of intermediate goods by the amount of *x*. A design is a durable good, so that the cost of purchasing it (P_R) is spread over perpetuity; thus, in each 'round' of production of a type of intermediate goods by the amount of *x*, the cost of using the design is a fraction *r* of P_R.

(21) $P_R = \eta$

where η is the amount of the final good required to produce a design. It is all too clear that an assumption is working here that a design is produced *immediately* (that is, in no time) with the application of the final good as input. (The stock of previous designs is also used as an input; however, it incurs no marginal cost for it is a public good.)

One thus has, from (20) and (21),

(22) $px = r\eta + \theta x$

Substituting (22) into (18), one has

(23) $Z = wL + r\eta N + \theta xN$

This is a counterpart of the 'single-layered' case (7): the aggregate income is composed of the aggregate wage, the aggregate interest on the 'assets' of the economy (which consists of the stock of designs, valued in terms of the final good) and the full depreciation of the intermediate goods.

Meanwhile, Processes *b1* and *b2*, taking account of (16) and (17), lead to the same relationship as

(8) $Y = C + F_1 + F_2$, with

(24) $F_1 = \theta xN$ and $F_2 = \eta \dot{N}$

In (8), $F_1 (= \theta xN)$ is the physical amount of the final good which is foregone for the production of intermediate goods; in (23), θxN stands for the value, measured in terms of the final good, of the used-up intermediate goods. They must be equal. Similarly, in (8), $F_2 (= \eta \dot{N})$ represents the physical amount of the final good which is foregone for the increase in the number of designs; in (23), ηN stands for the value, measured in terms of the final good, of the stock of (durable) designs, on which interest ensues. Thus, the value of the stock of the durable assets of the economy (*A*), measured in terms of the final good, is outright equal to the accumulated final good which is foregone for its production:

(25) $$A(\tau) \equiv \eta N(\tau) = \int_0^{\tau} \eta \dot{N}(t)dt$$

The value *Z* of the final output, as it is measured in terms of itself, cannot but be equal to the physical quantity *Y* of the final output; hence,

(26) $$wL + r\eta N = C + \eta\dot{N} \quad \text{or} \quad wL + rA = C + \dot{A}$$

One immediately notes that this is an exact counterpart of (11) in the case of the 'single-layered' economy. The only difference is that now the stock of 'assets' of the economy is the stock of designs ($A \equiv \eta N$) whilst previously it was the stock of capital goods (*K*). Starting from a structure of the economy different from the traditional Solovian one, the Barro and Sala-i-Martin model has built an economy which has exactly the same characteristic as the latter regarding the 'capital' of the economy; or, more generally, regarding the *time structure* of the economy.

This is a result done by a sleight of hand. In the traditional neoclassical model, there is (explicitly) only one layer of production, which involves the passage of positive time from the application of the final good as the investment good until the appearance of the final good as output; there is no positive time involved in the 'transformation' of the final good into the investment good, and this process is explained in the name of 'foregoing consumption'. In horizontal innovation models, there are potentially three layers of production. However, the number of production layers is reduced to one: no time runs either in the final good sector or in the R&D sector. A positive length of time passes only in the intermediate goods sector, from the instant of applying designs as one of the inputs till the point of intermediate goods being produced as the output (bizarrely, time does not apply to the other of the two inputs in this sector, the final good input). The potentially 'triple-layered' economy is reduced to a 'single-layered' one—not on any economic or logical ground but solely by pure assumption, in the name of modelling. In the world of horizontal innovations, time is vanished and that partially—hence, at odds with reality and with logic.[13]

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[13] The starting point of the horizontal innovation literature is Romer (1990). It is suggestive that its predecessor, Romer (1987), explicitly deals with a 'single-layered' economy where output is 'allocated between consumption … and investment in additional capital', and 'foregone output … [is] converted one-for-one into new capital' (1987, p. 60).

4. The return of time: a 'multi-layered' economy

Time discriminates neither in reality nor in logic. Production takes time—this is *reality*. If production in one sector of the economy takes time, then production in the other sectors must take time, too (especially if this 'production' is the process of 'real transformation', that is, transforming inputs into an output which is heterogeneous from the inputs); moreover, if one of inputs in a sector takes time to be used, then other inputs in the same sector must take time to be used, too—this is *logic*.[14] The treatment of time in horizontal innovation models is at variance with reality and violates logic. Reality and logic dictate a positive and undiscriminating existence of time in production.

In the case of the 'single-layered' economy, the 'transformation' of the final good into capital goods in a timeless setting *does* make sense (within its own logic). Here, the final good is in itself usable for multiple purposes; thus, if it is not used for consumption—that is, if it is foregone—then it is automatically used for investment. Foregone output is *in itself* an investment good; the input and the output are identical. However, the situation is different in horizontal innovation models. In the R&D sector, the input is the final good and the output is a design; they *are* different things. This means that some process of *real* transformation must exist; with reality, time must come in. Process *c* in Figure 2, to be real, must take place in time. Similarly in the final good sector, the inputs are labour and intermediate goods and the output is the final good. Heterogeneity between the inputs and the output should require, again, some process of real transformation—and thus time. Process *a* in Figure 2, to be real, must take place in time. (Recall that this process's counterpart in the 'single-layered' economy, Process *a* in Figure 1, *does* take place in time.)

Note further that the only sector in horizontal innovation models in which production takes time (Process *d* in Figure 2) is the counterpart of the production process in the 'single-layered' economy where 'transformation' is done immediately (Process *b* in Figure 1)—and we have said just above that the treatment of time for this sector in the 'single-layered' economy (that is, *no* time) makes sense. This sense-making must have had some appeal to authors of horizontal innovation. Observe how they comment regarding the production of intermediate goods—the only production process which takes time in their model—that '[i]n effect, the inventor of good *j* sticks a distinctive label on the homogenous flow of final output and, thereby, converts this

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[14] One can take up, for the sake of pure logic, the Barro and Sala-i-Martin assumption that a nondurable input is used up 'instantaneously' (that is, over a period of zero length). The zero-length elapse of time, then, should also apply to a durable input (designs). Park (2008) examines the various logical problems that will ensue in this context, in light of Cantor's mathematics of transfinite numbers, Newton's conception of time and Zeno's paradox, all related to the properties of the real numbers.

product into the *j*th type of intermediate good' (Barro and Sala-i-Martin, 2004, p. 291); or in more honest if cruder words, '[o]nce the design for a particular capital good has been purchased (a fixed cost), the intermediate-goods firm produces the capital good with a very simple production function: one unit of raw capital can be *automatically* translated into one unit of the capital good' (Jones, 1998, p. 104, emphasis added).[15] By describing production as involving only 'sticking labels' or as 'automatic translation', they strongly if implicitly suggest that the production of intermediate goods is timeless.[16] But, on the other hand, production in this sector involves the other input—designs; hence, a process of real transformation. The (unwitting) solution is to treat time schizophrenically: a design takes time to be used whilst the final good input takes no time.

Time should be restored in full to all the sectors and to all the inputs. Equations representing the economy must be correspondingly modified. Equations representing production, with full account of time, can be constructed either on the assumption that payment for the use of all inputs is made at the end of the production period (*post factum* payment) or on the assumption that payment for the use of inputs, except for labour, is made at the beginning of the period (*ante factum* payment). The result does not hinge a jot upon which assumptions. The following takes up the first assumption; though this will make the construction more cumbersome than taking the second assumption, it has the advantage of putting the underlying matter in much shaper relief.

The production of a design requires η units of the final good at the beginning of the production period. The supplier of the final good input, who is the producer of the final good, knows that she will get paid for the input at the end of the period when the design is produced. She will incorporate the cost of waiting into the price that she will charge the purchaser of the final good input (= the producer of the design). Thus, the *price* ($\tilde{\eta}$) of the final good input in the design sector is

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[15] All these authors set $\theta = 1$, perhaps in conformity with their feeling of what actually happens in Process *d*. Recall that, similarly, in the 'single-layered' economy, the usual—and conceptually the more consistent—setting for Process *b* is $\zeta_i = 1, \quad \forall i$; see footnote 6 above.

[16] This arouses a suspicion that intermediate goods are in fact undifferentiated from each other and also from the final good input. For there is a close similarity with the case of the 'single-layered' economy, where, for Process *b* in Figure 1, we have forced the values of ζ_i's different from each other and from unity; the more congenial case must be $\zeta_i = 1, \forall i$, implying that capital goods are homogeneous to each other and to the final good. See Park (2007*a*) for the argument that there is nothing in horizontal innovation models that provides theoretical criteria for distinguishing among different intermediate goods.

(27) $\tilde{\eta} = (1+r)\eta$

The design is produced and is ready to be handed over to the producer of intermediate goods. The producer of the design in her turn knows that she will get paid for the design one period of production after the handing over of the design. This will incur the cost of waiting to the producer of the design, and this cost will reflected in the price of the design: the price ($\tilde{P}_R$) will be

(28) $\tilde{P}_R = (1+r)\tilde{\eta}$

Meanwhile, the production of an intermediate good also requires the application of θ units of the final good input at the beginning of the production period. The supplier of the input, who gets paid at the end of the period, will charge

(29) $\tilde{\theta} = (1+r)\theta$

The producer of intermediate goods, even though they pay for the final good input at the end of the production period, will get paid for their output one period of production later. This means that the cost incurred by the final good input will be subject to a further cost, the cost of waiting; hence, the eventual cost of the final good input is $(1+r)\tilde{\theta}$. The total value of the *i*th type of intermediate goods, $\tilde{p}_i x_i$, that their producer will charge their user (= the producer of the final good), is determined by the ‘arbitrage’ condition, which will yield

(30) $\tilde{p}_i x_i = \tilde{p}x = r\tilde{P}_R + (1+r)\tilde{\theta}, \quad \forall i$

The producer of the final good will pay wL for labour and $\tilde{p}xN$ for the N types of intermediate goods at the end of their period of production. Then, the sum of the payments that the final good producer makes for the inputs takes the form similar to Barro and Sala-i-Martin’s.

(31) $Z = wL + \tilde{p}xN$

(One may go further from here, considering that there will be a lag of one period between the moment when the final good output is handed over to consumers, design producers and intermediate-good producers and the moment when these people pay for their purchases. Then the *revenue* for the final good producer will be $\tilde{Z} = (1+r)Z$. Consideration of (31) will suffice, however, as will be made clear shortly.)

Substituting (27), (28), (29) and (30) into (31), we have the following, which is the counterpart of (18):[17]

$$(32) \qquad Z = wL + r\left[(1+r)^2 \eta N\right] + \left[(1+r)^2 \theta x N\right]$$

This is the result obtained by going through the three 'layers' of production, all 'layers' being associated with a positive length of time. The magnitude in the first square brackets on the right-hand side of (32) is the *value* of the stock of designs measured in terms of the final good; that in the second square brackets is the aggregate *value* of the intermediate goods which are used up in production.

The national accounting for the use of the final good must be the same as before, for it refers to the relationship among physical quantities, without involving time: one has (8) with

$$(33) \qquad F_1 = \theta x N \quad \text{and} \quad F_2 = \eta \dot{N}$$

With time reinstituted in production, there holds no longer the quantitative identity either between the forgone output for the production of designs and the value of their stock, or between the foregone output for intermediate goods and the values of their stock. Time drives a wedge between the physical quantity of an input that is actually expended and its value.

The total amount of the foregone final good for intermediate goods is $F_1 (= \theta x N)$, whilst its aggregate value in the production of the final good, measured in terms of the final good, is $(1+r)^2 \theta x N$. The interest factor in the value term reflects the length of time which has elapsed, first, from the foregoing of consumption to the

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[17] The same expression is obtained even if one assumes the *ante factum* payment for inputs (except for labour). The counterpart 'price equations' will be

$$(28') \qquad P_R = (1+r)\eta$$

$$(30') \qquad px = rP_R + (1+r)\theta x, \quad \forall i$$

$$(31') \qquad Y = wL + (1+r)\, pxN$$

production of intermediate goods and, second, from their use in the production of the final good and the payment for their use.

The same reasoning applies to the case of designs. The amount $F_2(=\eta\dot{N})$ of the final good is foregone as an input for the production of designs. As this amount accumulates, the stock of designs grows to:

$$\eta N(\tau) = \int_0^\tau F_2(t)dt \tag{34}$$

But as the input ηN of the final good goes through the three stages of production, interest accrues on their stock correspondingly three 'times'—and then the value of the stock of designs (A) diverges from the accumulated final good foregone for the designs.

$$A(\tau) \equiv (1+r)^2 \eta N \neq \int_0^\tau \eta\dot{N}(t)dt \quad \text{or} \quad \dot{A}(\tau) \neq F_2 \tag{35}$$

The reader who is versed in capital theory will not fail to note that this will cause all sorts of problems that are well-known in that area of economic theory.

As before, we should have the necessary equality between Z (the total sum of the payments made for the final output, measured in terms of the final good itself) and Y (the physical amount of the final good that is produced):

$$wL + r\left[(1+r)^2 \eta N\right] + \left[(1+r)^2 \theta x N\right] = C + \eta\dot{N} + \theta x N \tag{36}$$

which some manipulation will transform into

$$wL + \left[r(1+r)^2 + r(2+r)\eta^{-1}\theta x\right]\eta N = C + \eta\dot{N} \tag{37}$$

It is difficult to miss the contrast with (26).

If the representative household maximises discounted utility over infinite lifetime on the basis of the utility function

$$U(t) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma} \tag{38}$$

with (37) as the constraint, the resulting Euler equation is

$$(39) \qquad \frac{\dot{C}}{C} = \frac{\phi - \rho}{\sigma}$$

where $\phi \equiv r(1+r)^2 + r(2+r)\eta^{-1}\theta x$; $\rho\,(>0)$ = the rate of time preference; σ^{-1} = the elasticity of intertemporal substitution. Of course, for Barro and Sala-i-Martin, who use (26) as the constraint, the Euler equation is

$$(40) \qquad \frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}.$$

Barro and Sala-i-Martin's r in (40) should have been ϕ; they *over*estimate the equilibrium rate of interest.

5. The revenge of time: complication

What is, then, the eventual result of time being fully reflected in the model under consideration? From maximising the profit of the final good sector, one gets the equality between the price of the *i*th intermediate good and its marginal product:

$$(41) \quad \tilde{p}_i = \alpha L^{1-\alpha} x_i^{\alpha-1}$$

Profit maximisation in the intermediate goods sector yields the following 'monopoly pricing' rule:

$$(42) \quad \tilde{p}_i = (1+r)^{-2} \alpha^{-1} \theta$$

As we have discussed above, the 'price equation' for the intermediate goods sector is

$$(43) \quad \tilde{p}_i x_i = r(1+r)^2 \eta + (1+r)^2 \theta x_i$$

These three relationships constitute an independent system with the corresponding three unknowns: $\tilde{p}$, x and r (by virtue of symmetric equilibrium, one can drop the subscript *i* from p_i and x_i). By manipulating them, one ends up with the following equation in *r*:

(44) $$(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\theta^{\frac{-\alpha}{1-\alpha}}L(1+r)^{\frac{-2}{1-\alpha}}-\eta r=0$$

It can be shown that equation (44) has a unique solution for r (hence, unique solutions for $\tilde{p}$, x_i, ϕ and $\dot{C}/C$).[18] However, in general, the equation cannot be solved algebraically.[19] Some uncomfortable implications may follow: for example, it is doubtful whether one can compare the above equilibrium (which is for the decentralised economy) with the centralised equilibrium in order to evaluate the externality effect of decentralised R&D—an essential element for policy considerations. The appeal of the model sharply drops with the heightened but due complexity.

The problem generalises. The model under consideration (and horizontal innovation models in general) has three sequentially connected sectors, and this *relatively simple* structure may have done the model the service of securing the existence of a unique equilibrium even when time is fully taken into account. Such a service may not be expected to come when one wishes to build a model that requires more than three sectors. With the number of sectors increasing, the compounding of interest may exert a rapidly compounding disturbing power on the equilibrium state—existence, uniqueness and stability—of the economy.

6. Epilogue

One cannot help but suspect that Barro and Sala-i-Martin have turned a blind eye to internal inconsistency—the reduction of a 'multi-layered' economy to a 'single-layered' one—for the simplicity of the equilibrium solution. Internal inconsistency is not confined to their model. Different horizontal innovation models use different settings (assumptions), but they invariably incur internal consistency of the kind similar to the

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[18] Expressing the left-hand side of (44) as $f(r;\alpha,\theta,\eta,L)$, one has

$$f(0;\alpha,\theta,\eta,L)>0\,;\ \frac{df}{dr}<0,\ \forall r\in[0,\infty)\,;\ \lim_{r\to\infty}\frac{df}{dr}=-\eta\ >-\infty\,.$$

Thus, there must be a unique $r^*>0$ such that $f(r^*;\alpha,\theta,\eta,L)=0$. On the basis of this, one can obtain unique values of $\tilde{p},x,\phi$ and $\dot{C}/C$, using relevant equations.

[19] Barro and Sala-i-Martin (2004) has, instead of (41) and (42),

(42') $p_i=\alpha^{-1}\theta$

(43') $p_i x_i = r\eta+\theta x_i$

Together with (41) (with p instead of $\tilde{p}$), the system solves for p, x and r uniquely and explicitly.

one disclosed above. For example, in Romer (1990)—the canonical model of horizontal innovation—time is *duly* taken into account in both the design sector and the intermediate goods sector, but this account is completely brushed away by his 'accounting measure of capital' in the final good sector (see Park, 2007*b*). The sleight of hand that horizontal innovation models exercise is tantamount to abandoning previous eminent economists' valiant and honest struggles of 'wrestling with time'. This is, however, no more than a symptom of malaise prevalent in mainstream economics, an aspect of an attitude which is getting increasingly customary there—the attitude of ignoring *economic* logic in the name of 'modelling'.

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