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Heterogeneity and Cyclical Unemployment*

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Abstract

We model worker heterogeneity in the rents from being employed in a Diamond-Mortensen-Pissarides model of matching and unemployment. We show that heterogeneity, reflecting differences in match quality and worker assets, reduces the extent of fluctuations in separations and unemployment. We find that the model faces a trade-off—it cannot produce both realistic dispersion in wages across workers and realistic cyclical fluctuations in unemployment.

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1. Introduction

Shimer (2005), Hall (2005), and Costain and Reiter (2003) each argue that matching models with flexible wages fail to explain business cycle fluctuations—the models generate much more procyclical wages and much less cyclical unemployment and job finding rates than observed. But, as discussed by Mortensen and Nagypal (2005) and Hagedorn and Manovskii (2008), this negative conclusion rests on an assumption that employment renders substantial economic rents relative to the monetary, home production, and leisure benefits to not being employed in the market. For example, Hagedorn and Manovski, by allowing the payout to unemployment to replace 95 percent that of employment, are able to rationalize the cyclical volatility of unemployment under the matching model with flexible wages and exogenous separations. So establishing the rents from employment is key to judging how well the matching model captures cyclical fluctuations. Judging the size of these rents *a priori* is problematic as they reflect, not only direct payments, but also individuals' valuations of leisure and home production.

We shed light on this question by considering endogenous separations. We introduce heterogeneity in reservation wages into a business cycle model of separations, matching, and unemployment. As in Mortensen and Pissarides (1994), we allow workers to face shocks to their employment matches, with bad draws possibly leading to endogenous separations. We depart from Mortensen and Pissarides by allowing for diminishing marginal utility in consumption and imperfect insurance as in Aiyagari (1994). As a result, willingness to trade work for search depends on the worker's wealth—workers

with lower savings, reflecting bad past earnings shocks, are less willing to separate. The heterogeneity in match quality and assets jointly determine the distribution of rents to being employed. In turn, this distribution drives both the level and cyclical nature of unemployment.

We find a trade-off between generating realistic dispersion in wages reflecting match quality and realistic cyclical fluctuations in unemployment. For instance, with the high replacement rate suggested by Hagedorn and Manovski, the model can generate reasonable average rates of separation and unemployment only if shocks to match quality are extremely small, so small that the cross-sectional standard deviation in wages from match quality is less than two percent. With Shimer's calibrated replacement rate of 40 percent, by contrast, substantial shocks to match quality are required to match average turnover and unemployment rates, with these shocks generating a cross-sectional standard deviation in wages from match quality of 18 percent. We argue that this latter figure, 18 percent, is consistent with micro data, in particular the importance of the match component to earnings dispersion estimated by Woodcock (2007) using matched worker-firm data. Thus we conclude that the calibrated search and matching model, with reasonable wage dispersion, fails to capture the cyclical nature of unemployment rates.

The model is presented in the next section then calibrated in Section 3. In Section 4 we examine the model's steady-state features. We show that both a high replacement rate and little heterogeneity, in match quality and assets, are key for producing an economy with many workers with low rents from employment—the scenario that generates a large response of unemployment to aggregate shocks. We require our benchmark econ-

omy to exhibit realistic separation and unemployment rates and a reasonable dispersion in wages reflecting match quality. In turn, this requires a relatively low replacement rate, comparable to Shimer's calibration, and significant match quality shocks. We consider an alternative economy that matches the average unemployment with a high replacement rate, but it requires extremely small shocks to match quality.

The model's cyclical predictions are presented in Section 5. The model can generate a very cyclical unemployment rate, but only if there is little dispersion in match quality. With little cross-sectional dispersion there is an important spike up in separations at the onset of a downturn. Secondly, again for low dispersion, the rents to vacancy creation are highly procyclical. Thirdly, the model generates a new avenue for cyclicity in unemployment—in response to higher expected unemployment duration, separations become skewed toward workers with higher assets and higher reservation match qualities. Because these workers generate smaller expected surplus to employers, this acts to further depress vacancy creation in a recession. However, for our benchmark model that displays reasonable dispersion in match wages, we find that separations, vacancies, and unemployment all exhibit much less cyclicity than seen in the data.

Besides Mortensen and Pissarides (1994), an antecedent to our model is Chang and Kim (2006, 2007). They show that the cross-sectional distributions of wealth and worker productivity play a critical role in determining the elasticity of aggregate labor supply in a competitive equilibrium. Nakajima (2007), Shao and Silos (2007), and Krusell, Mukoyama, and Sahin (2008) have also recently adopted diminishing marginal

utility in consumption and imperfect risk sharing into the Mortensen-Pissarides model.¹ However, only Shao and Silos allow for heterogeneous productivity; and none of these authors allows for endogenous separations.

2. Model

We build on the model of cyclical unemployment in Mortensen and Pissarides (1994). We depart from Mortensen and Pissarides by letting workers be risk averse, face a borrowing constraint, and value leisure, distinct from goods consumption, from being unemployed.

2.1. Environment

There is a continuum of infinitely-lived workers with total mass equal to one. Each worker has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} + B \cdot l_t \right\},$$

where $0 < \beta < 1$ is the discount factor, and $c_t (> 0)$ is consumption. The parameter B denotes the utility from leisure when unemployed. l_t is 1 when unemployed and otherwise zero. In Mortensen and Pissarides (1994), and many extensions, there is no

¹Other papers that entertain wealth effects in modeling search include Pissarides (1987), Gomez, Greenwood, and Rebelo (2001), and Hall (2006). Haefke and Reiter (2006) generate dispersion in reservation wages, while maintaining linear utility and no match-specific productivity, by assuming heterogeneity in individuals' value of home production. Several papers (Darby, Haltiwanger, and Plant, 1985, Baker, 1992, and Pries, 2007) have argued that lower job-finding rates during recessions may reflect a compositional shift toward workers who display lower job-finding rates. But these papers impose this shift exogenously, whereas our model, by allowing for wealth effects, predicts such a shift in recessions toward unemployed workers with high reservation match qualities.

valuation of leisure; so a marginal rate of substitution between leisure and consumption is not defined. Here the marginal rate of substitution ($c^{-\gamma}/B$) is decreasing in c . This provides the basis for a worker's reservation match quality to be increasing in consumption and thereby savings.

Each period a worker either works (employed) or searches for a job (unemployed). A worker, when working, earns wage w . If unemployed, a worker receives an unemployment benefit b . Each can borrow or lend at a given real interest rate r by trading the asset a . But there is a limit, \underline{a} , that one can borrow; that is $a_t > \underline{a}$. Real interest rate r is determined exogenously to fluctuations in this particular economy (small open economy).

There is also a continuum of identical agents we refer to as entrepreneurs (or firms). Entrepreneurs have the ability to create job vacancies with a cost κ per vacancy. Entrepreneurs are risk neutral (diversifying ownership of their investments across many vacancies and across economies) and maximize the discounted present value of profits

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \pi_t.$$

There are two technologies in this economy, one that describes the production of output by a matched worker-entrepreneur pair and another that describes the process by which workers and entrepreneurs become matched. A matched pair produces output

$$y_t = z_t x_t,$$

where z_t is aggregate productivity and x_t is idiosyncratic match-specific productivity (i.e., match quality). Both aggregate productivity and idiosyncratic productivity evolve

over time according to Markov processes, respectively $Pr[z_{t+1} < z'|z_t = z] = D(z'|z)$ and $Pr[x_{t+1} < x'|x_t = x] = F(x'|x)$. For newly formed matches, idiosyncratic productivity starts at the mean value of the unconditional distribution, which is denoted by \bar{x} .²

The number of new meetings between the unemployed and vacancies is determined by a matching function

$$m(v, u) = \eta u^{1-\alpha} v^\alpha$$

where v is the number of vacancies and u is the number of unemployed workers. The matching rate for an unemployed worker is $p(\theta) = m(v, u)/u = \eta\theta^\alpha$, where $\theta = v/u$ is the vacancy-unemployment ratio, the labor market tightness. The probability that a vacant job matches with a worker is $q(\theta) = m(v, u)/v = \eta\theta^{\alpha-1}$.

A matched worker-firm constitutes a bilateral monopoly. We assume the wage is set by bargaining between the worker and firm over the match surplus. This is discussed in the next subsection. The match surplus reflects the value of the match relative to the summed worker's value of being unemployed and the entrepreneur's value of an unmatched vacancy (which is zero in equilibrium). There are no bargaining rigidities; separations are efficient for the worker-firm pair, occurring if and only if match surplus

²With dispersion in initial match quality, then workers will turn down some new matches. This would, everything else equal, increase unemployment duration and the average rate of unemployment, especially for the economy calibrated below to reflect significant wage dispersion. Therefore, to maintain the target rate of unemployment, this extension would require a lower value of unemployment (parameter B) for our calibrated model. This would act to further reduce the model's ability to produce cyclical fluctuations in unemployment. Because this extension predictably reinforces our conclusion, that there is a severe tradeoff between generating realistic wage dispersion and unemployment cyclicality, we ignore it in order to streamline our model. Hornstein, Krusell, and Violante (2006) examine at length the tension between generating reasonable wage dispersion and average unemployment duration in a calibrated Diamond-Mortensen-Pissarides model.

falls below zero.

The timing of events can be summarized as follows.

1. At the beginning of each period matches from the previous period's search and matching are realized. Also aggregate productivity z and each match's idiosyncratic productivity x are realized.
2. Upon observing x and z , matched workers and entrepreneurs decide whether to continue as an employed match. Workers breaking up with an entrepreneur become unemployed. There is no later recall of matches.
3. For employed matches, production takes place with the wage reflecting worker-firm bargaining. Also at this time, unemployed workers and vacancies engage in the search/matching process.

2.2. Value functions

Consider a recursive representation, where W , U , J , and V denote respectively the values for the employed, unemployed, a matched entrepreneur, and a vacancy. All value functions depend on the measures of workers. In each labor market, two measures capture the distribution of workers: $\mu(a, x)$ and $\psi(a)$, respectively, represent the measures of employed workers and unemployed workers during the period.³ The evolution of these measures is given by \mathbf{T} , i.e., $(\mu', \psi') = \mathbf{T}(\mu, \psi, z)$. For notational convenience, let $\mathbf{s} = (z, \mu, \psi)$.

³Let \mathcal{A} and \mathcal{X} denote sets of all possible realizations of a and x , respectively. Then $\mu(a, x)$ is defined over σ -algebra of $\mathcal{A} \times \mathcal{X}$ while $\psi(a)$ is defined over σ -algebra of \mathcal{A} .

From the model discussion, it follows that the worker's value of being employed is

$$W(a, x, \mathbf{s}) = \max_{a'_e} \left\{ u(c_e) + \beta E [\max\{W(a'_e, x', \mathbf{s}'), U(a'_e, \mathbf{s}')\} | x, \mathbf{s}] \right\}, \quad (1)$$

subject to

$$c_e = (1+r)a + w - a'_e$$

$$a'_e \geq \underline{a}.$$

The value of being unemployed, recalling that $p(\theta)$ is the probability that an unemployed worker matches, is

$$U(a, \mathbf{s}) = \max_{a'_u} \left\{ u(c_u) + \beta(1-p(\theta(\mathbf{s})))E[U(a'_u, \mathbf{s}') | \mathbf{s}] + \beta p(\theta(\mathbf{s}))E[W(a'_u, \bar{x}, \mathbf{s}') | \mathbf{s}] \right\}, \quad (2)$$

subject to

$$c_u = (1+r)a + b - a'_u$$

$$a'_u \geq \underline{a}.$$

For an entrepreneur the value of a matched job is:

$$J(a, x, \mathbf{s}) = zx - w(a, x, \mathbf{s}) + \beta E [\max\{J(a'_e, x', \mathbf{s}'), V(\mathbf{s}')\} | x, \mathbf{s}]. \quad (3)$$

The value of a vacancy is:

$$V(\mathbf{s}) = -\kappa + \frac{1}{1+r} q(\theta(\mathbf{s})) \int E[J(a'_u, \bar{x}, \mathbf{s}') | \mathbf{s}] d\tilde{\psi}(a'_u) + \frac{1}{1+r} (1 - q(\theta(\mathbf{s}))) V(\mathbf{s}'), \quad (4)$$

where recall that κ is the vacancy posting cost and $q(\theta)$ is the probability that a vacancy is filled. $\tilde{\psi}(a'_u)$ denotes the measure of unemployed workers at the end of a period after decisions on asset accumulation are made.

2.3. Wage Bargaining

There is a setting for bilateral bargaining between a matched vacancy and worker. We follow much of the literature in assuming that wages reflect a Nash bargaining solution, such that

$$\underset{w}{\operatorname{argmax}} \ (W(a, x, \mathbf{s}; w) - U(a, \mathbf{s}; w))^{\frac{1}{2}} (J(a, x, \mathbf{s}; w) - V(\mathbf{s}; w))^{\frac{1}{2}} \quad (5)$$

subject to

$$S(a, x, \mathbf{s}) = W(a, x, \mathbf{s}) - U(a, \mathbf{s}) + J(a, x, \mathbf{s}) - V(\mathbf{s}),$$

for all (a, x, \mathbf{s}) .⁴

The Nash solution generates a wage that is increasing in a worker's assets, reflecting that being unemployed is less painful for a worker with greater assets. (Below see Figure 1.) In turn, this makes the vacancy creation decision depend on the assets of the unemployed. We believe these features potentially generalize to settings with wage posting by firms and directed search by workers. For instance, Acemoglu and Shimer (1999) model directed search by risk averse workers. They show that the distribution of posted wages exhibits a higher mean, with longer queues, if workers are less risk averse, as then workers are less willing to take lower wages in order to raise the probability of employment. We would expect increased assets for the unemployed, for given risk aversion, to exhibit comparative statics in this same direction in their setting.

⁴Rubinstein (1982) demonstrates in a stationary environment that the Nash solution can be interpreted as the outcome of a noncooperative game with sequential offers. In our stochastic setting without linear utility this interpretation does not literally hold (Coles and Wright, 1998.) We adopt the Nash solution, however, partly for comparability with the related literature.

2.4. Evolution of measures

The measures for workers employed and unemployed, $\mu(a, x)$ and $\psi(a)$, evolve as follows.

$$\begin{aligned} \mu'(A^0, X^0) &= \int_{A^0, X^0} \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{\{x' \geq x^*(a', s'), a' = a'_e(a, x, s)\}} dF(x'|x) d\mu(a, x) da' dx' \\ &\quad + p(\theta(s)) \int_{A^0} \int_{\mathcal{A}} \mathbf{1}_{\{x' = \bar{x}, a' = a'_u(a, s)\}} d\psi(a) da' dx' \end{aligned} \quad (6)$$

$$\begin{aligned} \psi'(A^0) &= \int_{A^0} \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{\{x' < x^*(a', s'), a' = a'_e(a, x, s)\}} dF(x'|x) d\mu(a, x) da' \\ &\quad + (1 - p(\theta(s))) \int_{A^0} \int_{\mathcal{A}} \mathbf{1}_{\{a' = a'_u(a, s)\}} d\psi(a) da' \end{aligned} \quad (7)$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

2.5. Equilibrium

The equilibrium consists of a set of value functions, $W(a, x, s)$, $U(a, s)$, $J(a, x, s)$, a set of decision rules for consumption $c_e(a, x, s)$, $c_u(a, s)$, asset holdings $a'_e(a, x, s)$, $a'_u(a, s)$, and separating $x^*(a, x, s)$, the wage schedule $w(a, x, s)$, the labor-market tightness $\theta(s)$, and a law of motion for the distribution, $(\mu', \psi') = \mathbf{T}(\mu, \psi, z)$. Equilibrium is defined by the following.

1. (Optimal Savings): Given θ , w , μ , ψ , and \mathbf{T} , a' solves the Bellman equations for W , U , J and V in (1), (2), (3), and (4).
2. (Optimal Separation): Given W , U , J , V , μ , ψ , and \mathbf{T} , x^* satisfies $S(a, x^*, s) = 0$.
3. (Nash Bargaining): Given W , U , J and V , w satisfies (5).

4. (Free Entry): Given w , x^* , J , μ , ψ , and \mathbf{T} , the vacancies are posted until $V = 0$.
5. (Rational Expectations): Given a'_e , a'_u and x^* , the law of motion for distribution $(\mu', \psi') = \mathbf{T}(\mu, \psi)$ is described in (6) and (7).

3. Model Calibration

We calibrate our model in order to present its predictions for business cycle fluctuations. But, prior to considering cycles, in Section 4 we display the model's steady-state features, in particular showing how the heterogeneity of worker's match quality and assets determine the distribution of rents to employment.

3.1. The benchmark economy

We consider two calibrated models that yield the same steady-state rates of separations and unemployment, but differ sharply in their predictions for the average level, and dispersion, in match rents. Our benchmark calibration reflects sizable rents to employment. These rents primarily reflect dispersion in wages due to important differences in match quality. We argue this cross-sectional dispersion is consistent with that estimated on matched employer, employee data (Woodcock, 2007). We also describe an alternative calibration that is designed to generate sizable cyclical fluctuations. But this calibration requires remarkably small dispersion in match quality.

Starting with preferences, we assume a relative risk aversion parameter γ equal to one. We choose a monthly discount factor β of 0.995 and an annualized real interest

rate of 6 percent. These together generate average assets equal to 18 months of labor earnings, which is about the median ratio of net worth to family earnings we calculate from the Survey of Income and Program Participation (SIPP) data. (See Bils, Chang, and Kim, 2007, for more details on statistics derived from the SIPP.) We set the borrowing constraint to six, so approximately six month labor income, as we see few households in the SIPP with unsecured debt exceeding this amount. The borrowing constraint has a relatively small impact on average asset holdings.

The key outcomes we target are the average rates of unemployment and separations. We target an average unemployment rate of 6 percent and a monthly separation rate of 2 percent. (A separation rate of 2 percent is consistent with what we see for the SIPP data.) These rates for unemployment and separations imply a steady-state job finding rate, of 0.313, a rate consistent with transition hazards reported by Meyer (1990). The vacancy posting cost κ is chosen so that the vacancy-unemployment ratio (θ) is normalized to 1 in the steady state. The matching technology is Cobb-Douglas; $m(v, u) = .313 v^\alpha u^{1-\alpha}$ hits the steady-state finding rate. We set the matching power parameter α to 0.5.

Remaining to calibrate are the payouts to being unemployed, which are unemployment insurance b and leisure utility B , and the magnitude of match-specific shocks. These are key determinants of rates of separations and unemployment. If unemployment is made more attractive, everything else equal, this clearly leads to higher separation and unemployment rates. We calibrate our benchmark economy to generate rents to employment comparable to that in Shimer (2005). To do so, we first considered a

special case of our model that, like Shimer's, has linear utility and no match-quality shocks or endogenous separations—separations occur exogenously at a rate of 2 percent monthly. We follow Shimer by calibrating unemployment insurance to $b = 0.4$, with $B = 0$. That economy generates capitalized match surplus ($S = W + J$) of 3.3, that is, a little over three months of match output. This in turn directly implies a vacancy creation cost κ of 0.52 (half of a month's output). We calibrate our benchmark economy to exhibit these same values, $S = 3.3$ and $\kappa = 0.52$. Keeping $b = 0.4$, we find this requires a value for leisure of $B = 0.15$. That is, a consumer views this leisure comparably, in terms of flow utility, to 15 percent higher consumption.

Greater match-quality shocks, like higher replacement rates, create more separations and higher average unemployment. We set the persistence of the match-specific shock to be quite high, $\rho_x = 0.97$. Finally, given the other parameters, we set the standard deviation of these match-quality shocks in order achieve the target separation and unemployment rates of 2 and 6 percent. This dictates $\sigma_x = 0.130$. We find this generates a standard deviation of wages across workers of 18 percent. We view this as a reasonable match to data, as it is consistent with the size of the match component in the dispersion of earnings estimated by Woodcock (2007). Woodcock allows for individual, employer, and match components in explaining dispersion in (ln)earnings for a large sample of matched employer-employee records across 37 states. He finds a variance of the match component in earnings that is nearly one-fifth the magnitude of overall earnings variance. If we assume this same ratio holds for (ln)wage rates, and allow for a standard deviation for (ln)wages of 0.40 to 0.45, this implies a standard

deviation in wages from match quality of 18 to 20 percent.⁵ This is extremely close to our benchmark model's standard deviation of wages of 18 percent.⁶

3.2. The high-volatility economy

For contrast, we consider a cyclically sensitive economy calibrated so that, in response to aggregate shocks to productivity, it exhibits a standard deviation of unemployment that is 9.5 times that in productivity—where 9.5 reflects the ratio of these standard deviations reported by Shimer (2005). To achieve this targeted cyclicality, while maintaining an average rate of 6 percent unemployment, we free up the leisure value of unemployment B and the variability of match-quality shocks σ_x , keeping other parameters at their benchmark values.⁷ The economic payoffs while unemployed are key, not only to the average rate of unemployment, but also to its cyclical volatility (Hagedorn and Manovski, 2008, and Mortensen and Nagypal, 2005)—less surplus to employment increases cyclical volatility of vacancies and unemployment. By contrast, greater volatility of match-specific productivity (higher σ_x) has opposite impacts on the level versus cyclical volatility of unemployment. Greater match-quality shocks create more separations and higher average unemployment, but actually reduce the cyclical

⁵Woodcock's sample reflects 49 million person-year observations over the years 1990 to 1999 for workers aged 25 to 65. The data reflect a matching of Census and state unemployment insurance data. The statement of a standard deviation for (ln)wages of 0.40 to 0.45 reflects CPS data for 1990 to 2002 for workers ages 25 to 65. These data show standard deviations in ln(wages) of 0.44 for men and 0.43 for women.

⁶The dispersion in wages for our model partially reflects dispersion in assets, as wages are increasing in assets. But most of the wage dispersion for the model reflects differences in match quality.

⁷It requires a very slightly different discount factor ($\beta = 0.9949$, versus 0.9948 for the benchmark) to hit average asset holding of 18 months earnings.

volatility of separations and unemployment. With greater match-quality shocks workers become sorted over time into matches with significant match surplus. This makes their separations less responsive to cyclical fluctuations in productivity. Because the level of unemployment is increasing in both B and σ_x , but its cyclicality responds oppositely to the two parameters, we can maintain unemployment's average rate of 6 percent, while increasing its cyclicality, by appropriately increasing B in conjunction with decreasing σ_x . We find that the combination $B = 0.51$, $\sigma_x = 0.014$ produces a standard deviation of unemployment that is 9.5 times that for productivity. We show that this economy, though generating realistic cyclicality, yields implausibly little cross-sectional wage dispersion, with a standard deviation of wages of only 1.9 percent.

Table 1 summarizes the calibrated parameters with values employed for both the benchmark and high-volatility economies.

4. Steady-state Statistics and the Distribution of Match Rents

We present statistics for the model's steady state to illustrate how a worker's assets and match quality determine his wage, reservation match quality, and the surplus from employment. We focus on the distribution of surplus from employment because this is key in determining cyclicalities of separations, vacancy creation, and unemployment for the model. We contrast the distribution of rents to employment from our benchmark model to those for the economy calibrated to generate high cyclical volatility in unemployment.

Starting with the benchmark economy, Figure 1 displays the values of the wage, $W - U$, and J as functions of a worker's assets. These relations are illustrated for three different values for match quality x . Higher values of match quality are directly associated with higher wages and capitalized value of employment W , while irrelevant for U . So both $W - U$ and J correspondingly increase with match quality. Focusing on assets, both W and U increase with assets. But having low assets particularly lowers the value of being unemployed, resulting in a lower bargained wage. Figure 1 displays this positive relation between assets and wages. Both $W - U$ and J (reflecting the higher wage) decrease in worker assets. The sharpest positive relation of the wage to assets, and opposite reaction in J , is concentrated at the very low end of assets, near or below zero.⁸ Focusing on firm rents J , we see that high assets lessens the expected rents of hiring a worker. In turn this provides a channel from assets, specifically the assets of the unemployed, to vacancy creation—high assets among the unemployed, everything else equal, reduces desired vacancies. This implies the cyclicality of assets for the unemployed will influence (oppositely) the cyclicality of vacancy creation.

The top panel of Figure 2 presents the distribution of assets separately for employed and unemployed workers. The model succeeds in generating a fairly wide dispersion in assets, given workers differ only in the history of the quality of employment matches and history of unemployment durations. Because the unemployed draw down assets to maintain consumption, they exhibit average assets of 21 percent less than the employed

⁸The assumptions of Nash bargaining and a coefficient of risk aversion of one imply that J equals $W - U$ times the worker's consumption. For this reason J decreases less than $W - U$ with worker assets. This is more relevant at low asset levels, where consumption responds more to assets. For instance, for $x = 1$, an increase in assets from 0 to 5 yields a drop in J of about two-thirds that in $W - U$.

(14.7 compared to 18.1). The unemployed exhibit lower consumption, by 9 percent, than the employed. The bottom panel of Figure 2 displays how a worker's critical value for match quality x^* depends positively on assets—the critical match quality increases with assets throughout the range of relevant asset holding. Projecting this policy for x^* on the distribution for assets in the top panel of Figure 2 yields the distribution for x^* . This distribution exhibits a standard deviation of 3.3 percent.

Statistics for unemployment, turnover, and assets for the benchmark economy are presented in Table 2. The table also reports that the cross-sectional standard deviation of $(\ln)\text{wages}$ is 18.0 percent. As discussed under calibrating, we perceive this statistic to be quite consistent with the importance of the employer/employee match component in earnings dispersion, as estimated by Woodcock (2007).

Figure 3 presents the distribution of workers' $\ln(\text{wages})$ relative to the critical wage, $\ln(w^*)$, at which the worker is indifferent to separating. (w^* is the bargained wage associated with critical match quality x^* .) This difference, $\ln(w) - \ln(w^*)$, reflects the flow rents associated with the employment match. These rents are significant for the benchmark economy, averaging 26 percent. If we consider a drop in match quality sufficient to reduce the wage by 10 percent, holding w^* unaffected, this would induce only about 16 percent of workers to separate. The standard deviation across workers of the differential $\ln(w) - \ln(w^*)$ equals 17.8 percent. This dispersion is largely driven by dispersion in the wage, not w^* , and in turn reflects the dispersion in match quality. Recall that $\ln(\text{wages})$ has a standard deviation of 18.0 percent. By contrast $\ln(w^*)$ has standard deviation of only 1.5 percent.

The magnitude of the differential $\ln(w) - \ln(w^*)$ is key to the economy's cyclical volatility. A negative aggregate shock induces only a small response in separations if few workers display wages close to the reservation wage w^* . Greater dispersion in $\ln(w) - \ln(w^*)$, absent search frictions, implies a less elastic aggregate labor supply response to aggregate shocks—in a search and matching model this is manifested by less response in separations. Secondly, a drop, say of one percent, in aggregate productivity represents a much smaller percentage hit to the expected payout to filling a vacancy if the average rents to employment are large. Therefore, considerable rents, such as depicted for the benchmark economy in Figure 3, will act to reduce the cyclicalilty of both separations and vacancy creation.

By contrast, the high-volatility economy displays much less dispersion in match quality and smaller rents to employment. Results for this model economy are given in Figures 4 and 5. The top panel of Figure 4 presents the distribution of assets separately for employed and unemployed workers; the bottom panel displays how a worker's critical match quality x^* depends on assets. Compared to the benchmark economy, the high-volatility economy generates a smaller dispersion of assets and, as a result, a smaller dispersion of x^* —the standard deviation of x^* is 0.8 percent for this economy, compared to 3.3 percent for the benchmark.

Statistics for the high-volatility economy are presented in the right-most column of Table 2. For the high volatility economy assets and consumption differ little between the employed and unemployed. Reflecting the small shocks to match quality, this economy exhibits a cross-sectional standard deviation for (ln)wages of only 1.9 percent,

which we view as unreasonably small. Figure 5 presents the distribution of workers' $\ln(\text{wages})$ relative to reservation wage $\ln(w^*)$. In order to match cyclical volatility of employment, this economy must exhibit a highly elastic aggregate labor supply. This is reflected in a distribution for the differential $\ln(w) - \ln(w^*)$ that is limited to near zero—it averages only 3.0 percent for workers, with a standard deviation equal to only 1.8 percent.⁹ A drop in match quality sufficient to reduce the wage by 10 percent, holding w^* unaffected, would induce nearly 100 percent of workers to separate. Thus, while we are able to generate large cyclical fluctuations with this model, we highlight that there is a severe tradeoff—achieving high cyclical volatility requires implausibly little dispersion in wages from match quality.

5. Business cycle predictions

We next characterize the business cycles properties of the model in response to exogenous shifts in aggregate productivity, contrasting results for the benchmark and high-volatility economies. For aggregate monthly productivity shocks we use $\rho_z = 0.95$ and $\sigma_z = 0.0077$. This yields a time series for (logged) TFP, after quarterly averaging and HP filtering, with autocorrelation of 0.84 and standard deviation of 2 percent. These coincide with the statistics reported by Shimer (2005) for U.S. quarterly labor productivity. We focus on discussing relative volatilities and correlations in describing the model results.

⁹As with the benchmark economy, this dispersion is driven by dispersion in the wage, not w^* . The standard deviation of $\ln(w^*)$ is only 0.6 percent. The correlation between $\ln(w)$ and $\ln(w^*)$ is 0.24. For the benchmark economy that correlation is 0.14.

With aggregate fluctuations, productivity z and the measures of workers, μ and ψ , are state variables for agents' optimization problems, as separation decisions depend on subsequent matching probabilities. These, in turn, depend on the next period's measures of workers. Because it is not possible to keep track of the evolution of these measures, we employ Krusell and Smith's (1998) "bounded rationality" method which approximates the distribution of workers by a limited number of its moments. In particular, we assume that agents make use of the average asset holdings of the economy and the fraction of workers who are employed. (The computational appendix gives more detail.). We generate 12,000 monthly periods for a model economy. After dropping the first 3,000 observations, we log and HP filter the data to produce the business cycle statistics.¹⁰

Key statistics are highlighted in Table 3. In addition to our benchmark and high-volatility economies, for comparison the table provides results for a model with linear utility, exogenous separations, and no shocks to match quality. We refer to this, in Column 2, as the Shimer model because it is similar to the model calibrated in Shimer (2005). Also for comparison, the first column reports the comparable statistics reported by Shimer for quarterly U.S. data for 1951-2003, where note that all standard deviations are expressed relative to that for labor productivity. Shimer points out that the natural log of the unemployment series exhibits volatility, measured by standard deviation, that is 9.5 times that in labor productivity, whereas for his calibrated model unemployment displays lower volatility by a factor of about one half. Comparing results for our

¹⁰We use H-P smoothing parameter of 9×10^5 on the monthly data to be comparable to Shimer's treatment.

Shimer model in Column 2 to the data essentially replicates this finding—here the relative standard deviation of unemployment to productivity falls short of the data by a factor of 16.

The cyclical results for our benchmark economy are given in Column 3. Unemployment is twice as volatile as for the Shimer economy, though still short of that in the data by a factor of eight. The increased volatility, compared to Shimer, largely reflects the impact of fluctuations in separations. Separations are both volatile and countercyclical for the model: the standard deviation for separations is nearly equal to that for unemployment, while the correlation between the rates of separations and unemployment is 0.34. (Separations lead the cycle for the model economy, and so are more highly correlated, 0.85, with the change in unemployment rate.) The correlation between Shimer’s data measure of separations and unemployment is even higher at 0.71; but separations for the data show a considerably lower standard deviation than that for unemployment. The finding rate for our benchmark model, like the data, is very procyclical (correlation with unemployment of -0.93 for the model compared to -0.95 for the data). But the volatility of the finding rate, as with the Shimer economy, falls far short of that for the data. Vacancies are actually less volatile and less cyclical for our model than for the Shimer economy. This reflects the model’s predicted increase in separations during contractions which, in turn, encourages vacancy creation. The standard deviation of vacancies is only 0.6 for the model, compared to the data’s 10.1, and the correlation with the unemployment rate is only -0.36 , compared to -0.89 for the data. Thus the model generates only a weak Beveridge curve relative to the Shimer

model, and especially relative to the data.

Finally, we turn to our high-volatility model, with results given in the last column of Table 3. The model by construction generates observed volatility in unemployment. Its standard deviation for unemployment is eight times that produced by our benchmark model. Because it exhibits many workers with little employment surplus, separations are much more volatile than for the benchmark model—the standard deviation of separations is 9 times higher. This model also generates much more cyclical vacancies. This primarily reflects that expected surplus of matches is only about one-tenth that for the benchmark economy. In other words, workers are highly concentrated at the margin. Therefore, a shock to aggregate productivity yields a much bigger percentage impact on expected surplus of matching. The high-volatility economy also generates a considerable skewing of separations during downturns toward workers with higher assets.¹¹ This shift toward workers with higher assets and higher reservation wages in recessions further drives down the value of vacancy creation. (This channel for volatility is distinctive to our model having both risk aversion and endogenous separations.) To separately quantify the impact of this cyclical sorting into unemployment by assets, we constructed a version of our high-volatility model where separations are exogenous, but display the same time series properties as the economy with endogenous separations.¹² We find that the selection of workers into the unemployment pool by assets

¹¹Both the benchmark and high-volatility economies show high correlations (about 0.77) between the unemployment rate and assets of the unemployed. But the impact of this cyclical selection on cyclicity of vacancies is considerably greater for the high-volatility economy, reflecting its much greater cyclicity of separations.

¹²We first estimate a two-variable VAR for productivity and the separation rate on data simulated from our model with endogenous separations, where the separation rate depends on current and lagged productivity as well as its own lag. We then employ the estimated VAR to generate shocks for

increases the volatility of unemployment by about 12 percent.

Despite matching cyclical volatility of unemployment, the high-volatility economy displays the qualitative shortcomings of our benchmark model. In particular, separations are far too cyclical relative to vacancies. This model generates an even weaker Beveridge curve correlation between unemployment and vacancies, -0.19 , than the benchmark economy. Finally, we repeat that this model can achieve its cyclicality for unemployment only by displaying an implausibly low cross-sectional dispersion for wages of just 1.9 percent.

6. Conclusions

We have introduced worker heterogeneity, in worker assets and match quality, into a model of separations, matching, and unemployment. We emphasize the trade-off between producing realistic dispersion in wages or realistic cyclical fluctuations in unemployment. We can generate very high cyclicality of unemployment, comparable to U.S. data, if shocks to match quality are small and payouts to unemployment are high. But we find this simultaneously implies dispersion of less than two percent in wages due to match quality. We consider this implausible, given estimates of wage dispersion controlling for worker and firm fixed effects (Woodcock, 2007). With lower payouts to unemployment, comparable to Shimer's calibration, and considerable match productivity shocks, we can generate a realistic dispersion in wages. But then the model falls drastically short in capturing cyclical fluctuations in unemployment of the separations as well as productivity for the model simulations.

magnitude displayed by the data.

How might the model be extended to overcome this conflict between realistic wage dispersion and realistic unemployment cyclicalities, while maintaining wage flexibility? This requires that the model produce little dispersion in the rents to employment, despite considerable dispersion in wages due to match productivity. One way to generate small employment rents, with significant wage dispersion, is to assume that productivity in the market and productivity in home tasks are highly correlated across workers; this weakens the link from a worker's relative productivity in the market to comparative advantage in market work. But it is difficult to make this case for differences in market productivity that reflect match quality—why would drawing a good employer match be associated with comparably higher home productivity? Another approach is to modify the environment to generate a stronger inverse relationship between a worker's match quality and the worker's marginal utility of consumption. Our model, because it assumes no insurance and limited borrowing, does generate higher consumption, and lower marginal utility of consumption, for workers with higher match wages. But we anticipate that breaking the link between match productivity and rents to employment would require extreme assumptions on preferences and/or the availability of asset markets.

A. Computational Algorithm

A.1. Steady-State Equilibrium

In steady state, the aggregate productivity z is constant at its mean and the measures of workers μ and ψ are invariant over time. Computing the steady-state equilibrium amounts to finding *i*) the value functions $W(a, x)$, $U(a)$ and $J(a, x)$, *ii*) the decision rules $a'_e(a, x)$, $a'_u(a)$ and $x^*(a)$, *iii*) the wage schedule $w(a, x)$, *iv*) the labor market tightness θ , *v*) the time-invariant measures $\mu(a, x)$ and $\psi(a)$ that satisfy the equilibrium conditions given in subsection 2.5. The detailed computational algorithm for steady state equilibrium is as follows.

1. Discretize the state space $\mathcal{A} \times \mathcal{X}$ over which the value functions and wages are computed. The stochastic process for the idiosyncratic productivity is approximated by the first-order Markov process of which transition probability matrix is computed using Tauchen's (1986) algorithm.
2. Assume an initial value of θ^0 .
3. Given θ^0 , we solve the Nash bargaining and individual optimization problems to approximate wages, value functions, and decision rules in the steady state, which will be used to compute the time-invariant measures.
 - (a) Assume an initial wage schedule $w^0(a, x; \theta^0)$ for each (a, x) node.
 - (b) Given $w^0(a, x; \theta^0)$, solve for the worker's value functions, $W(a, x; w^0)$ and $U(a; w^0)$, using equations (1) and (2) in the text. The value functions are

approximated using the iterative method. The utility maximization problems in the worker's value functions are solved through the Brent method. The decision rules $a'_e(a, x; w^0)$, $a'_u(a; w^0)$ and $x^*(a; w^0)$ are obtained at each iteration of the value functions.

- (c) Compute wages that satisfy the definition of $J(a, x, w^0)$ in (3) and the Nash bargaining solution in (5) in the text. Specifically, we solve for $w^1(a, x; \theta^0)$ for each (a, x) node that satisfies

$$w^1(a, x; \theta^0) = zx - J(a, x; w^0) + \beta(1 - \lambda)E [\max\{J(a'_e, x'; w^0), 0\}|x],$$

where $J(a, x; w^0)$ is computed using the first order condition for the Nash bargaining problem in (5):

$$J(a, x; w^0) = \left(\frac{1 - \alpha}{\alpha}\right) [W(a, x; w^0) - U(a; w^0)] c_e(a, x; w^0).$$

- (d) If $w^1(a, x; \theta^0)$ and $w^0(a, x; \theta^0)$ are close enough to each other, then move on to the step 4 to compute invariant measures and the corresponding labor market tightness, θ^1 . Otherwise, go back to the step 3a with a new guess for the wage schedule:

$$w^0(a, x; \theta^0) = \zeta_w w^1(a, x; \theta^0) + (1 - \zeta_w) w^0(a, x; \theta^0).$$

4. Using the converged decision rules $a'_e(a, x; w^0)$, $a'_u(a; w^0)$ and $x^*(a; w^0)$ given the converged wage schedule $w^0(a, x; \theta^0)$ from the step 3b and 3a, compute the time-invariant measures $\mu(a, x; \theta^0)$ and $\psi(a; \theta^0)$ by iterating the laws of motion for measures given in (6) and (7). Then, compute the labor market tightness θ^1 that

satisfies the free-entry condition using equation (4) and the converged measures:

$$\kappa = \beta q(\theta^1) \int J(a'_u, \bar{x}; \theta^0) d\tilde{\psi}(a'_u; \theta^0).$$

5. If θ^1 and θ^0 are close enough to each other, then we found the steady state.

Otherwise, go back to the step 3 with a new guess for the labor market tightness:

$$\theta^0 = \zeta_\theta \theta^1 + (1 - \zeta_\theta) \theta^0.$$

A.2. Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us to include the aggregate productivity, z , and the measures of workers, μ and ψ , as state variables for agents' optimization problems. In order to make match separation decisions at the end of a period, agents need to know their matching probabilities in the next period, $p(\theta_{t+1})$ and $q(\theta_{t+1})$, which in turn depends on the next period's measures of workers, $\mu_{t+1}(a, x)$ and $\psi_{t+1}(a)$. The laws of motion for the measures are given in equations (6) and (7). It is impossible to keep track of the evolution of these measures. We employ Krusell-Smith's (1998) "Bounded Rationality" method which approximates the distribution of workers by a number of its moments. We assume that agents in the economy make use of two first moments of the measures: the average asset holdings of the economy, $K = \int a d\mu(a, x) + \int a d\psi(a)$, and the number of employed workers, $E = \int d\mu(a, x)$. Let \hat{s} denote a vector of aggregate state variables in the approximation of equilibrium with fluctuations. Then $\hat{s} = (K, E, z)$. In addition we assume that the agents use log-linear rules in predicting the current θ , the future K and the future E .

1. Guess a set of prediction rules for the equilibrium labor market tightness (θ) in the current period, the average asset of the economy (K') and the number of employed workers (E') in the next period. This step amounts to setting the coefficients of the log-linear prediction rules:

$$\log \theta = b_{\theta,0}^0 + b_{\theta,1}^0 \log K + b_{\theta,2}^0 \log E + b_{\theta,3}^0 \log z$$

$$\log K' = b_{K,0}^0 + b_{K,1}^0 \log K + b_{K,2}^0 \log E + b_{K,3}^0 \log z$$

$$\log E' = b_{E,0}^0 + b_{E,1}^0 \log K + b_{E,2}^0 \log E + b_{E,3}^0 \log z.$$

As is the case in the steady state computation, we approximate the stochastic process for the aggregate productivity by the first-order Markov process of which transition probability matrix is computed using Tauchen's (1986) algorithm.

2. Given these prediction rules, we solve the individual optimization and wage bargaining problems. This step is analogous to step 3 in the steady state computation, so we omit the detailed description of computational procedure. However, the dimension of state variables is now much larger: (a, x, \hat{s}) . Computation of the conditional expectations involves the evaluation of the value functions not on the grid points along K and E dimensions since K' and E' are predicted by the log-linear rule above. We polynomially interpolate the value functions along the K dimension when necessary.
3. We generate a set of artificial time series data $\{\theta_t, K_t, E_t\}$ of the length of 9,000 periods. Each period, these aggregate variables are calculated by summing up 50,000 workers' decisions on asset accumulation and match separation, which are

simulated using the converged value functions, $W(a, x, \hat{s})$, $U(a, \hat{s})$, and $J(a, x, \hat{s})$, the decision rules, $a'_e(a, x, \hat{s})$, $a'_u(a, \hat{s})$ and $x^*(a, \hat{s})$ from the step 2, and the assumed prediction rules for θ , K' and E' from the step 1.

4. We obtain the new values for the coefficients (b^1 's) in the prediction functions through the OLS using the simulated data from the step 3. If b^0 and b^1 are close enough to each other, then we find the (limited information) rational expectations equilibrium with aggregate fluctuations. Otherwise, go back to the step 1 with a new guesses for the coefficients in the prediction functions:

$$b_{i,j}^0 = \zeta b_{i,j}^1 + (1 - \zeta_b) b_{i,j}^0,$$

where $i = \theta, K, E$ and $j = 0, \dots, 3$.

The converged prediction rules and their accuracy, measured by R^2 , for the benchmark calibration with $h = 1$ are as follows.

- Prediction for labor market tightness in the current period:

$$\log \theta = 1.934 - 0.05810 \log K + 0.4220 \log E + 0.14804 \log z, \quad R^2 = 0.9971$$

- Prediction for average asset holdings in the next period:

$$\log K' = 0.0096 + 0.9965 \log K - 0.0071 \log E + 0.0457 \log z, \quad R^2 = 0.9999$$

- Prediction for number of employed workers in the next period:

$$\log E' = -0.0182 - 0.0015 \log K + 0.6361 \log E + 0.0276 \log z, \quad R^2 = 0.9538$$

Overall, the estimated prediction rules are fairly precise as R^2 's are close to 1, while the prediction rule for average asset holdings provides the highest accuracy.

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Table 1: Parameter Values for Benchmark and High-Volatility Economies

Parameter	Description	Benchmark	High-volatility
γ	Relative risk aversion	1	same
r	Real interest rate (annualized)	6%	same
β	Discount factor	0.9948	0.9949
\underline{a}	Borrowing constraint	-6.0	same
θ	Steady state v/u ratio (normalized)	1	same
α	Matching technology $m(v, u) = .313 v^\alpha u^{1-\alpha}$	0.5	same
κ	Vacancy posting cost	0.522	0.0785
b	Unemployment benefit	0.4	same
B	Utility from leisure	0.15	0.506
ρ_x	Persistence of idiosyncratic productivity $\ln x$	0.97	same
σ_x	Standard deviation of innovation to $\ln x$	13.0%	1.38%

Table 2: Steady Statistics for Benchmark and High-Volatility Economies

	Benchmark	High-volatility
Unemployment rate	6%	Same
Separation rate	2%	Same
Finding rate	31%	Same
Average assets for employed	18.1	17.9
Average assets for unemployed	14.7	18.1
Standard deviation, assets	15.1	8.6
Average consumption for employed	1.19	1.07
Average consumption for unemployed	1.09	1.056
Standard deviation, ln(wage)	18.0%	1.9%

Note: See Table 1 for parameter values for two calibrations.

Table 3: Business Cycles for Benchmark and High-Volatility Economies

	U.S. Data	Shimer	Benchmark	High-volatility
Standard deviation (relative to productivity) for				
— Unemployment rate	9.5	0.6	1.2	9.6
— Separation rate	3.8	0	1.1	10.0
— Finding rate	5.9	0.7	0.8	5.5
— Vacancy rate	10.1	1.0	0.6	3.3
Correlation with unemployment rate for:				
— Separation rate	0.71	0	0.34	0.31
— Finding rate	-0.95	-0.83	-0.93	-0.98
— Vacancy rate	-0.89	-0.60	-0.36	-0.19

Note: Variables are in natural log form, e.g., unemployment rate refers to $\ln(\text{unemployment rate})$. Standard deviations are relative to productivity. Statistics for U.S. data are from Shimer (2005) which reflects the deviations from the H-P trend with smoothing parameter of 10^5 for 1951 to 2003. See Table 1 for parameter values for the two calibrations. The simulated data from the models are monthly deviation from the H-P trend with smoothing parameter 9×10^5 . The productivity shock used in the simulation exhibits the same persistence and standard deviation to the U.S. quarterly data reported in Shimer (2005).

Figure 1: Benchmark Economy: Wages and Value Functions

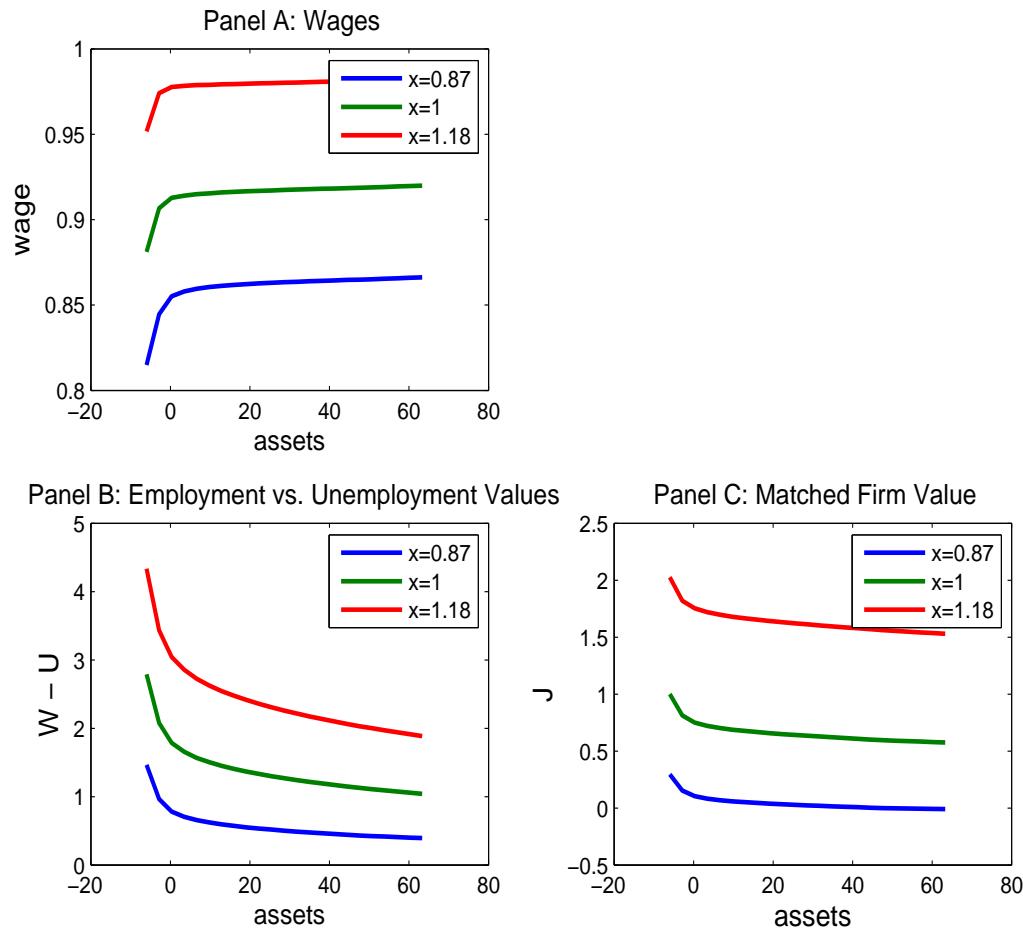


Figure 2: Benchmark Economy: Asset Distributions and Reservation Match Productivity

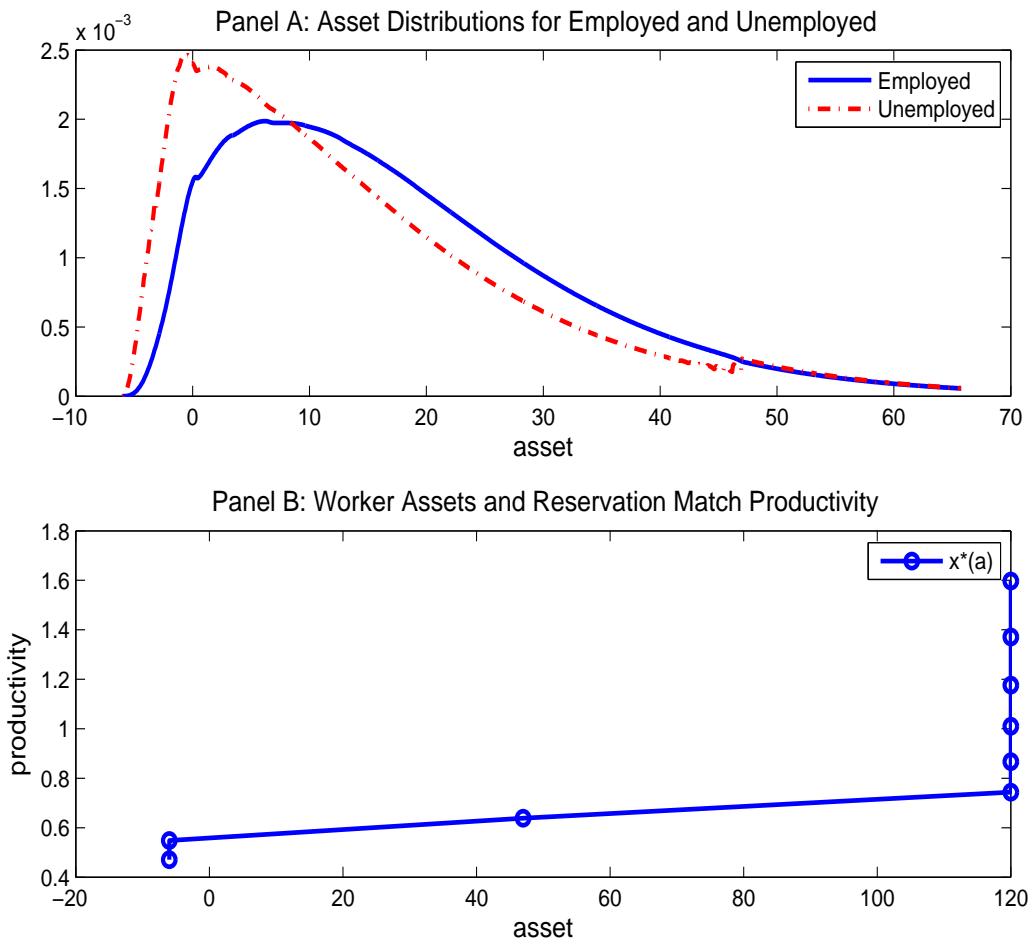


Figure 3: Benchmark Economy: Distribution of Surplus Match Quality ($\ln w - \ln w^*$)

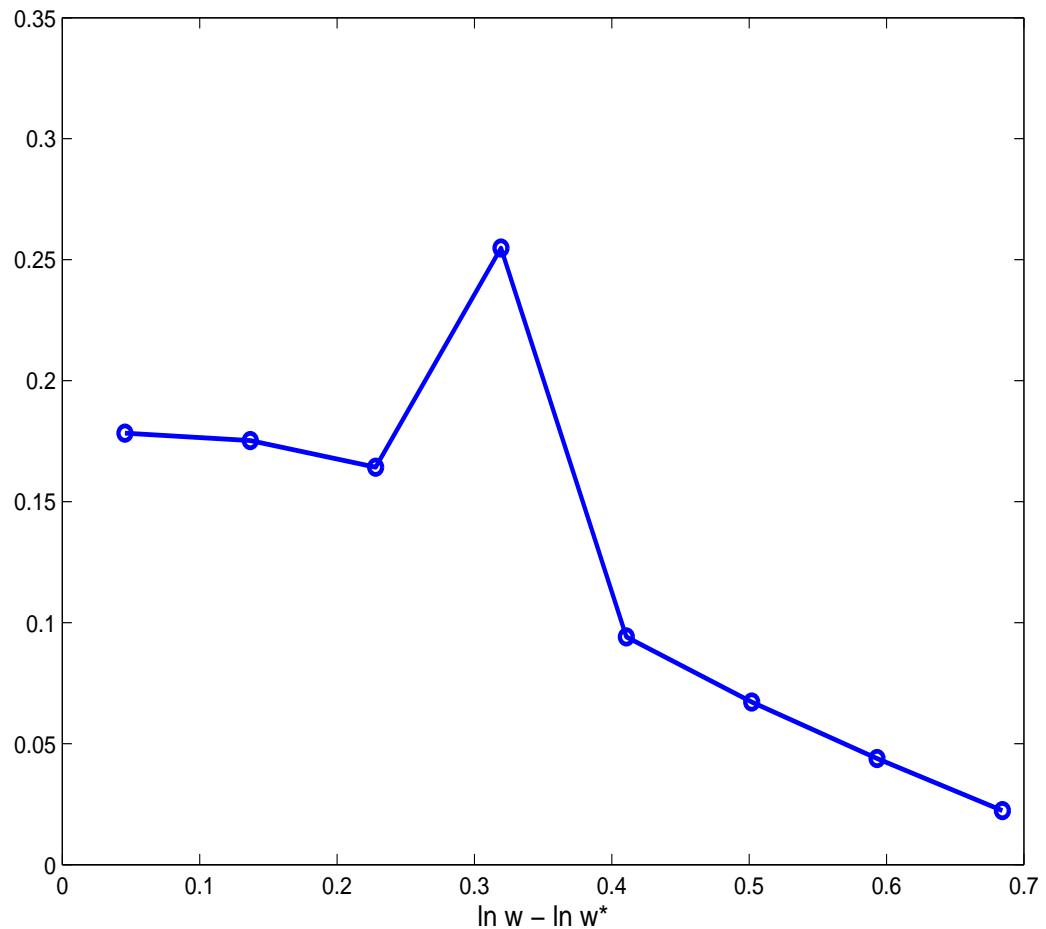


Figure 4: High-Volatility Economy

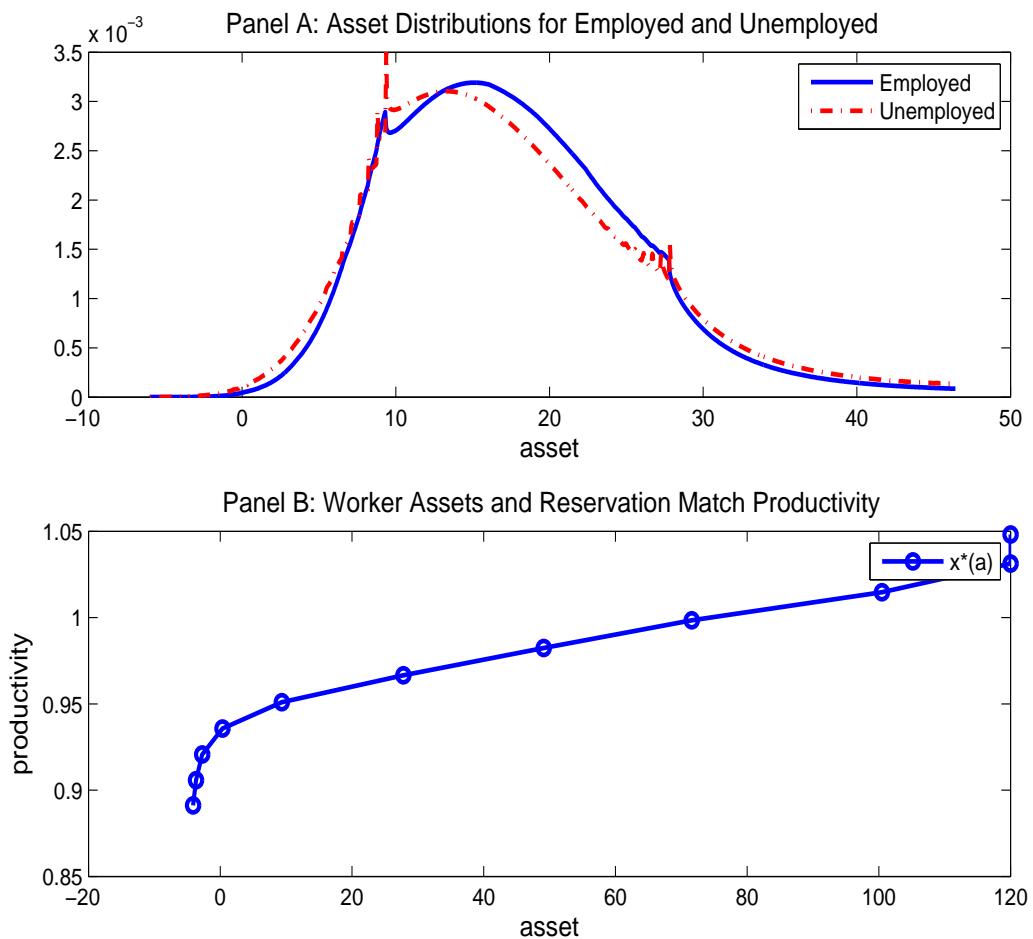


Figure 5: High-Volatility Economy: Distribution of Surplus Match Quality ($\ln w - \ln w^*$)

