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Unit of Account, Medium of Exchange, and Prices

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Abstract

The separation of a unit of account (UoA) from a medium of exchange (MoE) in the commodity-money system is investigated by considering explicitly a seller's choice with regard to posting price in terms of either an MoE or a UoA. If the likelihood of debasement of MoE or its rate is high enough and agents are sufficiently risk averse, there exists a monetary equilibrium in which price is quoted in terms of a UoA. Further, a UoA-posting equilibrium yields the flexible nominal price, whereas an MoE-posting equilibrium yields the sticky one. This suggests that in the fiat-money system where MoE and UoA are integrated, price would not be flexibly adjusted.

Keywords: debasement, medium of exchange, unit of account, nominal price rigidity
JEL classification: E31, E42, F33

1. Introduction

In the modern fiat-money system, the two functions of money—medium of exchange and unit of account—happen to coincide. However, there is wide agreement that there existed a dichotomy between the medium of exchange (e.g., silver coin) and the unit of account (or "money of account") in the commodity-money system; see, for example, White (1984), Glassman and Redish (1988), Rolnick, Velde and Weber (1997), Redish (2000), Sargent and Velde (2002), and Velde (2009). Indeed, historically, units of account preceded media of exchange in the sense that units of weight, such as the talent and the shekel, evolved into medium-of-exchange units when coins were minted that had specified relations to the weight (Shiller 2002). Spufford (1986, p. 19) also noted the problem associated with the dichotomy as follows:

> Money of account derived its name from its function. As a measure of value it was used almost exclusively for account purposes. Most financial transactions were first determined and expressed in money of account, although payments were naturally made subsequently in coin, ... Coin itself was valued as a commodity in terms of money of account, and like any other commodity, its value frequently varied.

However, there has been no study to address the following fundamental question in monetary economics: why was a medium of exchange (MoE) separated from a unit of account (UoA) in the commodity-money system? The goal of this paper is to provide a theoretical exposition for this question by considering explicitly a seller's choice with regard to posting price in terms of either an MoE (e.g., silver coin or penny) or a UoA (e.g., one ounce of silver) in the commodity-money system. We also examine the implications of separating a UoA from an MoE for the nominal price flexibility. This immediately suggests the nominal price rigidity in the fiat-money system where a UoA and an MoE are integrated.

Specifically, we assume that there is a given stock of silver and each ounce of a silver bar is given to an agent chosen randomly at the beginning of an initial period. The government operates a mint that stands ready to convert one ounce of a silver bar into silver coins (i.e., pennies) as an MoE and vice versa. There is also a UoA defined as one ounce of silver. In the beginning of each period, each seller (who does not hold a silver bar) posts the quantity of good that she is willing to produce per either penny or UoA. The debasement shock is followed such that the government will reduce the silver content of a penny with a positive probability. Each agent with a silver bar (potential buyer) then converts it into pennies using a free minting technology before she moves to the decentralized market for a pairwise trade with another agent.

We first show that, if the stock of silver is not too large and agents are sufficiently patient, there exists a symmetric stationary equilibrium in which the lowest price is posted in terms of either UoA or MoE (i.e., penny). Noting that the stock of silver represents the mass of buyers in the model economy, a relatively large stock of silver means the relatively small number of sellers who are then willing to exchange only a small quantity of good for either UoA or MoE.

Furthermore, if the likelihood of debasement or the debasement rate is high enough and agents are sufficiently risk averse, there exists a symmetric stationary equilibrium in which price is quoted in terms of UoA. In an MoE-posting equilibrium, the quantity of output posted by a seller in the beginning of a period depends on the likelihood of debasement as well as the debasement rate. That is, the higher debasement rate or the higher likelihood of debasement implies higher uncertainty of consumption in the MoE-posting equilibrium. On the other hand, the consumption in a UoA-posting equilibrium relies on neither the likelihood of debasement nor the debasement rate, and hence sufficiently risk-averse agents would prefer the UoA posting to the MoE posting. This implies a higher welfare in the UoA-posting equilibrium.

This result is consistent with a key fact during the Middle Ages, as documented by Rolnick, Velde and Weber (1997), that through 123 debasements in France (1285∼1490) and during the Great Debasement in England (1542∼1551), coins were valued in circulation by their intrinsic content (circulation by weight) rather than by their legal tender value (circulation by tale). Also, noting the inflationary nature of debasement in the sense that it reduces the penny-equivalent quantity of good, the UoA posting of price at the time of a relatively high debasement rate supports incidents during the German hyperinflation in the early 1920s that sellers quoted prices in terms of goldmarks, while payments were made in currency called Reichsmark (Wolf 2002).

Finally, a UoA-posting equilibrium yields the flexible nominal price (measured in terms of MoE), whereas an MoE-posting equilibrium yields the sticky one. In the equilibrium where price is quoted in terms of UoA, the number of pennies exchanged for a unit of good changes proportionally to the debasement rate. In the equilibrium where price is quoted in terms of pennies, however, the number of pennies exchanged for a unit of good does not vary with the debasement rate. This suggests that in the fiat-money system where MoE and UoA are integrated, price would not be flexibly adjusted.

Shiller (1998) claims that if an MoE is separated from a UoA that has no physical embodiment and is indexed to consumer price index, the effects of sticky prices on the macroeconomy would be substantially lessened. Once the price is quoted in terms of UoA that is indexed to the general price level, the quantity of good exchanged for a unit of MoE would be automatically adjusted to an increase in the price level. On the other hand, in an economy where MoE and UoA are integrated, price of good posted by sellers would not be adjusted immediately following inflation.[1]

The paper is organized as follows. Section 2 describes the model economy, followed by the equilibrium characterization in Section 3. Section 4 shows the properties of an equilibrium

[1] Jovanovic and Ueda (1997) show that if the parties to a contract expect that the price level will be measured with delay, they will not write a fully indexed contract because they know that they would choose to renegotiate it.

where price is posted in terms of UoA. Section 5 discusses the implications of our model. Section 6 summarizes the paper with a few concluding remarks, followed by Appendix which contains the proofs of the main results.

2. Model

The model builds on Camera and Winkler (2003), and Curtis and Wright (2004). Time is discrete and there is a $[0,1]$ continuum of agents who live forever with discount factor $\beta \equiv [1/(1+\rho)] \in (0,1)$. There are $K \geq 3$ types of agents who are uniformly distributed across types so that the measure of a given type $k \in K = \{1, 2, ..., K\}$ is $1/K$. There are also $K \geq 3$ types of divisible and perishable goods. A type k agent produces only good k, which incurs a disutility cost of $c(q) = q$ for producing $q \in \mathbb{R}_+$. A type k agent enjoys utility from consuming $q_i \in \mathbb{R}_+$ units of good $i \in \mathbb{K}$ according to $\mathcal{U}_k(q_i) = \alpha_{k,i} q_i^{\gamma}$, where $\gamma \in (0,1)$ and $1 = \alpha_{k,k+1} > \alpha_{k,k+2} > ... > \alpha_{k,K} > ... > \alpha_{k,k} = 0$ (modulo K). That is, a type k agent has greater valuation for good i than a type k' agent if $\alpha_{k,i} > \alpha_{k',i}$ and her own output does not provide utility ($\alpha_{k,k} = 0$).

At the beginning of an initial period, a fraction M of agents is chosen at random and each is given one ounce of a silver bar, whereas the remaining $(1-M)$ fraction of agents is given a production opportunity. In each succeeding period, a new production opportunity freely arises following consumption. An agent cannot store more than one ounce of a silver bar at a time. The government operates a mint that stands ready to convert one ounce of a silver bar into silver coins (i.e., pennies) and vice versa. Other than tangible pennies, there is an intangible object (with no physical embodiment) called a UoA that is defined as one ounce of silver; i.e., one ounce of silver is one UoA.[2]

The rest of the model is best described following the sequence of events within a period.

[2]Shiller (2002) noted: "Ultimately, the standard of value represented by this system was the silver *denarius* issued by Charlemagne in the late eighth century and early ninth centuries, coins that were no longer circulating, or ever seen, ... Charlemagne's *denarius* weighed one 240th of a troy pound, ..."

At the beginning of each period, a seller posts the quantity of good that she is willing to produce per either penny (q_m) or UoA (q_u) where a seller means an agent who does not hold a silver bar. Then the silver content of a penny is set by the government. Before its realization, individual agents only know that the "size" of a penny will be one ounce of silver with probability π and $\delta \in (0, 1)$ ounce of silver with probability $(1 - \pi)$, where $1/\delta$ is a positive integer. From the viewpoint of commodity-money system in medieval Europe, δ can be interpreted as debasement (i.e., reduction in the silver content of a penny). According to Rolnick, Velde and Weber (1997), there were 123 debasements of silver coins in France during $1285 \sim 1490$, among which 112 were the cases where the silver content of coins was decreased by more than 5%.

After the realization of penny-size shock, each agent with a silver bar (potential buyer) can convert it into pennies using a free minting technology and then moves to the decentralized market, where each agent of a given type is randomly matched with another agent. Agents in a pairwise meeting know each other's specialization type and penny holdings. However, trading histories are private and agents cannot commit to their future actions. Hence, all trade is *quid pro quo*. Further, a seller in a pairwise meeting cannot identify the weight and fineness of silver bar. Together with incomplete record-keeping and no commitment, this makes a penny essential as a medium of exchange.[3]

After the pairwise trades, free melting technology of pennies is available. We here assume that pennies minted in a specific date has a distinguishing feature and sellers do not take old pennies because of the risk of wear, tear and clipping. Therefore, all the agents with pennies melt them to restore one ounce of a silver bar before carrying it to the next period. Since the government can convert pennies into one ounce of a silver bar only, a seller is not willing to exchange her good for pennies that are not sufficient to restore one ounce of a silver bar. Each agent will then hold either 0 or 1 ounce of silver bar at the end of a period, which

[3]See, for example, Kocherlakota (1998), Wallace (2001), Corbae, Temzelides and Wright (2003), Aliprantis, Camera and Puzzello (2007), and Lagos and Wright (2008).

makes the distribution of silver holdings tractable.

3. Equilibrium

We focus on a symmetric and stationary equilibrium where agents' trading strategies are symmetric across the specialization types and constant over time. For simplicity, we analyze the case of $K = 3$ which is the smallest number to sustain a monetary equilibrium with the price posting by a seller.[4] In addition, we restrict our analysis to the equilibrium in which the price posted by a seller is the lowest so that trades occur in every pairwise meeting between the agents of different specialization types.[5]

Let $V_{s,k}$ and $V_{b,k}$ denote respectively the expected lifetime utility of a type $k \in \mathbb{K}$ seller and a type $k \in \mathbb{K}$ buyer at the end of a period. Then per-period gain for a type $k \in \mathbb{K}$ seller can be expressed as

$$\tilde{\rho} V_{s,k} = \max \left\{ \tilde{\rho} V_{s,k}^{u}, \tilde{\rho} V_{s,k}^{m} \right\} \tag{1}$$

where $\tilde{\rho} = \rho K$ is the discount rate normalized by the number of specialization types of agents. Also, $V_{s,k}^{u}$ and $V_{s,k}^{m}$ denote respectively the value of posting price by a seller in terms of UoA and pennies, which satisfy respectively the followings:

$$\begin{aligned} \tilde{\rho} V_{s,k}^{u} \quad = \quad & M \max_{q_u} \Bigg\{ \pi \sum_{j \in \mathbb{K}} \left(V_{b,k} - V_{s,k} - q_u \right) \sigma_{j,k}(q_u; 1) \\ & + (1-\pi) \sum_{j \in \mathbb{K}} \left(V_{b,k} - V_{s,k} - q_u \right) \sigma_{j,k}(q_u; \delta) \Bigg\} \end{aligned} \tag{2}$$

$$\begin{aligned} \tilde{\rho} V_{s,k}^{m} \quad = \quad & M \max_{q_m} \Bigg\{ \pi \sum_{j \in \mathbb{K}} \left(V_{b,k} - V_{s,k} - q_m \right) \sigma_{j,k}(q_m; 1) \\ & + (1-\pi) \sum_{j \in \mathbb{K}} \left(V_{b,k} - V_{s,k} - \delta^{-1} q_m \right) \sigma_{j,k}(q_m; \delta) \Bigg\}. \end{aligned} \tag{3}$$

[4] For more details, see Camera and Winkler (2003), and Curtis and Wright (2004).

[5] The conditions ensuring the existence of this equilibrium will be discussed in Section 4.

Here $\sigma_{j,k}(q_u;\delta)$ is the probability that a type $j \in \mathbb{K}$ buyer accepts the offer q_u made by a type $k \in \mathbb{K}$ seller after the realization of δ. Similarly, $\sigma_{j,k}(q_m;\delta)$ is the probability that a type $j \in \mathbb{K}$ buyer accepts the offer q_m made by a type $k \in \mathbb{K}$ seller. According to (2) and (3), the flow values of posting price by a seller in terms of UoA and pennies respectively consist of the net expected gains from trade with buyers depending on the likelihood of debasement and its rate.

Also, per-period gain for a type $k \in \mathbb{K}$ buyer consists of the following net expected gains from trade with sellers:

$$\begin{aligned}\tilde{\rho}V_{b,k} \;=\; (1-M)\max_{\sigma}\Bigg\{&\sum_{j\in\mathbb{K}}\sigma_{k,j}(q_u;\cdot)\left[V_{s,k}-V_{b,k}+\alpha_{k,j}(q_u)^{\gamma}\right]F_j(q_u)\\ &+\pi\sum_{j\in\mathbb{K}}\sigma_{k,j}(q_m;1)[V_{s,k}-V_{b,k}+\alpha_{k,j}(q_m)^{\gamma}]F_j(q_m)\\ &+(1-\pi)\sum_{j\in\mathbb{K}}\sigma_{k,j}(q_m;\delta)[V_{s,k}-V_{b,k}+\alpha_{k,j}(\delta^{-1}q_m)^{\gamma}]F_j(q_m)\Bigg\} \quad (4)\end{aligned}$$

where $F_j(q_u)$ and $F_j(q_m)$ denote the fraction of $j \in K$ type sellers posting the price in terms of UoA and pennies, respectively. Notice that in a symmetric equilibrium, $V_{s,k} = V_s$, $V_{b,k} = V_b$ for all $k \in \mathbb{K}$, and $F_j(\cdot) = F(\cdot)$ for all $j \in \mathbb{K}$.

Since it is the seller who posts the price, she only cares about a buyer's reservation price at which the buyer's net gain from trade is zero. First, if she wants to post price in terms of UoA, she considers meeting a buyer with either $\alpha = 1$ or $\alpha \in (0,1)$. For the former, there is no monetary equilibrium, whereas for the latter, q_u must satisfy

$$V_s^u - V_b + \alpha(q_u)^{\gamma} = 0. \quad (5)$$

If a seller chooses to post q_u that satisfies (5), $V_s^u - V_b + (q_u)^{\gamma} > 0$; i.e., trades occur in every

pairwise meeting between the agents of different types. Then, (2) can be rewritten as

$$\tilde{\rho} V_s^u = 2M(V_b - V_s^u - q_u). \tag{6}$$

With $F(q_u) = 1$ due to symmetry, (4) can be also simplified to

$$\tilde{\rho} V_b = (1 - M) \left[V_s^u - V_b + (q_u)^\gamma \right]. \tag{7}$$

From (6) and (7), we can obtain the net continuation payoff for the UoA posting of price

$$V_b - V_s^u = \frac{(1 - M)(q_u)^\gamma + 2Mq_u}{(1 + \tilde{\rho} + M)} \tag{8}$$

which, together with (5), yields

$$q_u = \left[\frac{\alpha(1 + \tilde{\rho} + M) - (1 - M)}{2M} \right]^{\frac{1}{1-\gamma}}. \tag{9}$$

Substituting (8) and (9) into (6), $\tilde{\rho} V_s^u$ becomes

$$\tilde{\rho} V_s^u = (2M\alpha - \mathbb{X}_u) (2M)^{\frac{1}{\gamma-1}} (\mathbb{X}_u)^{\frac{\gamma}{1-\gamma}} \tag{10}$$

where $\mathbb{X}_u \equiv \alpha(1 + \tilde{\rho} + M) - (1 - M)$.

Second, if the seller wants to post price in terms of pennies, she considers debasement with either $\delta = 1$ or $\delta \in (0, 1)$ as well as meeting a buyer with either $\alpha = 1$ or $\alpha \in (0, 1)$. Hence, the seller has 4 alternatives: (i) $V_s^m - V_b + \alpha(\delta^{-1} q_m)^\gamma = 0$, (ii) $V_s^m - V_b + \alpha(q_m)^\gamma = 0$, (iii) $V_s^m - V_b + (q_m)^\gamma = 0$, and (iv) $V_s^m - V_b + (\delta^{-1} q_m)^\gamma = 0$. For the last case, it is easy to show that there is no monetary equilibrium. Among (i)-(iii), trades occur in every pairwise meeting between the agents of different types if a seller chooses (ii). Hence, we consider the

following case:

$$V_s^m - V_b + \alpha(q_m)^\gamma = 0. \tag{11}$$

If a seller chooses to post q_m that satisfies (11), (3) and (4) can be respectively simplified to

$$\tilde{\rho} V_s^m = 2M(V_b - V_s^m - \bar{q}_m) \tag{12}$$

$$\tilde{\rho} V_b = (1-M)\left\{(2-\pi)(V_s^m - V_b) + [\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}](q_m)^\gamma\right\} \tag{13}$$

where $\bar{q}_m \equiv [\pi + (1-\pi)\delta^{-1}]q_m$. From (12) and (13), we have the net continuation payoff for the penny posting of price

$$V_b - V_s^m = \frac{(1-M)[\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}](q_m)^\gamma + 2M\bar{q}_m}{2 - \pi(1-M) + \tilde{\rho}} \tag{14}$$

which, together with (11), yields

$$q_m = \left[\frac{\alpha\left[2 - \pi(1-M) + \tilde{\rho}\right] - (1-M)[\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}]}{2M\bar{\delta}}\right]^{\frac{1}{1-\gamma}} \tag{15}$$

where $\bar{\delta} \equiv \pi + (1-\pi)\delta^{-1}$. Substituting (14) and (15) into (12), $\tilde{\rho} V_s^m$ becomes

$$\tilde{\rho} V_s^m = (2M\alpha - \mathbb{X}_m)\,(2M)^{\frac{1}{\gamma-1}}(\mathbb{X}_m/\bar{\delta})^{\frac{\gamma}{1-\gamma}} \tag{16}$$

where $\mathbb{X}_m \equiv \alpha[2 - \pi(1-M) + \tilde{\rho}] - (1-M)[\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}]$.

Now, we can complete the definition of a symmetric stationary monetary equilibrium in which trades occur in every pairwise meeting between the agents of different types.[6]

Definition 1 *A symmetric stationary monetary equilibrium with the lowest price is a set of* $\{V_s, V_b, q, F, (\sigma_{k,j})_{\forall k \in \mathbb{K}, j \in \mathbb{K}}\}$ *such that (i)* $V_s = \max\{V_s^u, V_s^m\}$ *where* V_s^u *is given by (10) and*

[6] We here ignore the knife-edge case of $V_s^u = V_s^m$ because randomization over UoA-posting and penny-posting (mixed strategy) is not a case of our interest.

V_s^m is given by (16); (ii) (V_b, q) satisfy (7) and (9) if $V_s = V_s^u$, and (13) and (15) if $V_s = V_s^m$; (iii) $F(q_u) = 1$ if $V_s = V_s^u$ and 0 if $V_s = V_s^m$; (iv) $\sigma_{k,j} = 1$ for all $(k, j) \in \mathbb{K} \times \mathbb{K}$ if $k \neq j$.

4. Separation of UoA from MoE

We first discuss the conditions that ensure the existence of a symmetric stationary equilibrium in which trades occur in every pairwise meeting between the agents of different types.

Lemma 1 *If $M < \bar{M}$, $\tilde{\rho} < \bar{\rho}$, and α is low enough, there exists a symmetric stationary monetary equilibrium in which the lowest price is posted in terms of either UoA or MoE, where $\bar{M}$ and $\bar{\rho}$ are defined in Appendix.*

Proof. See Appendix. ■

Lemma 1 suggests that if the stock of silver bar is not too large and agents are sufficiently patient, a symmetric stationary monetary equilibrium exists where buyers in every pairwise meeting accept the offer made in terms of either UoA or pennies by different types of sellers. Notice that the stock of silver bar is closely related to the sales opportunities because it represents the mass of buyers. Hence, if it is too large, a buyer would find it difficult to meet sellers. This then makes the value of silver relatively low so that the seller is willing to exchange a relatively small quantity of good for either UoA or MoE (i.e., pennies).

As regards α, it is easy to show that $[\partial q_m / \partial \alpha] > 0$ from (15), implying higher net benefit for a buyer $(V_b - V_s)$. In order to get some intuition, suppose that the seller deviates from q_m by producing $q'_m < q_m$, where q'_m satisfies $V_s - V_b + \alpha(\delta^{-1} q'_m)^\gamma = 0$. Then, as α increases, the benefit of one-period deviation from q_m as a seller $(q_m - q'_m) = (1 - \delta) q_m$ also increases and hence for a sufficiently high α, there would be an incentive for a seller to deviate from the lowest-price strategy.

Now, the following proposition characterizes the conditions for the existence of a UoA-posting equilibrium.

Proposition 1 *Under the assumptions made in Lemma 1, there exists a symmetric stationary monetary equilibrium with the lowest price posted in terms of UoA as long as $\pi < \pi_0$ or $\delta < \delta_0$ and agents are sufficiently risk averse, where π_0 and δ_0 are defined in Appendix.*

Proof. See Appendix. ■

Intuitively, as the debasement rate becomes larger (smaller δ), the seller is willing to produce a smaller quantity of output per penny, $[\partial q_m/\partial\delta] > 0$ from (15). Also, as the likelihood of debasement is higher (smaller π), the quantity of output that the seller is willing to produce per penny gets smaller, $[\partial q_m/\partial\pi] > 0$ from (15). This means that the smaller δ or π induces higher uncertainty of consumption in the penny-posting equilibrium because actual output transferred from a seller to a buyer in a pairwise meeting essentially depends on the likelihood of debasement (π) as well as the debasement rate (δ). However, the consumption in the UoA-posting equilibrium does not rely on (π, δ). Therefore, if the likelihood of debasement or its rate is high enough, sufficiently risk averse agents would prefer the UoA posting of price to the penny posting and hence the value of a silver bar in the UoA-posting equilibrium would be higher than that in the penny-posting equilibrium.

This result immediately suggests that welfare in the UoA-posting equilibrium should be higher than that in the penny-posting equilibrium under the assumptions of Proposition 1, where the welfare is defined as $\mathcal{W}^i = MV_b^i + (1-M)V_s^i$ for $i \in \{u, m\}$.

Corollary 1 *Under the assumptions made in Proposition 1, $\mathcal{W}^u > \mathcal{W}^m$.*

Proof. See Appendix. ■

The UoA posting of price at the time of a relatively high debasement rate (smaller δ) is consistent with a key fact on medieval monetary debasements. That is, Rolnick, Velde and Weber (1997) show that through 123 debasements in France (1285∼1490) and during the Great Debasement in England (1542∼1551), gold coins and, in some cases, silver coins were

valued in circulation by their intrinsic content (circulation by weight) rather than by their legal tender value (circulation by tale).

The results are also in line with Keynes' (1923) claim that as the volatility of MoE increases, its quality as a UoA is deteriorated and an alternative UoA separated from an MoE would emerge. Notice that from the viewpoint of fiat-money system, $\delta \in (0.1)$ can be interpreted as an inflation in the sense that $\delta < 1$ implies the reduction in penny-equivalent quantity of good.

Furthermore, in modern economies, we can find quite a few incidents that support our results. During the German hyperinflation in the early 1920s, sellers posted prices in terms of goldmarks, while payments were made by currency called Reichsmark (Wolf 2002). More recently, Brazil introduced a new UoA called *Unidade Real de Valor* following hyperinflation in the 1980s and early 1990s. It was entirely separated from the domestic MoE and linked to the U.S. dollar instead. Other Latin American countries such as Chile, Colombia, Ecuador, Mexico, and Uruguay also introduced separate UoA in the aftermath of high inflation (Shiller 2002).

5. Price Flexibility and Additional Implications

The UoA-posting equilibrium has a contrasting implication for the nominal price rigidity compared to the penny-posting equilibrium.

Proposition 2 *In an equilibrium with UoA posting, the number of pennies exchanged for a unit of good is adjusted in proportion to the debasement rate δ, whereas it does not in an equilibrium with penny posting.*

It is worth noting that in the equilibrium where price is quoted in terms of UoA, the number of pennies exchanged for a unit of good $(1/\delta q_u)$ changes proportionally to the debasement rate δ. That is, since q_u does not rely on δ, the number of pennies exchanged

for a unit of good with, say, $\delta = 1/2$ is twice as many as that with $\delta = 1$. On the other hand, in the equilibrium where price is quoted in terms of pennies, the number of pennies exchanged for a unit of good $[\delta^{-1}/(\delta^{-1}q_m) = 1/q_m]$ does not vary with δ. It suggests that in the fiat-money system where MoE and UoA are integrated, price quoted in terms of MoE would not be adjusted immediately following inflation.

These results as well as the model can be applied to various issues. First, the debasement can be interpreted as inflation in the fiat-money system. Then our model implies that in an economy where inflation is very high or volatile, introducing a stable UoA different from a domestic currency (MoE) would be desirable. That is, a stable UoA can facilitate domestic and international trades by reducing risk associated with volatility in the value of domestic currency (e.g., Rose 2000, Rose and Wincoop 2001). Other than the incidents of Germany and Latin American counties discussed above, most countries that adopted currency board or dollarization had experienced high inflation (e.g., Chang 2000, Alesina and Barro 2001, Cooper and Kempf 2001, and Edwards and Magendzo 2001). Put differently, those countries introduced a more stable foreign currency such as the U.S. dollar as an alternative UoA to a domestic currency. In a similar vein, our model can be used to explain imperfect substitutability among different currencies such as Euro, U.S. dollar, or Yen.

Finally, our model can also provide a theoretical framework in discussing the validity of commodity basket or currency basket as a UoA. For instance, Chile has adopted UF (*Unidad de Fomento*) as a UoA, which is indexed to the basket of consumer price index. Wolf (2002) and Rahn (2010) claim that the adoption of an optimal currency basket as an international UoA would reduce exchange risk substantially.

6. Concluding Remarks

We have provided a theoretical exposition for the separation of a medium of exchange (MoE) from a unit of account (UoA) by considering explicitly a seller's choice with regard to posting

price in terms of either an MoE (e.g., silver coin or penny) or a UoA (e.g., one ounce of silver) in the commodity-money system. We show that, as long as the likelihood of debasement or the debasement rate is high enough and agents are sufficiently risk averse, there exists a symmetric stationary monetary equilibrium in which price is posted in terms of UoA. This is consistent with a key fact during the Middle Ages that through 123 debasements in France and during the Great Debasement in England, coins were valued in circulation by their intrinsic content (circulation by weight) rather than by their legal tender value (circulation by tale).

The UoA-posting equilibrium also implies the flexible nominal price measured in terms of MoE in the sense that the number of pennies exchanged for a unit of good changes proportionally to the debasement rate. This is in a stark contrast to the sticky nominal price in the penny-posting equilibrium where the number of pennies exchanged for a unit of good does not vary with the debasement rate. This then suggests the nominal price rigidity in the fiat-money system where MoE and UoA are integrated.

Meanwhile, Fisher (1913) claimed that if an MoE is separated from a UoA, there should be some computational cost because of "laborious calculations in translations from the medium of exchange into the standard of deferred payments and back again." Shiller (2002) argued that at least in these ages, such an inconvenience would hardly matter due to advanced computing power. This point is not embedded in our model explicitly.

7. Appendix: Proofs

Proof of Lemma 1: We here use the *unimprovability* criterion to show the results.[7] The result follows from Claim 1 to Claim 6.

Claim 1 *If the price is posted in terms of UoA, q_u in (9) is unimprovable as long as $M < \bar{M}^u$*

[7]For more details, see footnote 8 in Williamson and Wright (1994): "A candidate policy is called unimprovable ... if the payoff from using it cannot be increased by deviating to a different decision at a single date and then reverting back to the candidate policy for the rest of time."

and $\tilde{\rho} < \bar{\rho}$.

Proof. Notice that from (2), $\tilde{\rho}V_s^u(q_u) = 2M(V_b - V_s^u - q_u)$ and $\tilde{\rho}V_s^u(q_u') = M(V_b - V_s^u - q_u')$ where q_u' satisfies $V_s^u - V_b + (q_u')^\gamma = 0$ and $\tilde{\rho}V_s^u(q_u')$ is a period net benefit of deviation from q_u to q_u' when other sellers post q_u. Then $\tilde{\rho}[V_s^u(q_u) - V_s^u(q_u')] = M(V_b - V_s^u) - M(2q_u - q_u') = M(V_b - V_s^u) - M(2 - \alpha^{1/\gamma})q_u$, which is positive if $(V_b - V_s^u) > (2 - \alpha^{\frac{1}{\gamma}})q_u$. Since $(V_b - V_s^u) = \alpha(q_u)^\gamma$ and $q_u = \{[\alpha(1+\tilde{\rho}+M) - (1-M)]/2M\}^{\frac{1}{1-\gamma}}$, $(V_b - V_s^u) - (2 - \alpha^{\frac{1}{\gamma}})q_u > 0$ can be rearranged as

$$M < \frac{(2 - \alpha^{\frac{1}{\gamma}})[1 - \alpha(1+\tilde{\rho})]}{(2 - \alpha^{\frac{1}{\gamma}})(1+\alpha) - 2\alpha} \equiv \bar{M}^u$$

where $\bar{M}^u \in (0,1)$ because $(2 - \alpha^{1/\gamma})(1+\alpha) - 2\alpha = 2 - \alpha^{1/\gamma}(1+\alpha) > 0$, $1 - \alpha(1+\tilde{\rho}) > 0$ due to $\tilde{\rho} < [(1-\alpha)/\alpha] \equiv \bar{\rho}$, and $[(2 - \alpha^{1/\gamma})(1+\alpha) - 2\alpha] - [(2 - \alpha^{1/\gamma})(1 - \alpha(1+\tilde{\rho}))] = 2\alpha[(2 - \alpha^{1/\gamma}) - 1] + \alpha(2 - \alpha^{1/\gamma})\tilde{\rho} > 0$. Therefore, when other seller post q_u, the individual strategy of posting q_u is unimprovable. ■

Claim 2 *If the price is posted in terms of pennies,* q_m *in (15) is unimprovable as long as* $M < \bar{M}^m$, $\tilde{\rho} < \bar{\rho}$, *and* α *is low enough.*

Proof. Notice that $\tilde{\rho}V_s^m(q_m) = 2M(V_b - V_s^m - \bar{\delta}q_u)$ and $\tilde{\rho}V_s^m(q_m') = M[(2-\pi)(V_b - V_s^m) - q_m'(\delta^{-1}(1-\pi) + \bar{\delta})]$ where q_m' satisfies $V_s^m - V_b + a(\delta^{-1}q_m')^\gamma = 0$. That is, q_m' is the next lowest price if α is low enough so that $\alpha < \delta^\gamma$. Now, $\tilde{\rho}V_s^m(q_m')$ is a period net benefit of deviation from q_m to q_m' when other sellers post q_m. Then, $\tilde{\rho}[V_s^m(q_m) - V_s^m(q_m')]$ can be rearranged as $M\pi\alpha(q_m)^\gamma - Mq_m[2(1-\pi)(\delta^{-1} - 1) + \pi(2-\delta)]$, which is positive if

$$M < \frac{\left[2\bar{\delta} - \pi\delta - 2(1-\pi)\right]\left[\hat{\delta} - \alpha(2 - \pi + \tilde{\rho})\right]}{(\alpha\pi + \hat{\delta})[2\bar{\delta} - \pi\delta - 2(1-\pi)] - \pi\alpha\bar{\delta}} \equiv \bar{M}^m \tag{17}$$

where $\hat{\delta} \equiv [\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}]$. Notice that the denominator of the right-hand side in (17) is positive; $(\alpha\pi + \hat{\delta})[2\bar{\delta} - \pi\delta - 2(1-\pi)] - \pi\alpha\bar{\delta} = \pi\alpha\bar{\delta}(1-\delta) + \hat{\delta}\bar{\delta}(1-\delta) + \hat{\delta}(1-\pi)(\delta^{-1} -$

$1) + \pi^2 + (1-\pi)[\pi\delta^{-\gamma} + a\pi(\delta^{-\gamma} - 1)] > 0$. And the numerator of it is also positive;

$$\begin{aligned}
& [2\bar{\delta} - \pi\delta - 2(1-\pi)][\hat{\delta} - \alpha(2 - \pi + \tilde{\rho})] \\
= \; & [2(\bar{\delta} - 1) + \pi(2-\delta)][\pi + (1-\pi)(1+\alpha)\delta^{-\gamma} - \alpha(2-\pi) - \alpha\tilde{\rho}] \\
> \; & [2(\bar{\delta} - 1) + \pi(2-\delta)][\pi + (1-\pi)(1+\alpha)\delta^{-\gamma} - \alpha(2-\pi) - (1-\alpha)] \\
= \; & [2(\bar{\delta} - 1) + \pi(2-\delta)](1+\alpha)(1-\pi)[\delta^{-\gamma} - 1] > 0
\end{aligned}$$

where the third inequality comes from $\tilde{\rho} < \bar{\rho} = [(1-\alpha)/\alpha]$. Further, the denominator of the right-hand side in (17) is greater than that of the numerator; that is, the denominator is larger than the numerator if $\alpha[2A + A\tilde{\rho} - \pi\bar{\delta}] > 0$, where $A \equiv [2\bar{\delta} - \pi\delta - 2(1-\pi)]$. It is the case because $2A + A\tilde{\rho} - \pi\bar{\delta} = 4\bar{\delta} - 2\pi\delta - 4(1-\pi) + 2\bar{\delta}\tilde{\rho} - \pi\delta\tilde{\rho} - 2\tilde{\rho}(1-\pi) - \pi\bar{\delta} = \tilde{\rho}[(2-\delta)\bar{\delta} - (1-\pi)] + 2(1-\pi)(\bar{\delta}-1)\bar{\delta}(\pi + 2 - 2\delta) > 0$. Hence, $\bar{M}^m \in (0,1)$ and for $M < \bar{M}^m$, the individual strategy of posting q_m is unimprovable if other sellers post q_m. ■

Claim 3 *$\bar{M}^u < \bar{M}^m$.*

Proof. The sign of $(\bar{M}^u - \bar{M}^m)$ depends on $(2 - \alpha^{1/\gamma})[1 - \alpha(1+\tilde{\rho})][(\alpha\pi + \hat{\delta})A - \pi\alpha\bar{\delta}] - A[(2-\alpha^{1/\gamma})(1+\alpha) - 2\alpha][\hat{\delta} - \alpha(2 - \pi + \tilde{\rho})]$, which can be rearranged as $(\alpha^{1/\gamma} - 1)2\alpha A\hat{\delta} - (2 - \alpha^{1/\gamma})\{\alpha(1+\alpha)A(2+\tilde{\rho}) + \pi\alpha(2+\tilde{\rho}) + \pi\alpha\bar{\delta}[1 - \alpha(1+\tilde{\rho})]\} < 0$. ■

Claim 4 *$V_s^u > 0$ and $V_s^m > 0$ as long as $M < \bar{M}_0$.*

Proof. From (10) and (16), $V_s^u > 0$ and $V_s^m > 0$ if $2M\alpha - \mathbb{X}_u > 0$ and $2M\alpha - \mathbb{X}_m > 0$. Since $\mathbb{X}_u - \mathbb{X}_m = \alpha(1+\tilde{\rho}+M) - (1-M) - \alpha[2 - \pi(1-M) + \tilde{\rho}] - (1-M)\hat{\delta} = (1-M)(1-\pi)(1+\alpha)(\delta^{-\gamma} - 1) > 0$, we have $\mathbb{X}_u > \mathbb{X}_m$. Hence, if $2M\alpha - \mathbb{X}_u = 2M\alpha - \alpha(1+\tilde{\rho}+M) + (1-M) = (1-M) - \alpha(1+\tilde{\rho}-M) > 0$, then $2M\alpha - \mathbb{X}_m > 0$. The inequality of $(1-M) - \alpha(1+\tilde{\rho}-M) > 0$ holds for M such that $M < \{[1 - \alpha(1+\tilde{\rho})]/(1-\alpha)\} \equiv \bar{M}_0$. ■

Claim 5 *$\bar{M}^u < \bar{M}_0$.*

Proof. The sign of $(\bar{M}^u - \bar{M}_0)$ depends on the sign of $(1-\alpha)(2-\alpha^{1/\gamma})[1-\alpha(1+\tilde{\rho})] - (1+\alpha)(2-\alpha^{1/\gamma})[1-\alpha(1+\tilde{\rho})] + 2\alpha[1-\alpha(1+\tilde{\rho})]$, which can be rearranged as $2\alpha(\alpha^{1/\gamma} - 1)[1-\alpha(1+\tilde{\rho})] < 0$. ■

Claim 6 *If $M < \bar{M} = \bar{M}^u$, $\tilde{\rho} < \bar{\rho} = (1-\alpha)/\alpha$, and $\alpha < \delta^\gamma$, there is a symmetric stationary monetary equilibrium in which the lowest price is posted in terms of either UoA or MoE.*

Proof. Obvious consequence of Claim 1-6. ■

Proof of Proposition 1: We first prove the case of $\pi < \pi_0$. We need to show that $V_s^u - V_s^m > 0$. Since $\tilde{\rho}V_s^u = \frac{2M}{\mathbb{X}_u}[2M\alpha - \mathbb{X}_u]\, q_u$ with $q_u = [\mathbb{X}_u/2M]^{1/(1-\gamma)}$ and $\tilde{\rho}V_s^m = \frac{2M}{\mathbb{X}_m}[2M\alpha - \mathbb{X}_m]\,\bar{\delta}q_m$ with $q_m = [\mathbb{X}_m/2M\bar{\delta}]^{1/(1-\gamma)}$, $\frac{1}{2M}(\tilde{\rho}V_s^u - \tilde{\rho}V_s^m)$ can be rearranged as

$$(2M)^{\frac{1}{\gamma-1}}\left[(\mathbb{X}_u)^{\frac{\gamma}{1-\gamma}}(2M\alpha - \mathbb{X}_u) - (\mathbb{X}_m)^{\frac{\gamma}{1-\gamma}}(2M\alpha - \mathbb{X}_m)\bar{\delta}^{\frac{\gamma}{\gamma-1}}\right].$$

Since $(\mathbb{X}_u)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_u) > 0$ and $(\mathbb{X}_m)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_m)\bar{\delta}^{\gamma/(\gamma-1)} > 0$ due to $\tilde{\rho}V_s^u > 0$ and $\tilde{\rho}V_s^m > 0$, respectively, in Lemma 1, the sign of $(\tilde{\rho}V_s^u - \tilde{\rho}V_s^m)$ depends on their relative magnitudes. In other words, $(\tilde{\rho}V_s^u - \tilde{\rho}V_s^m) > 0$ if

$$\bar{\delta} > \left[\frac{(\mathbb{X}_m)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_m)}{(\mathbb{X}_u)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_u)}\right]^{\frac{1-\gamma}{\gamma}}. \tag{18}$$

Since $\bar{\delta} = \pi + (1-\pi)\delta^{-1}$, $\bar{\delta}$ is strictly decreasing function with respect to π ($\partial\bar{\delta}/\partial\pi = (1-\delta^{-1}) < 0$) with $\bar{\delta} = \delta^{-1} > 1$ at $\pi = 0$ and $\bar{\delta} = 1$ at $\pi = 1$. Now notice that the denominator in the bracket of the right-hand side of (18) does not depend on π because $\mathbb{X}_u \equiv \alpha(1+\tilde{\rho}+M) - (1-M)$. For the numerator, let define $G(\pi) = (\mathbb{X}_m)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_m)$ where $\mathbb{X}_m \equiv \alpha[2-\pi(1-M)+\tilde{\rho}] - [\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}](1-M)$. Then

$$\frac{\partial G}{\partial \pi} = \left(\frac{1}{1-\gamma}\right)(\mathbb{X}_m)^{\frac{\gamma}{1-\gamma}}\left[2M\alpha\gamma(\mathbb{X}_m)^{-1} - 1\right]\frac{\partial \mathbb{X}_m}{\partial \pi}$$

with

$$\frac{\partial \mathbb{X}_m}{\partial \pi} = (1-M)\left[(1+\alpha)(\delta^{-\gamma}-1)\right] > 0.$$

Therefore, $(\partial G/\partial \pi) > 0$ if $2M\alpha\gamma > \mathbb{X}_m$ and $(\partial G/\partial \pi) < 0$ if $2M\alpha\gamma < \mathbb{X}_m$. That is, from

$$2M\alpha\gamma > \alpha[2-\pi(1-M)+\tilde{\rho}] - [\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}](1-M) \tag{19}$$

we have $(\partial G/\partial \pi) > 0$ if

$$\pi < \frac{(1+a\alpha(1-M)\delta^{-\gamma} + 2M\alpha\gamma - \alpha(2+\tilde{\rho})}{(1+\alpha)(1-M)(\delta^{-\gamma}-1)} \equiv \pi_u$$

and $(\partial G/\partial \pi) < 0$ if $\pi > \pi_u$. Furthermore, if $\pi = 1$ or $\pi = 0$, $V_s^u = V_s^m$ and hence, the right-hand side of (18) equals to 1. Notice that $(\mathbb{X}_m)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_m) = 2M\alpha(\mathbb{X}_m)^{\gamma/(1-\gamma)} - (\mathbb{X}_m)^{1/(1-\gamma)}$ is maximized at $\mathbb{X}^* = 2M\alpha\gamma$ where $\mathbb{X}^*$ comes from

$$2M\alpha\left(\frac{\gamma}{1-\gamma}\right)(\mathbb{X}^*)^{\frac{2\gamma-1}{1-\gamma}} = \left(\frac{1}{1-\gamma}\right)(\mathbb{X}^*)^{\frac{\gamma}{1-\gamma}}.$$

Then, at $\pi = \pi_u$, $2M\alpha\gamma = \mathbb{X}^* = \mathbb{X}_m$ and hence, the right-hand side of (18) is greater than 1. Finally, a low enough γ so that the right-hand side of (18) converges more rapidly to 1 as $\pi \to 1$ compared to the left-hand side ensures that there is a unique $\pi_0 \in (0, \pi_u)$ that satisfies the equality in (18). Then, strict inequality in (18) holds for $\pi \in (0, \pi_0)$.

We now prove the case of $\delta < \delta_0$. Since $\bar{\delta} = \pi + (1-\pi)\delta^{-1}$, $\bar{\delta}$ is strictly decreasing function with respect to δ ($\partial\bar{\delta}/\partial\delta = -(1-\pi)\delta^{-2} < 0$) with $\bar{\delta} = \infty > 1$ at $\delta = 0$ and $\bar{\delta} = 1$ at $\delta = 1$. For the numerator in the bracket of the right-hand side of (18), let define $H(\delta) = (\mathbb{X}_m)^{\gamma/(1-\gamma)}(2M\alpha - \mathbb{X}_m)$ where $\mathbb{X}_m \equiv \alpha[2-\pi(1-M)+\tilde{\rho}] - [\pi + (1-\pi)(1+\alpha)\delta^{-\gamma}](1-M)$. Then

$$\frac{\partial H}{\partial \delta} = \left(\frac{1}{1-\gamma}\right)(\mathbb{X}_m)^{\frac{\gamma}{1-\gamma}}\left[2M\alpha\gamma(\mathbb{X}_m)^{-1} - 1\right]\frac{\partial \mathbb{X}_m}{\partial \delta}$$

with

$$\frac{\partial \mathbb{X}_m}{\partial \delta} = \gamma(1-M)(1-\pi)(1+\alpha)\delta^{-1-\gamma} > 0.$$

Therefore, $(\partial H/\partial \delta) > 0$ if $2M\alpha\gamma > \mathbb{X}_m$ and $(\partial H/\partial \delta) < 0$ if $2M\alpha\gamma < \mathbb{X}_m$. That is, from (19), we have $(\partial H/\partial \delta) > 0$ if

$$\delta < \left[\frac{(1+\alpha)(1-M)(1-\pi)}{\alpha(2+\tilde{\rho}) - \pi(1+\alpha)(1-M) - 2M\alpha\gamma}\right]^{\frac{1}{\gamma}} \equiv \delta_u$$

and $(\partial H/\partial \delta) < 0$ if $\delta > \delta_u$. Notice that if $\delta = 1$, the right-hand side of (18) equals to 1 and if $\delta \to 0$, that of (18) goes to $-\infty$. Now, exactly the same arguments above give the result that there is a unique $\delta_0 \in (0, \delta_u)$ that satisfies the equality in (18) and for $\delta \in (0, \delta_0)$, strict inequality in (18) holds.

Proof of Corollary 1: Since $V_s^u > V_s^m$ in the UoA-posting equilibrium, (6) and (13) implies that $(V_b^u - V_s^u - q_u) > (V_b^m - V_s^m - \bar{q}_m)$. Therefore, it suffices to show that $q_u \geq \bar{q}_m$ because it immediately implies $V_b^u > V_b^m$ and consequently $MV_b^u + (1-M)V_s^u = \mathcal{W}^u > \mathcal{W}^m = MV_b^m + (1-M)V_s^m$. Notice that

$$\begin{aligned}
\bar{q}_m &= [\pi + (1-\pi)\delta^{-1}]q_m = \bar{\delta} q_m = \bar{\delta}\left[\frac{\mathbb{X}_m}{2M\bar{\delta}}\right]^{\frac{1}{1-\gamma}} = \bar{\delta}^{\frac{\gamma}{\gamma-1}}\left[\frac{\mathbb{X}_m}{2M}\right]^{\frac{1}{1-\gamma}} \\
&< \left[\frac{\mathbb{X}_m}{2M}\right]^{\frac{1}{1-\gamma}} < \left[\frac{\mathbb{X}_u}{2M}\right]^{\frac{1}{1-\gamma}} = q_u
\end{aligned}$$

where the first inequality in the second line is due to $\gamma \in (0,1)$ and $\bar{\delta} > 1$, and the last inequality is due to $\mathbb{X}_m < \mathbb{X}_u$ that is shown in Claim 4 of Lemma 1.

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