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Recognizability and Liquidity of Assets

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Abstract

This paper incorporates the recognizability of assets explicitly into the standard search model of exchange to determine the liquidity returns as an equilibrium outcome. Assuming that money is universally recognizable but bond is not, the two types of the single-coincidence meetings arise–one where both money and bond are accepted and the other where only money is accepted as medium of exchange–depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. The equilibrium restrictions imply that the liquidity differentials between money and bond tend to increase with the recognizability problem. With the relatively mild recognizability problem, there only exists an equilibrium where all the buyers bring the authentic bond to the decentralized market and sellers always accept the bond of unrecognized quality, and hence money and bond become equally liquid. As the recognizability problem becomes sufficiently severe, there only exists an equilibrium where some buyers bring the counterfeit bond, but sellers randomize between accepting and rejecting the bond of unrecognized quality. Money commands higher liquidity than bond by providing the additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond. The coexistence of money and bond requires a higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium.

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1 Introduction

An asset's liquidity–the degree to which it facilitates exchange–has been regarded as being essential to the determination of asset price. Townsend (1987), Bansal and Coleman (1996), Kiyotaki and Moore (2005, 2008), and Nah and Kim (2008) share the common implication that liquidity plays a significant role in accounting for the equity premium or the risk-free rate puzzle. Using a search-based model of exchange, Lagos (2008) shows that small liquidity differences have significant quantitative implications for asset prices in general, and the equity premium and risk-free rate puzzles in particular. Aruoba et al. (2007), Geromichalos et al. (2007), and Lagos and Rocheteau (2008) also use similar models to examine the implications of liquidity for asset prices and monetary policy. However, in these studies, either equal or differential liquidity between assets is exogenously assumed.

The goal of this paper is to derive endogenously the liquidity returns from holding assets, given the physical characteristics of the multiple assets, say money and bond. Specifically, in the spirit of Jevons (1875), King and Plosser (1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we note the recognizability of assets such that money is universally recognizable, whereas bond is not. Following Wallace's (1998) dictum, we incorporate the recognizability of assets explicitly into the standard search-based model of exchange such as Lagos and Wright (2005), and Lagos (2008) to determine the liquidity of assets as an equilibrium outcome.

Specifically, we assume that buyers have access to the bond counterfeit technology at no cost in the decentralized market. In a bilateral meeting, a seller can imperfectly recognize the quality of the buyer's bond. If a seller cannot recognize the quality of bond and she decides not to accept the bond of unrecognized quality, only money is accepted. If a seller cannot recognize the quality of bond but she decides to accept the bond of

unrecognized quality, both money and bond are accepted. It is worth noting that these two types of meetings in the decentralized market arise endogenously depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. This is in contrast to Lagos (2008) where an agent is placed in these two types of meetings due to some exogenous institutional restrictions.

We first show that there exist three types of equilibria where both money and bond are accepted as media of exchange. More importantly, the equilibrium restrictions imply that liquidity differentials between money and bond tend to increase with the recognizability problem. When the recognizability problem is not so severe, there only exists an equilibrium where all the buyers bring the authentic bond and sellers always accept the bond of unrecognized quality. This implies that money and bond become equally liquid in the sense that buyers obtain the identical expected marginal benefit from bringing additional money or bond to the decentralized market. This also means that money would not be essential if the sufficient liquidity service were provided by the ample supply of bond, as shown in Geromichalos et al. (2007), and Lagos and Rocheteau (2008).

As it becomes more difficult to recognize the quality of bond, there also arises an equilibrium where some buyers bring the counterfeit bond to the decentralized market, but sellers either accept or randomize between accepting and rejecting the bond of unrecognized quality. In an equilibrium where sellers accept the unrecognized bond, money commands higher liquidity than bond in the sense that, unlike bond, money provides the additional liquidity service when the counterfeit bond is recognized. In an equilibrium where sellers randomize the acceptance of the unrecognized bond, further liquidity service of money arises when sellers reject the bond of unrecognized quality as

well as when they recognize the counterfeit bond.

This implies that, in the presence of the recognizability problem with bond, the coexistence of money and bond requires higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium. This also means that, unlike the equilibrium with equal liquidity, money is essential due to its additional liquidity service. Finally, when the recognizability problem is sufficiently severe, there only exists an equilibrium where some buyers bring the counterfeit bond and sellers randomize between accepting and rejecting the bond of unrecognized quality.

These results are complementary to Rocheteau (2008) which links the partial liquidity of a real asset to the properties of its dividend process. Specifically, in the presence of asymmetric information on the dividend of the real asset, its liquidity is shown to decrease as the dispersion of the dividends across states increases. The imperfect substitutability between fiat money and the real asset also yields a rate-of-return differential, implying a positive liquidity premium. Thus, the relationship between the liquidity of a real asset and its physical characteristics appears to be robust regardless of the specific characteristics under consideration, recognizability or payoffs.

Our findings are also analogous to Lester et al. (2008) where if there are some agents with sufficiently high costs of verifying the authenticity of bond, both money and real assets circulate as media of exchange despite their liquidity differentials.[1] Moreover, the results of the paper generalize the recent studies on liquidity, asset prices and monetary policy (e.g., Geromichalos et al. 2007, Lagos 2008, and Lagos and Rocheteau 2008) in the sense that the liquidity differentials between assets and the essentiality of money are endogenously determined in the environment where money and bond differ in the

[1] As a related approach to modeling liquidity, Glosten and Milgrom (1985), and Kyle (1985) consider bilateral transactions with information asymmetries between a middleman and either a buyer or a seller who has more information about the expected future payoffs of an asset than the middleman. However, these models do not answer the questions of explaining the liquidity differences between money and other assets.

inherent properties such as recognizability.

The paper is organized as follows. Section 2 describes the model economy, followed by the equilibrium characterization in Section 3. Section 4 discusses the different types of equilibria with equal and differential liquidity between money and bond, including the relationship between recognizability and liquidity. Section 5 summarizes the paper with a few concluding remarks, followed by Appendix which contains the proofs of the propositions in the paper.

2 Model

The basic setup is Lagos and Wright (2005), and Lagos (2008) with the explicit consideration of the asset recognizability. There is a non-atomic unit measure set of infinitely-lived agents. Time is indexed by $t \in \mathbb{Z}_+$ and in each period t there are two markets, the decentralized and the centralized markets that open sequentially. There are three perishable and perfectly divisible consumption goods at each period: coconuts, apples and general goods. People cannot make any binding intertemporal commitments and their trading histories are private, and hence all the trades should be on the spot. Each agent maximizes the discounted expected utility with the discount factor $\beta \in (0, 1)$ between the decentralized market and the centralized market.[2]

In the model, there are two durable assets, money and a set of *Lucas trees*. The stock of perfectly divisible money is exogenously given by $\bar{M} > 0$, and the number of trees is also fixed and equal to the number of agents. Each tree yields d_t units of apples as dividends after the decentralized trades but before the current trades in the centralized markets where d_t follows an i.i.d. process realized at the beginning of each

[2]The discounting is not between the centralized market and the next decentralized market. As in Rocheteau and Wright (2003, 2005), all that matters is the total discounting between one period and the next.

period with $d_t \in \{d^h, d^l\}$, $d^h > d^l > 0$, and $\Pr\left[d_t = d^h\right] = \mu$, $\Pr\left[d_t = d^l\right] = 1-\mu$. Each tree has outstanding one durable and perfectly divisible consol bond that entitles the ownership of a share of tree as well as the dividends. In the spirit of Jevons (1875), King and Plosser (1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we assume that the quality of money is perfectly recognizable, whereas the quality of bond is not.

At the opening of the decentralized market, a half of the agents are endowed with $\varepsilon^h = (1+\varepsilon)A$ units of coconuts and the remaining half with $\varepsilon^l = (1-\varepsilon)A$ units of coconuts where $\varepsilon \in (0,1)$ and $A > 0$. We call the former as type-h agents and the latter as type-l agents. The realization of the stochastic individual endowments is i.i.d. across periods and agents. Note that the trades of coconuts for assets are motivated by these heterogenous endowments among agents. An agent gets utility $v(q)$ from consuming q units of coconuts where $v''(q) < 0 < v'(q)$ and $v'(\varepsilon^l)$ is sufficiently large.

After the realization of the endowment shock, type-l agents (as potential buyers) can access the bond counterfeit technology at no cost and they should decide whether to counterfeit bond or not. Those who do not counterfeit bond move to the decentralized market with money and authentic bond, while those who counterfeit bond move to the market with money and counterfeit bond. We assume for simplicity that potential buyers do not carry some combinations of authentic and counterfeit bonds to the decentralized market. It turns out that this assumption hardly affects the key relationship between the recognizability of an asset and its liquidity return, as will be discussed in Sections 3 and 4.

In the following bilateral meetings, each agent is randomly matched with another agent. Trades can occur in the single-coincidence meetings between a type-h agent (a potential seller) and a type-l agent (a potential buyer). People in a meeting can observe

each other's endowment and portfolio.

Further, a type-h agent receives a common-knowledge signal regarding the quality of bond held by the type-l agent. With probability $\theta_1 \in (0,1)$, the signal is informative and the quality of bond is revealed to a type-h agent; with probability $\theta_2 = 1-\theta_1$, the signal is uninformative and the quality of bond is not recognized. In an informative single-coincidence meeting, the terms of trade are determined by generalized Nash bargaining in which the buyer has bargaining power of $\eta \in (0,1)$. In an uninformative meeting, a type-h agent has to decide whether to accept the bond of unrecognized quality with the understanding that the terms of trade are equivalent to those in an informative meeting. It is worth noting that, as long as a type-h agent as a potential seller has some bargaining power $(1-\eta > 0)$ to obtain positive surplus in bilateral trade, the terms of payment will be agreed upon to compensate the risk that a potential seller takes by accepting the bond of unrecognized quality. Immediately after the bilateral trade, the counterfeit bond turns into a worthless red paper.

In the centralized market, the bearer of the authentic bond collects dividends (i.e., apples from a tree). An agent gets utility $u(c)$ from consuming c units of apples where $u''(c) < 0 < u'(c)$ and $u'(0) = \infty$. All agents can also produce, trade, and consume general goods. Each agent can produce one unit of general goods using one unit of labor which incurs one unit of disutility. An agent gets utility $U(y)$ from consuming y units of general goods where $U''(y) < 0 < U'(y)$ and $U'(0) = \infty$.

3 Equilibrium

To facilitate the description of an equilibrium, we first introduce some notations. Let $a_t = (b_t, m_t) \in \mathbb{R}^2_+$ denote a portfolio where b_t and m_t denote respectively bond and money holdings at the beginning of t. Let ϕ^g_t, ϕ^b_t, and ϕ^m_t denote respectively the

unit price of general good, bond (ex-dividend) and money in terms of apples. Also, let $V(a,d)$ denote the value function for an agent who enters the decentralized market with $a=(b,m)$ and dividend d, and $W(a,d)$ denote the value function when she enters the centralized market. In what follows, we will formulate an equilibrium in the recursive manner and work backward from the centralized market to the decentralized market.

3.1 Centralized Market

In the centralized market, agents produce general goods, consume apples and general goods, and adjust their portfolios. Hence, the problem for a representative agent entering the centralized market with $a_t=(b_t,m_t)$ is

$$W(a_t,d_t)=\max_{y_t,c_t,h_t,a_{t+1}}\{U(y_t)+u(c_t)-h_t+\mathbb{E}V(a_{t+1},d_{t+1})\}$$

$$\text{s.t. } \phi_t^g y_t+c_t=\phi_t^g h_t+(\phi_t^b+d_t)b_t+\phi_t^m m_t-\phi_t^b b_{t+1}-\phi_t^m m_{t+1},$$

$$c_t\geq 0,\ a_{t+1}\geq 0,\ y_t\geq 0,\ h_t\in[0,\bar{h}]$$

where $\bar{h}$ is an upper bound on h_t. We assume an interior solution for c_t, y_t and h_t.[3] Let $\phi_t=(\phi_t^b,\phi_t^m)$, and $\hat{\phi}_t=(\hat{\phi}_t^b,\hat{\phi}_t^m)$ with $\hat{\phi}_t^b\equiv(\phi_t^b+d_t)/\phi_t^g$ and $\hat{\phi}_t^m\equiv\phi_t^m/\phi_t^g$. Substituting h_t from the budget constraint, we have

$$W(a_t,d_t)=\hat{\phi}_t a_t+\max_{\{y_t,c_t,a_{t+1}\}}\left\{U(y_t)-y_t+u(c_t)-\frac{1}{\phi_t^g}(c_t+\phi_t a_{t+1})+\mathbb{E}V(a_{t+1},d_{t+1})\right\}.$$

The first order conditions are

$$U'(y_t)=1 \tag{1}$$

$$u'(c_t)=1/\phi_t^g \tag{2}$$

[3]An interior solution for c_t and y_t is guaranteed under the standard assumption on $u(c)$ and $U(y)$. It would be also true for h_t under the assumption as in Lagos and Wright (2005).

$$\phi_t^b/\phi_t^g \geq \partial V/\partial b_{t+1}, \ = \ \text{if } b_{t+1} > 0 \tag{3}$$

$$\phi_t^m/\phi_t^g \geq \partial V/\partial m_{t+1}, \ = \ \text{if } m_{t+1} > 0. \tag{4}$$

The envelope conditions are

$$\partial W(a_t, d_t)/\partial b_t = \hat{\phi}_t^b \tag{5}$$

$$\partial W(a_t, d_t)/\partial m_t = \hat{\phi}_t^m. \tag{6}$$

The conditions (1) and (2) imply that the consumption of general goods and apples does not depend on the current asset holdings a_t. The conditions (3) and (4) determine the portfolio to carry over the following period, $a_{t+1} = (b_{t+1}, m_{t+1})$, which is also independent of a_t. The conditions (5) and (6) imply that the value function $W(a_t, d_t)$ is linear in a_t. Further, a unique solution of $a_{t+1} > 0$ would be ensured by assuming sufficiently large η as shown in Lagos and Wright (2005), which implies that the portfolio distribution is degenerate at the beginning of each period.

3.2 Decentralized Market

The following two types of the bilateral meetings arise in the decentralized market: one is a meeting where both money and bond are traded for coconuts, and the other is a meeting where only money is traded for coconuts. If a seller can recognize the quality of bond or if she cannot recognize the quality of bond but decides to accept it, both assets can be accepted. If a seller cannot recognize the quality of bond and she decides not to accept the bond of unrecognized quality, then only money is accepted.[4]

Notice that these two types of meetings in the decentralized market are determined

[4]Although we assume that there is no direct cost in counterfeiting bonds, a potential buyer ends up bearing the cost of carrying the counterfeit bond by sacrificing some surplus in an informative single-coincidence meeting. For simplicity, the upper bound of this cost is considered here by assuming that a potential buyer does not carry some combinations of authentic and counterfeit bonds to the decentralized market.

endogenously depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond. This is in contrast to Lagos (2008) where an agent is placed in these two types of meetings due to some exogenous institutional restrictions.

Further, there are two sets of terms of trade in the decentralized market: $\left(q^{bm},\, p^{bm}\right)$ for the meeting where both money and bond are accepted, and $(q^m,\, p^m)$ for the meeting where only money is accepted. That is, a buyer hands over the portfolio $p^i = (p_1^i,\, p_2^i)_{i=bm,m} \in \mathbb{R}_+^2$ to a seller in exchange for $q^i \in \mathbb{R}_+^1$ units of coconuts where p_1^i and p_2^i denote respectively the amount of bond and money. In a bilateral meeting between a buyer with portfolio $a_t = (b_t, m_t)$ and a seller with portfolio $\breve{a}_t = (\breve{b}_t, \breve{m}_t)$, the terms of trade are determined by generalized Nash bargaining in which the buyer has bargaining power of $\eta \in (0,1)$ and the threat points are given by the continuation values.

Let $v_b(q) \equiv v(\varepsilon^l + q) - v(\varepsilon^l)$ and $v_s(q) \equiv v(\varepsilon^h) - v(\varepsilon^h - q)$. Then $(q^{bm},\, p^{bm})$ solves

$$\begin{aligned} \max \quad & \left[v_b(q^{bm}) + \beta W(a_t - p^{bm}, d_t) - \beta W(a_t, d_t)\right]^{\eta} \times \\ & \left[-v_s(q^{bm}) + \beta W(\breve{a}_t + p^{bm}, d_t) - \beta W(\breve{a}_t, d_t)\right]^{1-\eta} \end{aligned} \tag{7}$$

subject to $q^{bm} \in \mathbb{R}_+$ and $p^{bm} \leq \tilde{a}_t = (\tilde{b}_t, m_t) \leq a_t = (b_t, m_t)$ where $\tilde{a}_t = (\tilde{b}_t, m_t)$ is a buyer's portfolio carried to the decentralized market.[5] By using the linear property of W, we can simplify the bargaining problem as follows:

$$\max_{q^{bm} \in \mathbb{R}_+,\, p^{bm} \leq \tilde{a}} \left[v_b(q^{bm}) - \beta \hat{\phi}_t p^{bm}\right]^{\eta} \left[-v_s(q^{bm}) + \beta \hat{\phi}_t p^{bm}\right]^{1-\eta}. \tag{8}$$

Let $\omega_t \equiv \beta \hat{\phi}_t \tilde{a}_t$ denote the real balances of a buyer's portfolio brought to the bargaining

[5] As shown in Geromichalos et al. (2007), and Lagos and Rocheteau (2008), a buyer may be better off not bringing all the assets she owns to the bilateral meeting. However, given that money has value only as a medium of exchange, there is no reason to acquire money to carry it to the next centralized market. Hence, without loss of generality, we assume $\tilde{m} = m$.

table. Then, the solution to (8) is

$$q^{bm} = \begin{cases} \varepsilon A & \text{if } \omega_t \geq z(\varepsilon A) \\ z^{-1}(\omega_t) & \text{if } \omega_t < z(\varepsilon A) \end{cases}, \quad p^{bm} = \begin{cases} (\bar{b}_t, m_t) & \text{if } \omega_t \geq z(\varepsilon A) \\ (\tilde{b}_t, m_t) & \text{if } \omega_t < z(\varepsilon A) \end{cases}$$

where

$$z(q) = \left[\eta v_b'(q) v_s(q) + (1-\eta) v_s'(q) v_b(q)\right] / \left[\eta v_b'(q) + (1-\eta) v_s'(q)\right] \tag{9}$$

and $\bar{b}_t = \left[(z(\varepsilon A) - \beta \hat{\phi}_t^m m_t)/\beta \hat{\phi}_t^b\right]$.

Notice that $q^* \equiv \varepsilon A$ represents the efficient or first-best level of coconut consumption which equates the marginal utility of coconut consumption with its marginal disutility. If a buyer carries sufficiently large real balances of portfolio to the bargaining table so that $\omega_t \geq z(\varepsilon A)$, she gets εA units of coconuts in exchange for the real balances of $z(\varepsilon A)$. If $\omega_t < z(\varepsilon A)$, however, a buyer spends all the real balances in exchange for q units of coconuts which solves $z(q) = \omega_t$.

Similarly, (q^m, p^m) solves

$$\max_{q^m \in \mathbb{R}_+,\, p^m \leq (0,m)} \left[v_b(q^m) - \beta \hat{\phi}_t p^m\right]^{\eta} \left[-v_s(q^m) + \beta \hat{\phi}_t p^m\right]^{1-\eta}. \tag{10}$$

Let $\tilde{\omega}_t \equiv \beta \hat{\phi}_t^m m_t$ denote the real balances of a buyer's current money holdings. Then, the solution to (10) is

$$q^m = \begin{cases} \varepsilon A & \text{if } \tilde{\omega}_t \geq z(\varepsilon A) \\ z^{-1}(\tilde{\omega}_t) & \text{if } \tilde{\omega}_t < z(\varepsilon A) \end{cases}, \quad p^m = \begin{cases} (0, z(\varepsilon A)/\beta \hat{\phi}_t^m) & \text{if } \omega_t \geq z(\varepsilon A) \\ (0, m_t) & \text{if } \omega_t < z(\varepsilon A) \end{cases}$$

with $z(q)$ as defined in (9).

It is worth noting some key properties of the equilibrium terms of trade. First, in any equilibrium with $\eta \in (0,1)$, $z(\bar{q}) < z(\varepsilon A)$ where $\bar{q}$ is the quantity of coconuts that

maximizes the buyer's expected gain from a bilateral trade $\left[v(\varepsilon^l + q) - v(\varepsilon^l)\right] - z(q) = v_b(q) - z(q)$. It can be shown that, as q approaches εA from below, the buyer's expected gain from a bilateral trade, $v_b(q) - z(q)$, strictly decreases.[6] Noting that $v_b(q) - z(q)$ is maximized at $\bar{q}$, this implies $\bar{q} < \varepsilon A$. Further, $z'(q) > 0$ for all $q < \varepsilon A$ as in (11) implies $z(\bar{q}) < z(\varepsilon A)$:

$$z'(q) = \frac{\eta v_s'(v_b')^2 + (1-\eta)v_b'(v_s')^2 + \eta(1-\eta)(v_s''v_b' - v_b''v_s')(v_b - v_s)}{\left[\eta v_b' + (1-\eta)v_s'\right]^2} > 0. \tag{11}$$

Second, $q^m \leq q^{bm} \leq \bar{q} < \varepsilon A$ where $p^{bm} = (\tilde{b}, m)$ and $p^m = (0, m)$. The first inequality ($q^m \leq q^{bm}$) follows from $z'(q) > 0$ for $q < \varepsilon A$ and the strict inequality holds if $\tilde{b} > 0$. The second inequality ($q^{bm} \leq \bar{q}$) follows from the definition of $\bar{q}$. These inequalities also imply that sellers and buyers in the single-coincidence meetings are respectively willing to accept and transfer bond in the following sense:

$$\left[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})\right] \leq \beta\hat{\phi}_t(p^{bm} - p^m) \leq \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)\right]. \tag{12}$$

Notice that the first inequality in (12) represents a seller's participation constraint for accepting bond in a single-coincidence meeting, whereas the second inequality represents a buyer's participation constraint for transferring bond.

Now, the linearity of W and the bargaining solutions imply that the value function for a seller (type-h agent) entering the decentralized market with a degenerate portfolio $a_t = (b_t, m_t)$ satisfies

$$\begin{aligned} V_h(a_t, d_t) = {} & \frac{\theta_1}{2}\left[\sigma v_h^{bm} + (1-\sigma)v_h^m\right] + \frac{\theta_2}{2}\max_{\pi}\left\{\pi\left[\sigma v_h^{bm} + (1-\sigma)\hat{v}_h^{bm}\right] + (1-\pi)v_h^m\right\} \\ & + \frac{1}{2}v(\varepsilon^h) + \beta W(a_t, d_t). \end{aligned} \tag{13}$$

[6] As q approaches εA from below, the buyer's marginal gain from consuming additional unit of coconut decreases: $2\eta(1-\eta)\left\{\left[v''(\varepsilon^l + \varepsilon A)\right] / \left[v'(\varepsilon^l + \varepsilon A)\right]\right\}\left[2v(\varepsilon^l + \varepsilon A) - \left(v(\varepsilon^l) + v(\varepsilon^h)\right)\right] < 0$.

Here $\pi \in [0,1]$ denotes the probability with which a seller will accept the bond of unrecognized quality, $\sigma \in [0,1]$ denotes the fraction of buyers who bring the authentic bond, $v_h^{bm} = v(\varepsilon^h - q^{bm}) + \beta\hat{\phi}_t p^{bm}$, $v_h^m = v(\varepsilon^h - q^m) + \beta\hat{\phi}_t p^m$, and $\hat{v}_h^{bm} = v(\varepsilon^h - q^{bm}) + \beta\hat{\phi}_t \hat{p}^{bm}$ with $\hat{p}^{bm} = (0, p_2^{bm})$. The expected utility of a seller consists of the expected payoffs from the single-coincidence meetings where with probability θ_1 she recognizes the quality of bond and with probability θ_2 she does not recognize the quality of bond but accepts it with probability π or rejects it with probability $(1-\pi)$. In the meetings without single coincidence, a seller just consumes her coconut endowment ε^h.

The value function for a buyer (type-l agent) entering the decentralized market with the authentic bond satisfies

$$V_l^g(a_t, d_t) = \frac{\theta_1}{2} v_l^{bm} + \frac{\theta_2}{2}\left[\Pi v_l^{bm} + (1-\Pi) v_l^m\right] + \frac{1}{2} v(\varepsilon^l) + \beta W(a_t, d_t) \tag{14}$$

where $\Pi \in [0,1]$ denotes the buyer's belief that a seller will accept the bond of unrecognized quality, $v_l^{bm} = v(\varepsilon^l + q^{bm}) - \beta\hat{\phi}_t p^{bm}$, and $v_l^m = v(\varepsilon^l + q^m) - \beta\hat{\phi}_t p^m$. The expected utility of a buyer carrying the authentic bond consists of the expected payoffs from the single-coincidence meetings where with probability θ_1 the authentic bond is recognized by her trading partner and with probability θ_2 it is not recognized but accepted with probability Π or rejected with probability $(1-\Pi)$. In the meetings without single coincidence, she just consumes her coconut endowment ε^l. Similarly, the value function for a buyer entering the decentralized market with the counterfeit bond satisfies

$$V_l^f(a_t, d_t) = \frac{\theta_1}{2} v_l^m + \frac{\theta_2}{2}\left[\Pi \hat{v}_l^{bm} + (1-\Pi) v_l^m\right] + \frac{1}{2} v(\varepsilon^l) + \beta W(a_t, d_t) \tag{15}$$

where $\hat{v}_l^{bm} = v(\varepsilon^l + q^{bm}) - \beta\hat{\phi}_t \hat{p}^{bm}$. From (13), (14) and (15), the expected value of an agent entering the decentralized market with portfolio $a_t = (b_t, m_t)$, before knowing the

endowment shock, can be written as

$$V(a_t, d_t) = \frac{1}{2}\left[V_h(a_t, d_t) + \sigma V_l^g(a_t, d_t) + (1-\sigma)V_l^f(a_t, d_t)\right]. \tag{16}$$

An equilibrium is allocations $\{c_t, y_t, h_t, a_{t+1}\}_{t=0}^{\infty}$, a set of prices $\left\{\phi_t^g, \phi_t^b, \phi_t^m\right\}_{t=0}^{\infty}$, bilateral terms of trade $\{(q_t^i, p_t^i)_{i=bm,m}\}_{t=0}^{\infty}$, and the values of (σ, Π) such that

1. $\{c_t, y_t, h_t, a_{t+1}\}_{t=0}^{\infty}$ solve the individual maximization problem in the centralized market as summarized by (1) through (4) together with (5) and (6);

2. $\{(q_t^i, p_t^i)_{i=bm,m}\}_{t=0}^{\infty}$ are determined by generalized Nash bargaining as given by (8) and (10);

3. $\left\{\phi_t^g, \phi_t^b, \phi_t^m\right\}_{t=0}^{\infty}$ clear the centralized market: $c_t = d_t$, $b_t = 1$ and $m_t = \bar{M}$ for all $t \in \mathbb{Z}_+$; and

4. For the value functions $V_h(a_t, d_t), V_l^g(a_t, d_t), V_l^f(a_t, d_t)$ given by (13), (14) and (15) respectively, (σ, Π) satisfies (i) $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) > 0$ implies $\sigma = 1$, $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) < 0$ implies $\sigma = 0$, and $\sigma \in (0, 1)$ implies $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) = 0$; (ii) Given σ, $\pi = \Pi$ must solve the maximization problem in (13).

There are potentially 9 types of equilibria depending on $\sigma \in \{0, \Phi, 1\}$ and $\Pi(\pi) \in \{0, \Phi, 1\}$ where Φ denotes the open interval $(0, 1)$. Note that the agents with low coconut endowment (i.e., buyers) should choose whether to counterfeit bond $\sigma \in [0, 1]$, while the agents with high coconut endowment (i.e., sellers) should choose whether to accept the bond of unrecognized quality $\Pi(\pi) \in [0, 1]$. In doing so, they take as given the strategies of others. Here, $\pi = \Phi$ stands for the case in which a seller is indifferent between accepting and rejecting the bond of unrecognized quality, so that

she randomizes between them with an arbitrary probability.[7] Similarly, $\sigma = \Phi$ stands for the case in which a buyer is indifferent between carrying the counterfeit and the authentic bond so that $V_l^g(a_t, d_t) = V_l^f(a_t, d_t)$.

From (2), (3), (4), and (16), potential liquidity services of money and bond imply $(\phi_t^m/\phi_t^g) \geq \beta\mathbb{E}\left(\phi_{t+1}^m/\phi_{t+1}^g\right)$ and $\left(\phi_t^b/\phi_t^g\right) \geq \beta\mathbb{E}\left[(\phi_{t+1}^b + d_{t+1})/\phi_{t+1}^g\right]$ in any equilibrium. In order to focus on the role of recognizability in the liquidity differentials between money and bond, we restrict attention to equilibria with $(\phi_t^m/\phi_t^g) > \beta\mathbb{E}\left(\phi_{t+1}^m/\phi_{t+1}^g\right)$ and $\left(\phi_t^b/\phi_t^g\right) > \beta\mathbb{E}\left[(\phi_{t+1}^b + d_{t+1})/\phi_{t+1}^g\right]$ where bond as well as money provides liquidity service and hence $q^m < q^{bm}$ and $\tilde{b} > 0$.

Proposition 1 *There exist three types of equilibria where both money and bond are accepted as media of exchange:* $(\sigma, \Pi) = (1, 1)$, $(\sigma, \Pi) = (\Phi, 1)$, *and* $(\sigma, \Pi) = (\Phi, \Phi)$.

Proof. See Appendix. ■

There exists no equilibrium with $\Pi = 0$ where no bond of unrecognized quality is accepted. Intuitively, with $0 < \sigma < 1$, some sellers have to accept the bond of unrecognized quality in order for buyers to bring the counterfeit bond to the decentralized market. When $\sigma = 1$ so that no buyer carries the counterfeit bond, rejecting any bond of unrecognized quality would not be a best response. Similarly, with $\Pi = 0$, $\sigma = 0$ would not be optimal because the authentic bond is still accepted as long as a seller can recognize the quality of bond with a positive probability $\theta_1 > 0$. However, if $\theta_1 = 0$, there exists an $(\sigma, \Pi) = (0, 0)$ equilibrium where only money is accepted as a medium of exchange in a bilateral trade.

[7] In the model economy with a continuum of agents, the probability of accepting the unrecognized bond by the average seller can be interpreted as the proportion of sellers accepting the unrecognized bond.

4 Liquidity of Money and Bond

In the first type of equilibrium where $(\sigma, \Pi) = (1, 1)$, money and bond are equally liquid in the sense that there is no counterfeit bond and both money and bond are equally acceptable even when the quality of bond is not recognized. On the other hand, the other two types of equilibria where $(\sigma, \Pi) = (\Phi, 1)$ and $(\sigma, \Pi) = (\Phi, \Phi)$ imply the differential liquidity between money and bond.

This is a generalization of the recent studies on the implications of liquidity for asset prices and monetary policy where either equal or differential liquidity is exogenously assumed. For instance, assuming that both money and real assets (e.g., capital or equity) in fixed supply are equally liquid, Geromichalos et al. (2007), and Lagos and Rocheteau (2008) show that money is essential as long as the supply of real asset is sufficiently low. Motivated by legal or other institutional restrictions, Lagos (2008) assumes the liquidity differentials between bond and stock to examine the liquidity implications for asset prices.

An exception is Lester et al. (2008) in which the acceptability of real assets is determined endogenously by the acquisition of costly verification technology. However, they only consider the pure strategy of rejecting the real assets of unrecognized quality, which can be regarded as a limiting case of the $(\sigma, \Pi) = (\Phi, \Phi)$ equilibrium.

4.1 Equal Liquidity

In the equilibrium with $(\sigma, \Pi) = (1, 1)$, $\sigma = 1$ requires $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) > 0$ for $\Pi(\pi) = 1$. From (14) and (15), this condition implies the following restriction on the recognizability of bond:

$$\theta_1 > \left\{ \beta \hat{\phi}_t (p^{bm} - p^m) / \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right] \right\} \equiv \bar{\theta}_{1,1}. \tag{17}$$

Further, from (13) with $\sigma = 1$, $\pi = 1$ would be a best response if $[v(\varepsilon^h - q^{bm}) + \beta\hat{\phi}_t p^{bm}] > [v(\varepsilon^h - q^m) + \beta\hat{\phi}_t p^m]$. This inequality and (17) with $\theta_1 < 1$ imply $[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] < \beta\hat{\phi}_t(p^{bm} - p^m) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$, which is consistent with the participation constraints in (12) with $q^m < q^{bm}$ ($\tilde{b} > 0$).

The value function (16) for a representative agent can be written as

$$V(a_t, d_t) = \frac{1}{4}\left[v(\varepsilon^l + q^{bm}) + v(\varepsilon^h - q^{bm}) + v(\varepsilon^l) + v(\varepsilon^h)\right] + \beta W(a_t, d_t). \tag{18}$$

Then (2), (3), (4) and (18) imply that asset prices satisfy

$$u'(d_t)\phi_t^b = \beta\mathbb{E}\left[u'(d_{t+1})\mathcal{L}_{1,t+1}(\phi_{t+1}^b + d_{t+1})\right]$$

$$u'(d_t)\phi_t^m = \beta\mathbb{E}\left[u'(d_{t+1})\mathcal{L}_{1,t+1}\phi_{t+1}^m\right]$$

where

$$\mathcal{L}_{1,t+1} \equiv 1 + \left(\frac{1}{4}\right)\frac{v'(\varepsilon^l + q_{t+1}^{bm}) - v'(\varepsilon^h - q_{t+1}^{bm})}{z'(q_{t+1}^{bm})} \tag{19}$$

with $z'(q)$ as defined in (11). Notice that the factor $\mathcal{L}_{1,t+1}$ can be interpreted as the liquidity returns from holding money or bond in the equilibrium where all the buyers bring the authentic bond ($\sigma = 1$) and sellers always accept the bond of unrecognized quality ($\Pi = 1$). That is, as in Lagos (2008), $\mathcal{L}_{1,t+1}$ represents the liquidity of money or bond in terms of the expected marginal benefit that the buyers obtain from bringing the value of an additional apple's worth of money or bond to the decentralized market.

Not surprisingly, there is no difference in the liquidity between money and bond as long as the recognizability problem with bond is not so severe in the sense that the quality of bond is recognized with a high probability given by (17). It implies that money would no longer be valued as liquidity if the ample supply of real asset (e.g.,

trees) provides sufficient liquidity service. This is consistent with Geromichalos et al. (2007), and Lagos and Rocheteau (2008) in which money is not essential when money and real asset are equally liquid and the fixed supply of real asset is sufficiently large.

4.2 Differential Liquidity

In the $(\sigma, \Pi) = (\Phi, 1)$ equilibrium, some buyers carry the counterfeit bond to the decentralized market, but sellers always accept bond even when they cannot recognize the quality of bond. First, $\sigma \in (0,1)$ requires $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) = 0$ for $\pi(\Pi) = 1$. From (14) and (15), this condition implies the following restriction on θ_1:

$$\theta_1 = \left\{ \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}) / \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right] \right\} \equiv \bar{\theta}_{1,2}. \tag{20}$$

Further, from (13) with a given $\sigma \in (0,1)$, $\Pi(\pi) = 1$ would be a best response if $\left[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm}) \right] < \sigma \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm})$. This inequality and (20) with $\theta_1 < 1$ imply $\left[\left(v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm}) \right) / \sigma \right] < \beta \hat{\phi}_t (p^{bm} - p^m) < \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right]$, which is consistent with the participation constraints in (12) with $q^m < q^{bm}$ $(\tilde{b} > 0)$.

The value function (16) for a representative agent can be written as

$$\begin{aligned} V(a_t, d_t) = & \frac{(\theta_1 \sigma + \theta_2)}{4} \left[v(\varepsilon^l + q^{bm}) + v(\varepsilon^h - q^{bm}) \right] + \frac{\theta_1 (1 - \sigma)}{4} \left[v(\varepsilon^l + q^m) + v(\varepsilon^h - q^m) \right] \\ & + \frac{1}{4} \left[v(\varepsilon^l) + v(\varepsilon^h) \right] + \beta W(a_t, d_t). \end{aligned} \tag{21}$$

Then (21) with (2), (3), and (4) imply that asset prices satisfy the followings:

$$u'(d_t) \phi_t^b = \beta \mathbb{E} \left[u'(d_{t+1}) \mathcal{L}_{2,t+1}^b (\phi_{t+1}^b + d_{t+1}) \right]$$

$$u'(d_t) \phi_t^m = \beta \mathbb{E} \left[u'(d_{t+1}) \mathcal{L}_{2,t+1}^m \phi_{t+1}^m \right]$$

where

$$\mathcal{L}^b_{2,t+1} = 1 + \left(\frac{1}{4}\right) \frac{(\theta_1\sigma + \theta_2)\left[v'(\varepsilon^l + q^{bm}_{t+1}) - v'(\varepsilon^h - q^{bm}_{t+1})\right]}{z'(q^{bm}_{t+1})} \tag{22}$$

$$\mathcal{L}^m_{2,t+1} = \mathcal{L}^b_{2,t+1} + \left(\frac{1}{4}\right) \frac{\theta_1(1-\sigma)\left[v'(\varepsilon^l + q^m_{t+1}) - v'(\varepsilon^h - q^m_{t+1})\right]}{z'(q^m_{t+1})} \tag{23}$$

with $z'(q)$ defined in (11).

Now, the factors $\mathcal{L}^b_{2,t+1}$ and $\mathcal{L}^m_{2,t+1}$ denote respectively the liquidity returns from holding bond and money in the equilibrium where some buyers bring the counterfeit bond to the decentralized market ($\sigma = \Phi$), but sellers still accept the bond of unrecognized quality ($\Pi = 1$). Notice that $1 < \mathcal{L}^b_{2,t+1} < \mathcal{L}^m_{2,t+1}$ where the second inequality comes from the additional liquidity service provided by money when the counterfeit bond is recognized with probability θ_1.

This also implies that, in the presence of the recognizability problem with bond, the coexistence of money and bond requires a higher full (liquidity augmented) return for bond than money, implying a positive liquidity premium. More specifically, since $b_{t+1} > 0$ and $m_{t+1} > 0$, (3) and (4) should hold with equality. Simple manipulations of these equalities then imply

$$\mathbb{E}\phi^m_{t+1}/\phi^m_t = \left(\mathcal{L}^b_{2,t+1}/\mathcal{L}^m_{2,t+1}\right)\left[\mathbb{E}(\phi^b_{t+1} + d_{t+1})/\phi^b_t\right].$$

Since $\mathcal{L}^b_{2,t+1} < \mathcal{L}^m_{2,t+1}$, $\left(\mathbb{E}\phi^m_{t+1}/\phi^m_t\right) < \left[\mathbb{E}(\phi^b_{t+1} + d_{t+1})/\phi^b_t\right]$ of which the difference represents the liquidity premium. Notice that, unlike the $(\sigma, \Pi) = (1, 1)$ equilibrium with equal liquidity, money is essential in the $(\Phi, 1)$ equilibrium due to its additional liquidity service in the presence of the recognizability problem with bond.

When $(\sigma, \Pi) = (\Phi, \Phi)$, some buyers carry the counterfeit bond to the decentralized market and some sellers accept the bond of unrecognized quality. Now, $\sigma \in (0, 1)$

requires $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) = 0$ for a given $\pi(\Pi) \in (0, 1)$. From (14) and (15), this condition implies the following equilibrium value for $\pi \in (0, 1)$:

$$\pi = \frac{\theta_1 \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) - \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}) \right]}{(1 - \theta_1) \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm})}. \tag{24}$$

Then $\pi < 1$ implies the restriction on θ_1 as follows:

$$\theta_1 < \left\{ \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}) / \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right] \right\} \equiv \bar{\theta}_{1,3}. \tag{25}$$

Further, for a given $\sigma \in (0, 1)$, any $\pi \in (0, 1)$ would be a best response if a seller is indifferent between accepting and rejecting the unrecognized bond. This requires from (13) that $\sigma[v(\varepsilon^h - q^{bm}) + \beta \hat{\phi}_t p^{bm}] + (1 - \sigma)[v(\varepsilon^h - q^{bm}) + \beta \hat{\phi}_t \hat{p}^{bm}] = v(\varepsilon^h - q^m) + \beta \hat{\phi}_t p^m$, which can be rewritten as

$$\sigma = \left[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm}) \right] / \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}). \tag{26}$$

Because $\theta_1 < 1$ and $\sigma < 1$, (25) and (26) imply $[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})] < \beta \hat{\phi}_t (p^{bm} - p^m) < [v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)]$, which is consistent with the participation constraints in (12) with $q^m < q^{bm}$ ($\tilde{b} > 0$).[8]

The value function (16) for a representative agent can be written as

$$\begin{aligned} V(a_t, d_t) \;=\; & \frac{(\theta_1 \sigma + \theta_2 \pi)}{4} \left[v(\varepsilon^l + q^{bm}) + v(\varepsilon^h - q^{bm}) \right] \\ & + \frac{[\theta_1 (1 - \sigma) + \theta_2 (1 - \pi)]}{4} \left[v(\varepsilon^l + q^m) + v(\varepsilon^h - q^m) \right] \\ & + \frac{1}{4} \left[v(\varepsilon^l) + v(\varepsilon^h) \right] + \beta W(a_t, d_t). \end{aligned} \tag{27}$$

[8] Strictly speaking, (25) with $\theta_1 < 1$ implies $\beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}) \leq \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right]$. But (24) with $\pi \in (0, 1)$ rules out the case of $\left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right] = \beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm})$. Hence, we have $\beta \hat{\phi}_t (p^{bm} - \hat{p}^{bm}) < \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m) \right]$.

Now, (27) with (2), (3) and (4) imply the following equilibrium asset prices:

$$u'(d_t)\phi_t^b = \beta\mathbb{E}\left[u'(d_{t+1})\mathcal{L}_{3,t+1}^b(\phi_{t+1}^b + d_{t+1})\right]$$

$$u'(d_t)\phi_t^m = \beta\mathbb{E}\left[u'(d_{t+1})\mathcal{L}_{3,t+1}^m\phi_{t+1}^m\right]$$

where

$$\mathcal{L}_{3,t+1}^b \equiv 1 + \left(\frac{1}{4}\right)\frac{(\theta_1\sigma + \theta_2\pi)\left[v'(\varepsilon^l + q_{t+1}^{bm}) - v'(\varepsilon^h - q_{t+1}^{bm})\right]}{z'(q_{t+1}^{bm})} \tag{28}$$

$$\mathcal{L}_{3,t+1}^m \equiv \mathcal{L}_{3,t+1}^b + \left(\frac{1}{4}\right)\frac{[\theta_1(1-\sigma) + \theta_2(1-\pi)]\left[v'(\varepsilon^l + q_{t+1}^m) - v'(\varepsilon^h - q_{t+1}^m)\right]}{z'(q_{t+1}^m)} \tag{29}$$

with $z'(q)$ as defined in (11).

Notice that the liquidity factors $\mathcal{L}_{3,t+1}^b$ and $\mathcal{L}_{3,t+1}^m$ represent respectively the liquidity returns from holding bond and money in the equilibrium where some buyers bring the counterfeit bond to the decentralized market ($\sigma = \Phi$) and sellers randomize between accepting and rejecting the bond of unrecognized quality ($\Pi = \Phi$). These have the property of $1 < \mathcal{L}_{3,t+1}^b < \mathcal{L}_{3,t+1}^m$ with the same qualitative interpretations as in the $(\sigma, \Pi) = (\Phi, 1)$ equilibrium including a positive liquidity premium, $\left(\mathbb{E}\phi_{t+1}^m/\phi_t^m\right) < \left[\mathbb{E}(\phi_{t+1}^b + d_{t+1})/\phi_t^b\right]$.

The difference is that money provides the additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond. This also makes money essential as in the $(\sigma, \Pi) = (\Phi, 1)$ equilibrium. Finally, it can be verified that when $\Pi = 1$ the above liquidity factors, $\mathcal{L}_{3,t+1}^b$ and $\mathcal{L}_{3,t+1}^m$, are reduced respectively to $\mathcal{L}_{2,t+1}^b$ and $\mathcal{L}_{2,t+1}^m$, while they are reduced to $\mathcal{L}_{1,t+1}^b = \mathcal{L}_{1,t+1}^m = \mathcal{L}_{1,t+1}$ when $\sigma = \Pi = 1$.

4.3 Recognizability and Liquidity

The equilibrium restrictions on the recognizability parameter θ_1 as given by (17), (20), and (25) imply that the equal liquidity equilibrium with $(\sigma, \Pi) = (1, 1)$ exists for $\theta_1 > \bar{\theta}_{1,1}$, while the differential liquidity equilibrium with $(\sigma, \Pi) = (\Phi, 1)$ exists for $\theta_1 = \bar{\theta}_{1,2}$ and the one with $(\sigma, \Pi) = (\Phi, \Phi)$ exists for $\theta_1 < \bar{\theta}_{1,3}$. More importantly, the equilibrium relationship between the recognizability parameter θ_1 and the liquidity differentials is obtained from the following result:

Proposition 2 *$\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$ as long as buyers have sufficient bargaining power so that $z''(q) > 0$ for all $q \in (0, \varepsilon A)$. This implies that (i) if $\theta_1 \geq \bar{\theta}_{1,3}$, only the equal liquidity equilibrium with $(\sigma, \Pi) = (1, 1)$ exists; (ii) if $\theta_1 \leq \bar{\theta}_{1.1}$, only the differential liquidity equilibrium with $(\sigma, \Pi) = (\Phi, \Phi)$ exists; (iii) if $\theta_1 \in (\bar{\theta}_{1,1}, \bar{\theta}_{1,3})$, at least both $(1, 1)$ and (Φ, Φ) equilibria exist; and (iv) if $\theta_1 = \bar{\theta}_{1,2}$, all the three types of equilibria exist.*

Proof. See Appendix. ■

That is, as the recognizability problem becomes more severe, the liquidity differentials between money and bond tend to increase. When the recognizability problem is not so severe, there only exists an equilibrium with equally liquid money and bond. As it becomes more difficult to recognize the quality of bond, there also arises an equilibrium where money commands higher liquidity than bond. Finally, when the recognizability problem is sufficiently severe, there only exists an equilibrium where money commands higher liquidity than bond.

This is somewhat analogous to Lester et al. (2008). When the recognizability of real assets requires the acquisition of costly verification technology which varies with individual agents, they show that if there are some agents with sufficiently high costs

of verification, both money and real assets circulate as media of exchange despite their liquidity differentials.

5 Concluding Remarks

In order to understand the reasons for the liquidity differentials between assets, we have extended Lagos' (2008) search model of exchange and asset prices by explicitly incorporating the recognizability of asset. Following Jevons (1875), King and Plosser (1986), Williamson and Wright (1994), and Banerjee and Maskin (1996), we assume that, unlike money, bond is not universally recognizable. The two types of the single-coincidence meetings arise in the decentralized search market–one where both money and bond are accepted and the other where only money is accepted as a means of payment–depending on a seller's strategy of accepting or rejecting the bond of unrecognized quality and a buyer's strategy of carrying the counterfeit bond.

We have shown that the liquidity differentials between money and bond tend to increase with the recognizability problem of bond. When the recognizability of bond is not so serious, there only exists an equilibrium where all the buyers bring the authentic bond to the decentralized market and sellers always accept the bond of unrecognized quality, and hence money and bond become equally liquid. As the recognizability problem becomes more severe, there also arises an equilibrium where some buyers bring the counterfeit bond, but sellers either accept or randomize between accepting and rejecting the bond of unrecognized quality. Money commands higher liquidity by providing the additional liquidity service when sellers reject the bond of unrecognized quality as well as when they recognize the counterfeit bond. The coexistence of money and bond then requires a higher liquidity-augmented return for bond, implying a positive liquidity premium.

Finally, in the presence of an interest-bearing asset (e.g., bond), the essentiality of money is followed in an environment where the fixed supply of bond is assumed to be sufficiently small. A future research is to account for the coexistence puzzle by allowing for the endogenous supply of bond as an interest-bearing asset.

6 Appendix

Proof of Proposition 1:

For the combinations of σ and $\Pi(\pi)$, which are marked as "N" in Table 1, there exists no equilibrium where both money and bond are accepted as media of exchange.

Table 1: Candidate Equilibria

	$\sigma = 0$	$\sigma = \Phi$	$\sigma = 1$
$\Pi = 0$	N	N	N
$\Pi = \Phi$	N	Differential Liquidity	N
$\Pi = 1$	N	Differential Liquidity	Equal Liquidity

(1) $\sigma = 0$ and $\Pi(\pi) = 0$: $\sigma = 0$ requires $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) < 0$ for a given $\pi(\Pi) = 0$. From (14) and (15), this condition can be written as $\beta\hat{\phi}_t(p^{bm} - p^m) > \left[v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)\right]$. It implies $q^{bm} > \bar{q}$, and hence $\tilde{b} = 0$.

(2) $\sigma = 1$ and $\Pi(\pi) = 0$: From (13) with $\sigma = 1$, $\pi(\Pi) = 0$ would be a best response if $\left[v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm})\right] > \beta\hat{\phi}_t(p^{bm} - p^m)$. This violates the seller's participation constraint for accepting bond, and hence $\tilde{b} = 0$.

(3) $\sigma = \Phi$ and $\Pi(\pi) = 0$: $\sigma \in (0, 1)$ requires $V_l^g(a_t, d_t) - V_l^f(a_t, d_t) = 0$ for a given $\pi(\Pi) = 0$. From (14) and (15), this condition can be written as $v(\varepsilon^l + q^{bm}) - \beta\hat{\phi}_t p^{bm} = v(\varepsilon^l + q^m) - \beta\hat{\phi}_t p^m$. This equality holds if and only if $q^{bm} = q^m$ and $p^{bm} = p^m$ with $\tilde{b} = 0$.

(4) $\sigma = 0$ and $\Pi(\pi) = \Phi$: From (13) with $\sigma = 0$, $\pi(\Pi) = \Phi$ would be a best response if $v(\varepsilon^h - q^{bm}) + \beta\hat{\phi}_t\hat{p}^{bm} = v(\varepsilon^h - q^m) + \beta\hat{\phi}_t p^m$. Again this equality holds if and only if $q^{bm} = q^m$ and $p^{bm} = p^m = \hat{p}^{bm}$ with $\tilde{b} = 0$.

(5) $\sigma = 1$ and $\Pi(\pi) = \Phi$: From (13) with $\sigma = 1$, $\pi(\Pi) = \Phi$ would be a best response if $v(\varepsilon^h - q^{bm}) + \beta\hat{\phi}_t p^{bm} = v(\varepsilon^h - q^m) + \beta\hat{\phi}_t p^m$. The exactly same argument in (3) gives the result of $\tilde{b} = 0$.

(6) $\sigma = 0$ and $\Pi(\pi) = 1$: From (13) with $\sigma = 0$, $\pi(\Pi) = 1$ would be a best response if $v(\varepsilon^h - q^m) - v(\varepsilon^h - q^{bm}) < \beta\hat{\phi}_t(\hat{p}^{bm} - p^m)$. This cannot hold because $v(\varepsilon^h - q^m) > v(\varepsilon^h - q^{bm})$ with $\tilde{b} > 0$ but $\hat{p}^{bm} = p^m$.

Proof of Proposition 2:

It suffices to show that $\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$. The proof consists of several claims.

Claim 1 *For a given $q^{bm} > q^m$, $\bar{\theta}_1$ is strictly increasing in q^m.*

Proof. Since $v(\cdot)$ is strictly concave and $z''(q) > 0$ due to the assumption of $\eta \approx 1$, $z'(q)/v'(\varepsilon^l + q)$ is strictly increasing in q. Then, for a given $q^{bm} > q^m$,

$$\frac{z'(q^m)}{v'(\varepsilon^l + q^m)} < \frac{z(q^{bm}) - z(q^m)}{v(\varepsilon^l + q^{bm}) - v(\varepsilon^l + q^m)}$$

which gives the result. ■

Claim 2 *$q_1^m < q_2^m < q_3^m$ where the subscript 1,2, and 3 refer respectively to the equilibrium with $(\sigma, \Pi) = (1, 1)$, $(\sigma, \Pi) = (\Phi, 1)$, and $(\sigma, \Pi) = (\Phi, \Phi)$.*

Proof. In the equilibrium with $(\sigma, \Pi) = (\Phi, 1)$, the first-order condition with respect to m_{t+1} can be written as

$$\beta\hat{\phi}_{t+1}^m + \frac{\theta_1(1-\sigma)}{4}\xi(q_2^m)\beta\hat{\phi}_{t+1}^m = \hat{\phi}_t^m \tag{30}$$

where $\xi(q) \equiv \left[v'(\varepsilon^l + q) - v'(\varepsilon^h - q)\right](1/z'(q))$ denotes the value of an additional unit of money in a pairwise meeting. With the fixed supply of money, $\hat{\phi}^m_{t+1} = \hat{\phi}^m_t$ in equilibrium. Therefore, (30) can be rearranged as

$$\frac{\theta_1(1-\sigma)}{4}\xi(q_2^m) = \frac{1}{\beta} - 1. \tag{31}$$

The corresponding first-order condition in the equilibrium with $(\sigma, \Pi) = (\Phi, \Phi)$ is

$$\frac{[\theta_1(1-\sigma) + \theta_2(1-\pi)]}{4}\xi(q_3^m) = \frac{1}{\beta} - 1. \tag{32}$$

Noting that $z''(q) > 0$ implies $\xi'(q) < 0$, the left-hand side of (31) and (32) is strictly decreasing, and $\xi(0)$ is large enough because $v'(\varepsilon^l)$ is sufficiently large. Hence, there exists a unique solution of q_2^m and q_3^m which satisfy (31) and (32), respectively. Now, $\theta_1(1-\sigma) < [\theta_1(1-\sigma) + \theta_2(1-\pi)]$ implies $q_2^m < q_3^m$. Finally, in the equilibrium with $(\sigma, \Pi) = (1, 1)$, the smaller value of money holding brought to the decentralized market than in the $(\sigma, \Pi) = (\Phi, 1)$ equilibrium implies $q_1^m < q_2^m < q_3^m$. ■

Claim 3 *There exists a unique q^{bm} such that $q_1^{bm} = q_2^{bm} = q_3^{bm}$.*

Proof. Notice that q^{bm} is a function of real balances $\kappa_{t+1} = \tilde{\phi}^m_{t+1} m_{t+1} + (\tilde{\phi}^b_{t+1} + \tilde{d}_{t+1}) b_{t+1}$ where $\tilde{\phi}^m = \phi^m/\phi^g$, $\tilde{\phi}^b = \phi^b/\phi^g$, and $\tilde{d} = d/\phi^g$. When both money and bond are accepted, the portfolio choice problem becomes

$$\max_{(m_{t+1},\, b_{t+1})} \left\{ \begin{array}{c} -\tilde{\phi}^m_t m_{t+1} - \tilde{\phi}^b_t b_{t+1} + \frac{1}{4}[v(\varepsilon^l + q^{bm}) + v(\varepsilon^h - q^{bm}) + \\ v(\varepsilon^l) + v(\varepsilon^h)] + \beta\tilde{\phi}^m_{t+1} m_{t+1} + \beta(\tilde{\phi}^b_{t+1} + \tilde{d}_{t+1}) b_{t+1} \end{array} \right\}. \tag{33}$$

The first-order conditions with respect to m_{t+1} and b_{t+1} imply respectively $\tilde{\phi}^m_t = \left[\frac{1}{4}\xi(q^{bm}) + 1\right]\beta\tilde{\phi}^m_{t+1}$ and $\tilde{\phi}^b_t = [\frac{1}{4}\xi(q^{bm}) + 1]\beta(\tilde{\phi}^b_{t+1} + \tilde{d}_{t+1})$, which can be combined to

have $\tilde{\phi}_t^b/(\tilde{\phi}_{t+1}^b + \tilde{d}_{t+1}) = \tilde{\phi}_t^m/\tilde{\phi}_{t+1}^m$. Then, substituting $\tilde{\phi}_t^m = \tilde{\phi}_t^b\tilde{\phi}_{t+1}^m/(\tilde{\phi}_{t+1}^b + \tilde{d}_{t+1})$ and $\tilde{\phi}_t^b = \tilde{\phi}_t^m(\tilde{\phi}_{t+1}^b + \tilde{d}_{t+1})/\tilde{\phi}_{t+1}^m$ into (33), we can express (33) as the choice problem of real balances:

$$\max_{\kappa_{t+1}} \left\{ -\kappa_{t+1} + \frac{1}{4}\left[v(\varepsilon^l + q^{bm}(\kappa_{t+1})) + v(\varepsilon^h - q^{bm}(\kappa_{t+1})) + v(\varepsilon^l) + v(\varepsilon^h)\right] + \beta\kappa_{t+1} \right\}.$$

The first-order condition is $\xi[q^{bm}(\kappa_{t+1})] = 4\beta^{-1}(1-\beta)$ in which the left hand side is strictly decreasing due to $\xi'(q) < 0$ implied by $z''(q) > 0$ and $\xi(0)$ is large enough by the assumption on $v'(\varepsilon^l)$. Hence, there is a unique solution for κ_{t+1} which then pins down a unique q^{bm}. Since this is true in all the three types of equilibria, $q_1^{bm} = q_2^{bm} = q_3^{bm}$. ■

Claim 4 $\bar{\theta}_{1,1} < \bar{\theta}_{1,2} < \bar{\theta}_{1,3}$.

Proof. Obvious consequence of the claim 1, 2 and 3. ■

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