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# Unobserved Investment, Signaling, and Welfare

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# Unobserved Investment, Signaling, and Welfare*

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**Abstract**

We consider a model in which each worker selects a public signal following a private investment on his quality type. Signaling then contributes to social welfare through its influence on the quality choice. We offer a rationale for the argument that there are too many high-type workers in separating equilibrium and the inefficiency can be reduced in pooling equilibrium. On the other hand, pooling equilibrium can generate too few high-type workers and the inefficiency is reduced in separating equilibrium.

**Keywords and Phrases**: Investment, Endogenous quality, Signaling, Welfare
**JEL Classification Numbers**: **D63, I21, J24**

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# 1 Introduction

We consider a competitive market in which each seller makes a private investment to determine its product quality. Combined with the endogenous quality choice, the adverse selection problem then leads to an inefficient outcome; given a single price, no seller has an incentive to make the investment. But the inefficiency can be reduced if sellers' types can be revealed through signaling, especially with no ex-post bargaining for the non-contractible investment. In separating equilibrium, different investment choices can be treated differently in the market.

While signaling is a natural solution to the market inefficiency with the dynamic adverse selection, the literature has neglected the "inefficiency" of signaling. As shown, separating equilibrium is always better than no signal, unlike a typical signaling model with no endogenous quality choice. The question, however, is whether we can order different separating and pooling equilibria in terms of social welfare, and more importantly, whether a socially optimal investment is achieved in separating or pooling equilibrium. This question becomes valid only by observing the new role of pooling equilibrium in the dynamics. The standard negative correlation between quality and signaling cost implies that even a pooling equilibrium can reduce the inefficiency, based on the investment return, not from quality revelation but from signaling-cost saving. Furthermore, the single-crossing implies that as the pooling's signal level increases, high-quality type workers can save a larger signaling cost, a greater incentive to make the investment in the pooling.

In this paper, we address the questions using a labor-market model in which each worker endogenously selects his own quality-type through a private investment and then chooses a public signal in the market.[1] In the model, the worker decides whether to make the investment and become high type, or to remain as low type, by comparing the future benefit through signaling with the investment costs that are drawn from a distribution of

[1] We adopt a fairly standard labor-market model that can be broadly extended for the setting in which unobserved attribute or quality is endogenously selected by sellers and signal conveys information about the sellers' endogenous choice. For example, a firm may undertake a private investment and select the quality of its product before it chooses its price or advertising level as a public signaling, or a firm may make a private investment and position its underlying value in the market before it reveals its capital structure as a public signaling, or a country can make a private investment and arrange a domestic objective of setting an environmental standard before it reveals domestic policies to its trading partners.

workers' inborn cost types[2]. An equilibrium consists of a proportion of workers who make the investment to be high type, referred to as the investment ratio, and a signaling form, either separating or pooling. We raise the question: from a welfare perspective, can we rationally say that there are too many high-type workers when the workers' selection of types is endogenously made in their interests? In separating equilibrium, some workers choose to become high type for their own benefits while causing no welfare loss to the remaining workers. Thus, with the separating equilibrium concept alone, it is impossible to argue that there are too many high-type workers, even when most of workers make the costly investment to be treated differently from a very small fraction of the remaining workers.

We show, however, that there exist circumstances under which separating equilibrium generates too many high-type workers. The use of pooling equilibrium is essential for the finding due to the accompanying feature: affecting all workers, the aforementioned role of pooling signal entails a *trade-off* between the generation of high-type workers and the signaling costs of low-type workers. To complete the proof, it is necessary to find a way to make connection between separating and pooling equilibrium in the dynamic setting. In particular, we identify circumstances under which the following statements are valid: (i) there exists a pooling equilibrium that approximates the "best" separating equilibrium in terms of the investment ratio and social welfare; (ii) this pooling equilibrium has overinvestment; there exists an optimal pooling equilibrium that reduces the inefficiency of overinvestment.

Our finding is presented with a strong result; it is a single condition that makes those two, by nature, independent statements, such a pooling's existence and its welfare dominance, hold. The condition is that pooling signal reaches a "saturation point" such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. As such, the tension observed in pooling signal implies that it is socially

[2]To focus on the effect of information transmission on the private investment, any direct effect of signal on investment is controlled in this model. Moreover, it fits better for some applications; e.g. advertisement or price itself has no direct effect on product quality. Some signaling models allow that an increase in education level directly promotes productivity (see Weiss (1983), Noldeke and Van Damme (1990), Swinkels (1999) and Kremer and Skrzypacz (2007) among others). It is commonly assumed in previous models that the distribution of workersítypes is exogenously determined and that productivity-enhancing actions are publicly observable; thus, previous models disregard our central concern.

preferred to reduce signaling costs than to increase high-type workers. In this case, separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal and treat high-type workers differently in the market. The inefficiency of overinvestment can then be reduced in pooling equilibrium where workers use the same signal without having to be treated differently. On the other hand, there also exist circumstances under which pooling signal remains sufficiently effective in generating the investment ratio further. In this case, pooling equilibrium generates too few high-type workers, relative to separating, and the inefficiency of underinvestment can be reduced in separating equilibrium. Interestingly, the pooling's ineffectiveness serves for the pooling's dominance; its effectiveness for the separating's dominance.

In practice, it is commonly argued that there are too many college graduates typically based on limited job openings. It is, however, difficult to support the argument perhaps for two main reasons. First, despite limited job openings, high school graduates may choose to go to college for their own benefits. Indeed, there exists a significant wage gap between college-educated and high-school-only workers in real data.[3] Second, a fundamental question of whether and how the signal (college degree) contributes to human capital is rarely discussed or answered in the argument.[4] In regard to the specific issue, our model broadly indicates that, despite the significant wage gap, if the capacity for education to increase the aggregate human capital reaches a saturation point over the education level between high school and college, then it becomes reasonable to argue that signaling costs of college degree are too high, and there are too many college graduates, from a welfare perspective. The same argument can be applied to welfare analysis of other signaling applications.

[3]The college wage premium substantially increased between 1980 and 2005 in the US, and it has been studied by a vast body of literature (see, for example, Taber (2001), Fang (2006), Goldin and Katz (2007a, 2007b), Walker and Zhu (2008) and Cunha, Karahan, and Soares (2011) among many others).

[4]Since the classical papers of Spence (1973, 1974), the "information-conveying" aspect of signaling has produced a large body of literature (see Kreps and Sobel (1994) and Riley (2001) for literature survey). The information-conveying aspect of education has also been empirically tested (Wolpin (1977), Riley (1979), Lang and Kropp (1986), Tyler, Murnane and Willett (2000), Bedard (2001)). For example, using a unique data set containing the General Educational Development (GED) test scores, Tyler, Murnane and Willett (2000) identify the signaling value of the GED, net of human capital effects. They observe that there are substantial signaling effects for young white dropouts, estimated at about 20% earnings gain after 5 years.

Our model is related to a few existing models. Fang (2001) contains an investment stage before workers select signaling, and highlights an economic role of "social culture" by showing that there exists a separating equilibrium in which the seemingly irrelevant activity, social culture, becomes an endogenous signaling instrument for the workers who invested in skills. In his model, however, there is no overinvestment, since pooling equilibrium is inferior to separating equilibrium in which workers make the investment to be treated differently. Using a signaling setting in which the market (receiver) observes an informative grade in addition to the regular signal, Daley and Green (2014) show that some degree of pooling emerges in equilibria, and that if the market's prior belief that the sender is high type approaches one, then the equilibrium converges to the complete-information outcome, pooling with no costly signaling. In their discussion of the possibility that there is an ex ante privately-observed investment, they predict that the investment remains inefficiently low even in the presence of informative grades given that it takes additional resources to be treated differently as high type. In our model, the receiver's belief is endogenously supported only if signaling is large enough to support the belief. We find that there are important welfare implications that have been ignored by the existing information-conveying argument, showing that signaling may overly generate high-type workers.

Recent papers by Hermalin (2013) and Kawai (2014) consider a situation in which an investment in an asset made by a seller endogenously determines the value of the asset, and a potential buyer cannot observe the seller's investment decision made prior to trade. In those models, there is a key trade-off between the provision of ex ante incentive for investment and the achievement of ex post efficiency in trade: if trade is sure to happen, then the seller has no incentive to invest ex ante, and if no trade is anticipated, then the seller has incentive to invest for her own benefit. In equilibria, investment and trade occur both with a positive probability when the buyer cannot observe the seller's investment, or receive any signal of it. In particular, Hermalin (2013) observes that a holdup problem arises when the buyer has all the bargaining power and the problem may cause overinvestment. Our model considers a similar situation in which ex ante investment generates asymmetric information between sellers and buyers, but it allows that signaling is a natural option available for sellers and that trade surely occurs in a competitive market.[5]

[5] Goldlücke and Schmitz (2014) consider an ex ante investment as well, but in a different context where

This paper is organized as follows. The model is introduced in Section 2, and the existence of separating and pooling equilibria is provided in Section 3. In Section 4, we offer a rationale for the assertion that there may be too many, or too few, high-type workers. In Section 5, we discuss government policies to implement the optimal investment. We provide numerical examples in Section 6, and concluding remarks in Section 7. All the proofs are collected in an appendix.

# 2 Model

Consider a labor market with a unit mass of workers and two firms. Each worker chooses both his private investment and public signal. Each firm makes an wage offer to hire workers. The investment is binary; if the worker invests, he becomes qualified $q = H$; if not, unqualified $q = L$. An investment cost $c$ captures a worker's endowments, e.g., intellect, health and other parental environments. Following the investment, each worker chooses a signal $e \in \mathbb{R}_+$. Upon observing the signal, two risk-neutral firms engage in a Bertrand-type competition. The inborn cost $c$ is drawn from a differentiable distribution function $G(c)$ with its support $[\underline{c}, \overline{c}]$, $\overline{c} > \underline{c} \geq 0$ and density $g > 0$ for all $c$.

If firm $i$ hires a worker with signal $e$ given a wage $w_i \in \mathbb{R}_+$, the worker obtains $u_q(w_i, e) - c$, for high quality-type $q = H$; $u_q(w_i, e)$ for low quality-type $q = L$, and the firm obtains $y_q - w_i$ with $y_H > y_L$. The payoff $u_q(w_i, e)$ is differentiable, strictly increasing in $w$, strictly decreasing in $e$ and satisfies the Spence-Mirrlees property (SMP), i.e., for any pair $e' > e$,

$$u_H(w_i, e') - u_H(w_i, e) > u_L(w_i, e') - u_L(w_i, e). \tag{1}$$

In addition, it is assumed to satisfy no "cross effect" between $q$ and $w_i$, i.e., for any pair $w_i' > w_i$,

$$u_H(w_i', e) - u_H(w_i, e) = u_L(w_i', e) - u_L(w_i, e), \tag{2}$$

meaning that the utility gain associated with wage increase is type-irrelevant, which holds for all separable utility functions, $u_q(w_i, e) = v(w_i) - c_q(e)$ with any increasing function

a seller can make an observable investment to improve his product specialized for a buyer, showing that a seller's signaling motive can alleviate the ex ante underinvestment (i.e., the hold-up problem). A key insight of their model is that if the seller has private information about the fraction of the ex post surplus that he can realize on his own, then his large investment can serve as signal of having the strong outside options that affect the buyer's take-it-or-leave-it offer.

$v(\cdot)$. For no signal $e = 0$, it is reasonable to assume that the level of utility is type-irrelevant:

$$u_H(w_i, 0) = u_L(w_i, 0), \tag{3}$$

which, combined with SMP, implies $u_H(w_i, e) > u_L(w_i, e)$ for all $e > 0$.

The game's formal timeline is as follows. First, Nature chooses $c$. After observing $c$, each worker chooses private type $q$. Then, each worker chooses public signal $e$. It follows that the two firms simultaneously make wage offers. Finally, each worker accepts the highest wage; one firm randomly if indifferent.

# 3 Equilibrium and existence

Each firm forms the (common posterior) belief $\mu(e)$ that a worker with $e$ is qualified $q = H$. The Bertrand-type competition yields an identical equilibrium wage offer $w(e) = \mu(e) y_H + (1 - \mu(e)) y_L$. Each worker's signaling strategy (at time 3) is a mapping $E : \{H, L\} \to \mathbb{R}_+$, and his investment strategy (at time 2) is a mapping $Q : [\underline{c}, \overline{c}] \to \{H, L\}$. It is clear that, by comparing the investment cost with its benefit through signaling, the equilibrium investment strategy $Q$ takes a "cutoff strategy" with a threshold cost $k$ such that workers with a cost $c < k$ $(c > k)$ make the investment (no investment). A strategy profile $\{(Q(c), E(q)), w(e)\}$ is a *perfect Bayesian equilibrium* if in each time line, the strategy of each player is the best response to the other players' strategies, and the belief is updated by the Bayes' rule where possible.[6] An equilibrium is called an *interior* equilibrium if its threshold has an interior value, $k \in (\underline{c}, \overline{c})$, and a *boundary* equilibrium otherwise.

We first examine interior equilibria. By the Bayes' rule, on the equilibrium path, for separating equilibria with $e_H \equiv E(H) \neq e_L \equiv E(L)$, $\mu(e_H) = 1$ and $\mu(e_L) = 0$ and the wage for high-type (low-type) workers is $y_H$ $(y_L)$; and for pooling equilibria with $e \equiv E(H) = E(L)$, $\mu(e) = \lambda$, where $\lambda$ denotes the proportion of high-type workers, and

[6] Formally, a set of strategies $\{(Q(c), E(q)), (w_i(e))_{i=1}^2\}$ and a belief function $\mu(e)$ constitute a perfect Bayesian equilibrium if

(i) $(Q(c), E(q))$ is optimal for the worker given $(w_i(e))_{i=1}^2$;

(ii) $\mu(e)$ is derived from $E(q)$ via the Bayes' rule where possible;

(iii) $(w_i(e))_{i=1}^2$ is a Nash equilibrium of the simultaneous move game in which both firms make wage offers to the worker knowing that $q = H$ with probability $\mu(e)$.

the wage for both types $\mathbb{E}^{\lambda}[y] = \lambda y_H + (1-\lambda) y_L$. Note that the proportion of high-type workers, $\lambda$, is *endogenously* determined by the workers' investment decision in this model. For separating equilibria, incentive compatibility conditions are satisfied, with $e_L = 0$, such that

$$u_H(y_H, e_H) \geq u_H(y_L, 0) \text{ and } u_L(y_L, 0) \geq u_L(y_H, e_H),$$

and similarly for pooling,

$$u_H(\mathbb{E}^{\lambda}[y], e) \geq u_H(y_L, 0) \text{ and } u_L(\mathbb{E}^{\lambda}[y], e) \geq u_L(y_L, 0).$$

A separating signal $e_H$ must be in an interval, $e_H \in [\underline{e}_H, \overline{e}_H]$, where $\underline{e}_H$ and $\overline{e}_H$ are respectively defined by binding constraints:

$$u_H(y_H, \overline{e}_H) = u_H(y_L, 0) \text{ and } u_L(y_L, 0) = u_L(y_H, \underline{e}_H). \tag{4}$$

The inequalities, $\overline{e}_H > \underline{e}_H > 0$, follow from (3), $u_L(y_L, 0) = u_H(y_L, 0)$, and $y_H > y_L$. A pooling signal $e$ must be in an interval, $e \in [0, \overline{e}(\lambda)]$, where the upper bound $\overline{e}(\lambda)$ is defined by the binding constraint

$$u_L(\mathbb{E}^{\lambda}[y], \overline{e}(\lambda)) = u_L(y_L, 0). \tag{5}$$

There is no overlap in the use of signal in two equilibria, $\overline{e}(\lambda) < \underline{e}_H$, for all $\lambda < 1$.[7]

Consider next the investment stage. For separating equilibria, a worker's choice $q = H$ yields utility $u_H(y_H, e_H) - c$, and his choice $q = L$ yields $u_L(y_L, 0)$, which leads to a separating equilibrium threshold:

$$k_s = u_H(y_H, e_H) - u_L(y_L, 0). \tag{6}$$

For pooling equilibria, a worker's choice $q = H$ yields $u_H(\mathbb{E}^{\lambda}[y], e) - c$, and his choice $q = L$ yields $u_L(\mathbb{E}^{\lambda}[y], e)$, which leads to a pooling equilibrium threshold:

$$k_p = u_H(\mathbb{E}^{\lambda}[y], e) - u_L(\mathbb{E}^{\lambda}[y], e).$$

Since the utility gain from wage increase is type-irrelevant as in (2),

$$u_H(\mathbb{E}^{\lambda}[y], e) - u_L(\mathbb{E}^{\lambda}[y], e) = u_H(0, e) - u_L(0, e),$$

[7] For $\lambda < 1$, we have $\overline{e}(\lambda) < \underline{e}_H$ from $u_L(y_H, \underline{e}_H) = u_L(y_L, 0) = u_L(\mathbb{E}^{\lambda}[y], \overline{e}(\lambda))$ and $y_H > \mathbb{E}^{\lambda}[y]$.

the threshold $k_p$ equals

$$k_p = u_H(0, e) - u_L(0, e). \tag{7}$$

The role of the signaling action $e$ differs in separating and pooling equilibrium. Namely, for separating, an increase in separating signal $e_H$ discourages workers from becoming high type, while causing no welfare loss to low type, whereas an increase in pooling signal $e$ affects all workers, in such a way that an increase in $e$ encourages workers to become high type. From the negative correlation between quality and signaling cost, high-quality type can enjoy a lower signaling cost even in the pooling, and furthermore, by SMP, this gap is larger for a higher pooling signal $e$.

**Lemma 1** *The separating threshold $k_s$ is a strictly decreasing function of $e_H \in [\underline{e}_H, \overline{e}_H]$, whereas the pooling threshold $k_p$ is a strictly increasing function of $e \in [0, \overline{e}(\lambda)]$.*

An interior separating equilibrium is defined as a pair $(k_s^*, e_H^*)$ that satisfies

$$k_s^* = u_H(y_H, e_H^*) - u_L(y_L, 0) \in (\underline{c}, \overline{c}) \text{ and } e_H^* \in [\underline{e}_H, \overline{e}_H]. \tag{8}$$

An interior pooling equilibrium is defined as a pair $\left(k_p^*, e^*\right)$ that satisfies

$$k_p^* = u_H(0, e^*) - u_L(0, e^*) \in (\underline{c}, \overline{c}) \text{ and } e^* \in [0, \overline{e}\left(G\left(k_p^*\right)\right)], \tag{9}$$

where the proportion of high-type workers in the population, $G(k_p^*)$, is endogenous.

In a boundary equilibrium, workers are treated equally by the same wage and thus they select no costly signal, $e = 0$. Given that workers are treated equally and select no signal, we have $u_H(w, 0) - u_L(w, 0) = 0$ from the assumption in (3), which shows that workers have no incentive to make the investment.

**Lemma 2** *A unique boundary equilibrium $(k_b^*, e_b^*)$ exists with $G(k_b^*) = e_b^* = 0$.*

In the model, there can be two types of interior equilibria, separating and pooling, and there is a unique boundary equilibrium. We henceforth restrict attention to non-trivial interior equilibria; in what follows, a separating (pooling) equilibrium refers to an interior separating (pooling) equilibrium.[8]

[8] In our analysis below, the boundary equilibrium with no investment is considered only under the ban on signaling.

We proceed to examine the separating equilibrium with the least costly signal $\underline{e}_H$ that satisfies Cho-Kreps' intuitive criterion (Cho and Kreps (1987)). Since the threshold $k_s$ is strictly decreasing in $e_H \in [\underline{e}_H, \overline{e}_H]$, the separating equilibrium with $\underline{e}_H$ has the highest level $\overline{k}_s$,

$$\overline{k}_s = u_H\left(y_H, \underline{e}_H\right) - u_L\left(y_L, 0\right) = u_H(y_H, \underline{e}_H) - u_L(y_H, \underline{e}_H),$$

where the second equality follows from the definition of $\underline{e}_H$, $u_L\left(y_L, 0\right) = u_L\left(y_H, \underline{e}_H\right)$ in (4). Given the assumption that the utility gain from wage increase is type-irrelevant, the threshold becomes

$$\overline{k}_s = u_H\left(0, \underline{e}_H\right) - u_L(0, \underline{e}_H). \tag{10}$$

We now adopt two useful notations. First, we define a function

$$\Delta\left(e\right) \equiv u_H\left(0, e\right) - u_L\left(0, e\right),$$

to capture how signal $e$ determines the quality-type relevant gain that workers expect when making the investment. Notice that, for any pooling signal $e$ and the separating signal $\underline{e}_H$, we can use the same function $\Delta$ to represent investment thresholds:[9]

$$k_p = \Delta(e) \text{ and } \overline{k}_s = \Delta(\underline{e}_H).$$

Second, we define a distribution function

$$D(e) \equiv G(\Delta\left(e\right)),$$

to examine how signal $e$ generates the proportion of high-type workers in the population that is hereafter referred to as the *investment ratio*. The function $D(e)$ is strictly increasing in pooling signal $e \in [0, \overline{e}\left(\lambda\right)]$ for all $D(e) \in (0, 1)$. The slope $D'\left(e\right)$ is sufficiently steep (flat) if an increase in pooling signal $e$ is sufficiently effective (ineffective) in increasing the investment ratio further. For instance, the slope $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$ may be steep (flat) for $e \geq e^*$, if the population density $g(c)$ is high (low) for $c \geq \Delta(e^*)$, and (or) if the magnitude of $\Delta'(e)$ is large (small) for $e \geq e^*$.[10]

[9] Note that $\Delta$ captures the type-relevant gain *net of income effect*. We also know that pooling signal cannot exceed the level $\underline{e}_H$ and that the separating signal $\underline{e}_H$ has the feature in (10). Thus, for any pooling signal and the separating signal $\underline{e}_H$, we can use the same function $\Delta$.

[10] Recall that SMP implies $\Delta'(e) > 0$. In broad terms, the magnitude of $\Delta'(e)$ refers to the degree of SMP.

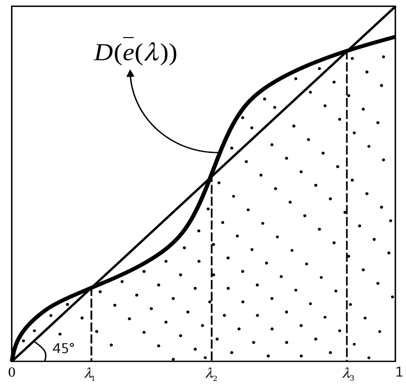


Figure 1: Two intervals of equilibrium proportions

We next use those functions, $\Delta(e)$ and $D(e)$, and establish the existence of equilibria. There exists a separating equilibrium with $\underline{e}_H$ if and only if $\Delta(\underline{e}_H) \in (\underline{c}, \overline{c})$, or equivalently $D(\underline{e}_H) \in (0, 1)$. In the separating equilibrium, the signal $\underline{e}_H$ motivates the workers with cost types below $\overline{k}_s = \Delta(\underline{e}_H)$ to make the investment and results in the investment ratio $D(\underline{e}_H) = G(\Delta(\underline{e}_H))$. We also establish the existence of a pooling equilibrium using the correspondence

$$\{x \in [0, 1] : x = D\left(e\right) \text{ for } e \in [0, \overline{e}\left(\lambda\right)]\}. \tag{11}$$

The correspondence has the maximum value $D\left(\overline{e}\left(\lambda\right)\right) = G\left(\Delta\left(\overline{e}\left(\lambda\right)\right)\right)$ for the highest pooling signal $\overline{e}\left(\lambda\right)$ given $\lambda$. The following proposition shows that there exists a pooling equilibrium with some $e \in [0, \overline{e}\left(\lambda\right)]$ if and only if the function $D\left(\overline{e}\left(\lambda\right)\right)$ reaches the 45 degree line for some $\lambda \in (0, 1)$. This existence condition means that, given $\lambda \in (0, 1)$, the highest pooling signal $\overline{e}\left(\lambda\right)$ motivates the workers with cost types below $k_p = \Delta(\overline{e}\left(\lambda\right))$ to make the investment and results in the investment ratio $D(\overline{e}\left(\lambda\right))$ becoming at least as high as $\lambda$.

**Proposition 1** *(i) There exists a separating equilibrium with $\underline{e}_H$ if and only if $\Delta\left(\underline{e}_H\right) \in (\underline{c}, \overline{c})$, or equivalently $D\left(\underline{e}_H\right) \in (0, 1)$.*

*(ii) There exists a pooling equilibrium if and only if $D\left(\overline{e}\left(\lambda\right)\right) \geq \lambda$ for some $\lambda \in (0, 1)$.*

Figure 1 depicts the case with two sets of equilibrium proportions, $[0, \lambda_1]$ and $[\lambda_2, \lambda_3]$, where the dotted area below the curve $D(\overline{e}(\lambda))$ represents the correspondence in (11).

Since $u_L(\mathbb{E}^{\lambda}[y], \overline{e}(\lambda)) = u_L(y_L, 0)$ where $\mathbb{E}^{\lambda}[y] = y_L + \lambda(y_H - y_L)$, the highest signal $\overline{e}(\lambda)$ is strictly increasing for all $\lambda \in (0,1)$ with boundary values, $\overline{e}(0) = 0$ and $\overline{e}(1) = \underline{e}_H$. Thus, using the same function $\Delta$ for interior and boundary values, we can find that $\Delta(\overline{e}(\lambda))$ is strictly increasing for all $\lambda \in (0,1)$ with $\Delta(\overline{e}(0)) = 0$ and

$$\Delta(\overline{e}(1)) = \Delta(\underline{e}_H) = \overline{k}_s,$$

and that $D(\overline{e}(\lambda))$ is strictly increasing for all $D(\overline{e}(\lambda)) \in (0,1)$ with the vertical intercept

$$D(\overline{e}(1)) = D(\underline{e}_H) = G(\overline{k}_s).$$

If the wage gap, $y_H - y_L$, becomes larger given $y_L$, then $D(\overline{e}(\lambda))$ shifts up since $\overline{e}(\lambda)$ increases given $\lambda > 0$. The function shifts more if the gain from making the investment, $\Delta(e)$, is larger. In the following proposition, we impose a condition on the slope of $D(\overline{e}(\lambda))$: the slope is sufficiently small such that a pooling equilibrium exists and generates the investment ratio that approaches the separating's highest investment ratio $G(\overline{k}_s)$. If $\Delta(\overline{e}(1)) > \overline{c}$, then the condition immediately holds: if $\Delta(\overline{e}(1)) > \overline{c}$, then $D(\overline{e}(\lambda))$ is perfectly flat on the top, $D(\overline{e}(\lambda)) = 1$ on $[\lambda', 1]$ for some $\lambda' \in (0,1)$, and thus a pooling equilibrium exists and generates the investment ratio that approaches 1. As we confirm in Section 4, the condition on the slope of $D(\overline{e}(\lambda))$ plays a key role in our justification for the assertion that there are too many high-type workers for a welfare perspective.

**Proposition 2** *(i) If $\Delta(\overline{e}(1)) > \overline{c}$, then there exists a pooling equilibrium with $\lambda$ sufficiently close to 1.*

*(ii) Suppose $\underline{c} < \Delta(\overline{e}(1)) \leq \overline{c}$. If there exists a sufficiently small $\lambda' > 0$ such that $dD(\overline{e}(\lambda))/d\lambda$ is sufficiently small on $[\lambda', 1]$, then there exists a pooling equilibrium. In addition, if $dD(\overline{e}(\lambda))/d\lambda$ converges to zero, the investment ratio in the pooling equilibrium converges to $G(\overline{k}_s)$.*

Notice that the slope of $D(\overline{e}(\lambda))$ depends on the slope $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$. The condition on the slope of $D(\overline{e}(\lambda))$, stated in Proposition 2 (ii), is likely to hold if the wage gap, $y_H - y_L$, is sufficiently large given $y_L$ so that $D(\overline{e}(\lambda))$ is above 45 degree line for some $\lambda$, and an increase in pooling signal $e$ is ineffective in increasing the investment ratio so that the slope $D'(e)$ is sufficiently flat above a certain level.

# 4 Welfare analysis: distortions in investment

First, we find that any separating or pooling equilibrium generates a strictly higher welfare than the ban on signaling (no signaling), in contrast to the standard case with a fixed $\lambda \in (0,1)$. The ban on signaling leads to a pooling equilibrium in which workers receive the same wage and thus select no signal, and given $u_H(w,0) - u_L(w,0) = 0$, workers make no investment. Therefore, the ban on signaling results in the boundary equilibrium with social welfare $u_L(y_L, 0)$.

If the ban on signaling is lifted, then the no-investment problem can be solved. In a separating equilibrium with $e_H$, the workers with $c \in (\underline{c}, k_s)$ select high type to be treated differently with a higher wage $y_H$, $u_H(y_H, e_H) - c > u_L(y_L, 0)$, while the remaining workers have utility $u_L(y_L, 0)$. In a pooling equilibrium with $e$, although all workers are treated equally by the same wage, the workers with $c \in (\underline{c}, k_p)$ select high type to reduce signaling costs, $u_H(\mathbb{E}^\lambda[y], e) - u_L(\mathbb{E}^\lambda[y], e) = u_H(0,e) - u_L(0,e) > c$, while the remaining workers have utility $u_L(\mathbb{E}^\lambda[y], e) \geq u_L(y_L, 0)$.

**Lemma 3** *In contrast to a fixed $\lambda$, with the investment, any separating or pooling equilibrium generates a strictly higher welfare than the ban on signaling.*

Then, we purse the question of how much investment is optimal. Since employers earn zero profits in the competitive market and workers have surplus, a separating equilibrium generates the social welfare:

$$\begin{aligned} U_s(k_s) &= \int_{\underline{c}}^{k_s} [u_H(y_H, e_H) - c] dG(c) + \int_{k_s}^{\overline{c}} u_L(y_L, 0)\, dG(c) \\ &= u_L(y_L, 0) + \int_{\underline{c}}^{k_s} [k_s - c] dG(c), \end{aligned}$$

where the second equality follows from $k_s = u_H(y_H, e_H) - u_L(y_L, 0)$. The social welfare consists of two parts: the utility $u_L(y_L, 0)$ that is secured for all workers and the surplus of investment that is available only for the workers with cost types below $k_s$. Integrating by parts, we can rewrite $U_s(k_s)$ as

$$U_s(k_s) = u_L(y_L, 0) + \int_{\underline{c}}^{k_s} G(c) dc. \tag{12}$$

Thus, in a separating equilibrium, an increase in the workers' investment unambiguously raises the welfare $U_s(k_s)$. Since $k_s$ is strictly decreasing in $e_H \in [\underline{e}_H, \overline{e}_H]$, the welfare is highest at $U_s(\overline{k}_s)$ when $e_H = \underline{e}_H$.

On the other hand, a pooling equilibrium has the social welfare:

$$U_p(k_p) = \int_{\underline{c}}^{k_p} [u_H(\mathbb{E}^\lambda[y], e) - c] dG(c) + \int_{k_p}^{\overline{c}} u_L(\mathbb{E}^\lambda[y], e) dG(c).$$

Using $k_p = u_H(\mathbb{E}^\lambda[y], e) - u_L(\mathbb{E}^\lambda[y], e)$ and integration by parts, we find that the social welfare consists of the utility $u_L(\mathbb{E}^\lambda[y], e)$ that is secured for all workers and the surplus of investment that is available only for the workers with cost types below $k_p$:

$$U_p(k_p) = u_L(\mathbb{E}^\lambda[y], e) + \int_{\underline{c}}^{k_p} G(c) dc, \text{ where } \lambda = G(k_p) \text{ and } e = \Delta^{-1}(k_p). \tag{13}$$

In a pooling equilibrium, an increase in $k_p$ has two competing effects: an increase in $k_p$ raises the expected wage and the surplus of investment, but it increases signaling costs of workers who remain as low type,

$$U_p'(k_p) = \frac{\partial U_p}{\partial k_p} + \frac{\partial U_p}{\partial e}\frac{de}{dk_p} = \left(\frac{\partial u_L}{\partial w} g(k_p)(y_H - y_L) + G(k_p)\right) + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)}, \tag{14}$$

where $1/\Delta'(e)$ follows from the inverse function $e = \Delta^{-1}(k_p)$. Thus, pooling signal affects all workers and entails a tension between the generation of high-type workers and the signaling costs of low-type workers.

To relate this tension to our main findings in Proposition 3, we here report the conditions on $g(k_p)$ and $\Delta'(e)$ under which $U_p(k_p)$ is strictly decreasing in $k_p$. Notice that the conditions remain valid for any wage gap, $y_H - y_L$.

**Lemma 4** *In a pooling equilibrium, if $g(k_p)$ or $\Delta'(e)$ is sufficiently small (large) at $e = \Delta^{-1}(k_p)$, then $U_p(k_p)$ is strictly decreasing (increasing) in $k_p$.*

We finally ask the main question: can we rationally say that there are too many high-type workers from a welfare perspective? In a separating equilibrium, the workers with lower cost types make the investment to receive a higher wage while causing no welfare loss to the remaining workers. Therefore, with the use of separating equilibrium alone, it is impossible to assert that there are too many high-type workers, even when most of workers make the costly investment to be treated differently from a very small fraction of the remaining workers.

We begin by deriving the difference in the welfare functions in (13) and (17):

$$U_p(k_p) - U_s(\overline{k}_s) = u_L(\mathbb{E}^{\lambda}[y], e) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p} G(c)dc, \tag{15}$$

where $u_L(\mathbb{E}^{\lambda}[y], e) - u_L(y_L, 0) \geq 0$ with equality only if $e = \overline{e}(\lambda)$ from (4). We also recall that the separating equilibrium with $\underline{e}_H$ generates the investment ratio $G(\overline{k}_s) = D(\overline{e}(1)) \in (0, 1)$. Denote this investment ratio by $\overline{\lambda} \equiv G(\overline{k}_s) = D(\overline{e}(1))$. We next impose a condition on the slope of $D(e)$: the slope $D'(e)$ is sufficiently flat for $e \geq e^* \equiv \overline{e}(\lambda^*)$ such that a fixed point $\lambda^* = D(\overline{e}(\lambda^*))$ approximates the investment ratio $\overline{\lambda} = D(\overline{e}(1)) \in (0, 1)$. Defining the threshold $k_p^*$ by $G(k_p^*) = \lambda^*$, we have

$$U_p(k_p^*) - U_s(\overline{k}_s) = u_L(\mathbb{E}^{\lambda^*}[y], e^*) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p^*} G(c)dc = \int_{\overline{k}_s}^{k_p^*} G(c)dc < 0. \tag{16}$$

The condition on the slope $D'(e)$ leads to two important points. First, $k_p^*$ approaches $\overline{k}_s$ and thus $U_p(k_p^*)$ approaches $U_s(\overline{k}_s)$. Second, the pooling equilibrium with $e^*$ has overinvestment, since the conditions on $g(k_p)$ and $\Delta'(e)$ reported in Lemma 4 imply that the social welfare $U_p(k_p)$ is strictly decreasing in $k_p$ when $D'(e) = g(\Delta(e)) \cdot \Delta'(e)$ is sufficiently small. Thus, the condition on $D'(e)$ means that there is $k_p^{**}$ such that $k_p^{**} < k_p^*$ and $U_p(k_p^{**})$ is greater than $U_s(\overline{k}_s)$. As we show in Lemma 6, once a superior form of signaling is found to be pooling, the government can support the pooling equilibrium as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.

**Proposition 3** *Given $D(\overline{e}(1)) > 0$, if there exists a sufficiently small $\lambda' > 0$ such that the slope $D'(e)$ is sufficiently small on $\{e : D(e) = \lambda,\ \lambda \in [\lambda', 1]\}$, then a pooling equilibrium maximizes social welfare.*

In summary, due to the restriction on the slope $D'(e)$, we can make the following statements: (i) Proposition 2 ensures that, for any separating equilibrium, there exists a pooling equilibrium that approximates the separating equilibrium in terms of the investment ratio and social welfare; (ii) Lemma 4 implies that this pooling equilibrium has overinvestment; and (iii) it follows from Proposition 1 and Lemma 4 that there exists an optimal pooling equilibrium that restricts the inefficiency of overinvestment. Therefore, there exist circumstances under which there are too many high-type workers from a welfare perspective.[11] Intuitively, the condition on $D'(e)$ corresponds to a situation in which pooling signal has a saturation point such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. Under the condition, the tension observed in pooling signal implies that it is socially preferred to reduce signaling costs than to increase high-type workers. In this case, separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal and treat high-type workers differently in the market. The inefficiency of overinvestment can then be reduced in pooling equilibrium where workers use the same signal without having to be treated differently.

We can also identify circumstances under which there are too few high-type workers from a welfare perspective. If the slope of $D(e)$ is sufficiently steep for some range, then it is uncertain whether a pooling equilibrium exists, and even when a pooling equilibrium exists, it may generate too few high-type workers. To formalize this argument, we find that given $G(\overline{k}_s) = D(\overline{e}(1)) \in (0, 1)$, if $\lambda^*$ that satisfies $\lambda^* = D(\overline{e}(\lambda^*))$ is sufficiently smaller than $D(\overline{e}(1))$, then the term $u_L(\mathbb{E}^{\lambda^*}[y], e^*)$ in (16) approaches $u_L(y_L, 0)$, but $k_p^*$ does not approach $\overline{k}_s$. Then $U_s(\overline{k}_s)$ is greater than $U_p$ for any potential pooling equilibrium. This condition represents the situation in which pooling signal remains sufficiently effective in generating the investment ratio further. In this case, there are too few high-type workers in pooling equilibrium, and the inefficiency of underinvestment can be reduced in separating equilibrium. The following proposition reports this finding.

**Proposition 4** *Given $D(\overline{e}(1)) > 0$, if the maximum $\lambda^*$ that satisfies $\lambda^* = D(\overline{e}(\lambda^*))$ is sufficiently smaller than $D(\overline{e}(1))$, then a separating equilibrium maximizes social welfare.*

# 5 Equilibrium selection and regulation

The government can affect the workers' investment level, $\overline{k}_s$ or $k_p$, and thus social welfare through its regulation. For this analysis, we assume that the government regulation determines a signal range $[\underline{E}, \overline{E}]$ such that workers can only select signal within the interval,

[11] As our numerical examples show in the next section, the inefficiency of overinvestment may be reduced in pooling equilibrium, $k_p < \overline{k}_s$ and $U_p(k_p) > U_s(\overline{k}_s)$, even when the slope of the function $D(\overline{e}(\lambda))$ is moderately small.

where $\overline{E} > \underline{E} \geq 0$. We also assume that the regulation implements equilibria that satisfy the Cho-Kreps' intuitive criterion. Specifically, the regulation $S$ (the regulation $P$) refers to the regulation under which there exists a separating (pooling) equilibrium that satisfies the Cho-Kreps' criterion.

We first show that an increase in the minimum education $\underline{E}$ decreases the social welfare $U_s(\overline{k}_s)$, which means that the regulation $S$ must satisfy $\underline{E} = 0$ to maximize $U_s(\overline{k}_s)$. To observe that with the boundary condition on $\underline{E}$, the separating signal $\underline{e}_H$ that satisfies the Cho-Kreps' criterion is determined by $u_L(y_L, \underline{E}) = u_L(y_H, \underline{e}_H)$ and thus $\underline{e}_H$ is an increasing function of $\underline{E}$,

$$\frac{d\underline{e}_H}{d\underline{E}} = \frac{\partial u_L\left(y_L, \underline{E}\right)/\partial \underline{E}}{\partial u_L(y_H, \underline{e}_H)/\partial \underline{e}_H} > 0.$$

The corresponding threshold,

$$\overline{k}_s = u_H(y_H, \underline{e}_H) - u_L(y_L, \underline{E}),$$

is also a function of $\underline{E}$, while it is not clear whether $\overline{k}_s$ is increasing or decreasing in $\underline{E}$. Using the welfare function

$$U_s(\overline{k}_s) = u_L(y_L, \underline{E}) + \int_{\underline{c}}^{\overline{k}_s} G(c)dc,$$

we can show that an increase in $\underline{E}$ decreases the social welfare,

$$\frac{dU_s(\overline{k}_s)}{d\underline{E}} = [1 - G(\overline{k}_s)]\frac{\partial u_L(y_L, \underline{E})}{\partial \underline{E}} + G(\overline{k}_s)\frac{\partial u_H\left(y_H, \underline{e}_H\right)}{\partial \underline{e}_H}\frac{d\underline{e}_H}{d\underline{E}} < 0,$$

due to the additional signaling costs imposed on workers whether their inborn cost types are below or above $\overline{k}_s$. Thus, the regulation $S$ must satisfy $\underline{E} = 0$ to achieve the welfare

$$U_s(\overline{k}_s) = u_L\left(y_L, 0\right) + \int_{\underline{c}}^{\overline{k}_s} G(c)dc \text{ with } \overline{k}_s = \Delta(\underline{e}_H), \tag{17}$$

where $\underline{e}_H$ is defined by (4), $u_L(y_L, 0) = u_L(y_H, \underline{e}_H)$. The significance of the boundary condition $\underline{E} = 0$ is that the *optimal* regulation $S$ has the welfare $U_s(\overline{k}_s)$ in (17) and the investment ratio $D(\overline{e}(1))$. Our analysis of the regulation $S$ is hereafter based on these $U_s(\overline{k}_s)$ and $D(\overline{e}(1))$, assuming that the boundary conditions, $\underline{E} = 0$ and $\overline{E} \geq \underline{e}_H$, are satisfied. Moreover, under the regulation $S$ with the boundary conditions, $\underline{E} = 0$ and $\overline{E} \geq \underline{e}_H$, we find that the separating equilibrium with $\underline{e}_H$ is a unique equilibrium that satisfies the Cho-Kreps' criterion.

**Lemma 5** *The regulation $S$ with these boundary conditions supports the separating equilibrium with $\underline{e}_H$ as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.*

In Lemma 5, we show further that the government can target the separating equilibrium with $\underline{e}_H$ since this equilibrium is a unique equilibrium that satisfies the Cho-Kreps' criterion under the regulation $S$ with the boundary conditions. This finding is immediate from the standard result that there exists no pooling equilibrium that satisfies the Cho-Kreps' criterion, although any pooling signal $e$ satisfies $e < \underline{e}_H$ and thus is available in the interval $[\underline{E}, \overline{E}]$.

We next consider the regulation $P$. The regulation $P$ may be associated with circumstances in which the government opts to implement a pooling point $e^*$ in the interval $[\underline{E}, \overline{E}]$. Suppose that a pair $(k_p^*, e^*) = (u_H(0, \overline{E}) - u_L(0, \overline{E}), \overline{E})$ satisfies (9) and thus is a pooling equilibrium. As we show in the Appendix, the government can then support the pooling equilibrium with $e^*$ as a unique equilibrium that satisfies the Cho-Kreps' criterion. Intuitively, the regulation $P$ allows no signal above $\overline{E}$ to which a high-type worker can deviate, and there exists no separating equilibrium given that separating signal $e_H$ must satisfy $e_H > \overline{e}(\lambda^*) \geq e^* = \overline{E}$.

**Lemma 6** *For any pooling equilibrium, the regulation $P$ can support the pooling equilibrium as a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.*

We also find that the government can affect the workers' investment level through its subsidy or tax policy. Formally, for any separating (pooling) equilibrium, there is a subsidy (tax) policy under which the separating (pooling) equilibrium is a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.

**Proposition 5** *For any separating (pooling) equilibrium, there is a subsidy (tax) policy under which the separating (pooling) equilibrium is a unique equilibrium that satisfies the Cho-Kreps' intuitive criterion.*

It is worthwhile to note that the separating equilibrium arises, *without actually subsidizing* any worker: otherwise, the government may face its budget balance problem. The policy is carefully chosen for the pooling case as well, so that the equilibrium arises, without actually taxing any worker: otherwise, the worker may not participate in the game.

# 6 Numerical examples

In this section, we use numerical analysis and report circumstances under which the inefficiency of overinvestment can be reduced in pooling equilibrium, $k_p < \overline{k}_s$ and $U_p(k_p) > U_s(\overline{k}_s)$.

We use the utility function, $u_q(w, e) = w - c_q(e)$ for $q \in \{L, H\}$, where $c_L(e) = e^2$ and $c_H(e) = ae^2$ for $a \in (0, 1)$. Then, $k_p = \Delta(e) = u_H(0, e) - u_L(0, e) = (1 - a)e^2$. From $u_L(\mathbb{E}^\lambda[y], \overline{e}(\lambda)) = u_L(y_L, 0)$, we have $\overline{e}(\lambda) = \sqrt{B\lambda}$, where $B$ denotes the wage gap, $B \equiv y_H - y_L$. From $u_L(y_L, 0) = u_L(y_H, \underline{e}_H)$, we find $\underline{e}_H = \sqrt{B} = \overline{e}(1)$ and $\overline{k}_s = \Delta(\underline{e}_H) = (1 - a)B$. We consider an exponential CDF:

$$G(c; \tau) = \frac{1 - e^{-\tau c}}{1 - e^{-\tau}}, \; c \in [0, 1] \text{ and } \tau > 0.$$

We then have

$$D(e) = G(\Delta(e); \tau) = \frac{1 - e^{-\tau(1-a)e^2}}{1 - e^{-\tau}} \text{ and } D(\overline{e}(\lambda)) = \frac{1 - e^{-\tau(1-a)B\lambda}}{1 - e^{-\tau}}.$$

The welfare comparison between the two signaling forms in (15) becomes

$$U_p(k_p) - U_s(\overline{k}_s) = B\left(\frac{1 - e^{-\tau k_p}}{1 - e^{-\tau}}\right) - \frac{k_p}{1 - a} + \left(\frac{k_p - \overline{k}_s + e^{-\tau k_p} - e^{-\tau \overline{k}_s}}{1 - e^{-\tau}}\right).$$

For a fixed $\overline{k}_s = (1 - a)B = 0.6$, Proposition 3 indicates that, if $(1 - a)$ is sufficiently small, or if the exponential parameter $\tau$ is sufficiently large, then there exists a pooling equilibrium that is superior to any feasible separating. For different parameters, we identify $D(\overline{e}(1)) = G(\overline{k}_s; \tau)$, fixed points $\lambda^* = D(\overline{e}(\lambda^*))$, and thresholds $k_p^*$ corresponding to $\lambda^* = G(k_p^*; \tau)$. Table 1 summarizes the outcomes.

**Table 1**. Fixed point values

| $(1-a)$ | $B$ | $\overline{k}_s$ | $\tau$ | $D(\overline{e}(1))$ | $\lambda^*$ | $k_p^*$ |
|---|---|---|---|---|---|---|
| 0.6 | 1 | 0.6 | 3 | 0.8784 | 0.8055 | 0.5458 |
| 0.3 | 2 | 0.6 | 3 | 0.8784 | 0.8055 | 0.5458 |
| 0.2 | 3 | 0.6 | 3 | 0.8784 | 0.8055 | 0.5458 |
| 0.3 | 2 | 0.6 | 2 | 0.8082 | 0.5797 | 0.4334 |
| 0.3 | 2 | 0.6 | 3 | 0.8784 | 0.8055 | 0.5458 |
| 0.3 | 2 | 0.6 | 4 | 0.9262 | 0.9016 | 0.5798 |

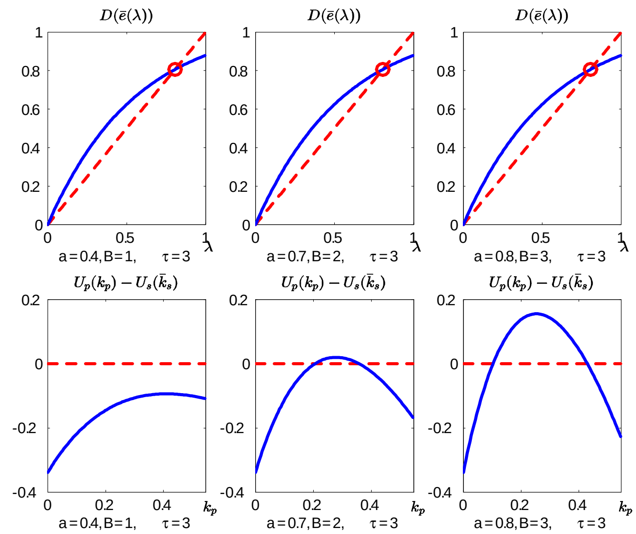


Figure 2: When $(1-a)$ decreases

For $e = \bar{e}(\lambda^*)$, $u_L(\mathbb{E}^{\lambda^*}[y], e) - u_L(y_L, 0) = 0$ in (15) and $\lambda^* = G\left(k_p^*; \tau\right) < G\left(\bar{k}_s; \tau\right) = D(\bar{e}(1))$. Thus, for $k_p = k_p^*$, we have

$$B\left(\frac{1-e^{-\tau k_p^*}}{1-e^{-\tau}}\right) - \frac{k_p^*}{1-a} = 0, \text{ and } U_p\left(k_p^*\right) - U_s\left(\bar{k}_s\right) < 0.$$

However, for $(1-a)$ sufficiently small, there exist pooling equilibria with $k_p < k_p^*$ and $U_p(k_p) - U_s(\bar{k}_s) > 0$. Table 2 reports this result.

| **Table 2**. Change in $(1-a)$ | | | | | |
|---|---|---|---|---|---|
| $(1-a)$ | $B$ | $\bar{k}_s$ | $\tau$ | $U_p(k_p^*) - U_s(\bar{k}_s)$ | $U_p(k_p) - U_s(\bar{k}_s)$ for $k_p = 0.3$ |
| 0.6 | 1 | 0.6 | 3 | $-0.1087$ | $-0.1066$ |
| 0.3 | 2 | 0.6 | 3 | $-0.1707$ | 0.0180 |
| 0.2 | 3 | 0.6 | 3 | $-0.2327$ | 0.1425 |

Figure 2 illustrates the outcomes when $(1-a)$ decreases (i.e., $\Delta'(e) = 2(1-a)e$ decreases) while holding $\bar{k}_s = (1-a)B$ and $\tau$ fixed. The function $D(\bar{e}(\lambda))$ then remains the same, but the differential $U_p(k_p) - U_s(\bar{k}_s)$ shifts up on $[0, k_p^*]$ and results in $U_p(k_p) - U_s(\bar{k}_s) > 0$. Table 3 reports that for the exponential parameter $\tau$ sufficiently small, there exist pooling

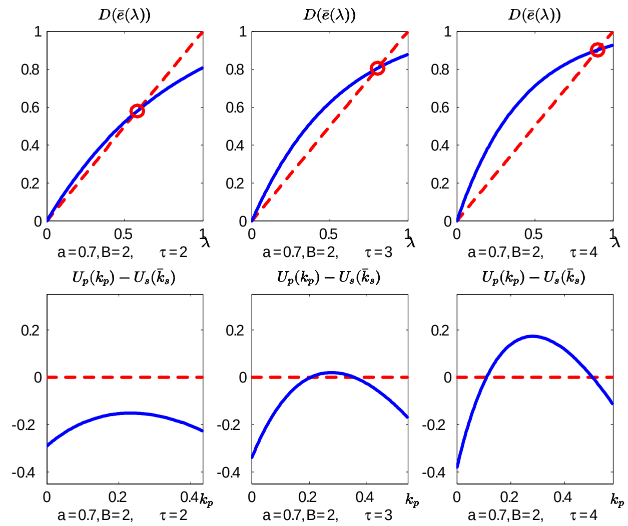


Figure 3: When $\tau$ increases

equilibria with $k_p < k_p^*$ and $U_p(k_p) - U_s(\overline{k}_s) > 0$.

**Table 3.** Changes in the exponential parameter

| $(1-a)$ | $B$ | $\overline{k}_s$ | $\tau$ | $U_p(k_p^*) - U_s(\overline{k}_s)$ | $U_p(k_p) - U_s(\overline{k}_s)$ for $k_p = 0.3$ |
|---|---|---|---|---|---|
| 0.3 | 2 | 0.6 | 2 | $-0.2276$ | $-0.1602$ |
| 0.3 | 2 | 0.6 | 3 | $-0.1707$ | 0.0180 |
| 0.3 | 2 | 0.6 | 4 | $-0.1143$ | 0.1717 |

Figure 3 illustrates the outcomes when $\tau$ increases. An increase in $\tau$ shifts $G(\Delta(e);\tau)$ such that $D(\overline{e}\,(\lambda))$ shifts up with a flatter slope for larger $\lambda$, and the differential $U_p(k_p) - U_s(\overline{k}_s)$ shifts up on $[0, k_p^*]$. As a result, there exist pooling equilibria with $k_p < k_p^*$ such that $U_p(k_p) - U_s(\overline{k}_s) > 0$.

# 7 Conclusions

In this paper, we examine a situation in which each worker endogenously determines the quality of labor through a private investment decision, and the consequent asymmetric information in the market in return causes a moral hazard problem in the investment stage. We consider a model in which signaling is a natural option for workers and socially beneficial due to its influence on the workers' investment. We offer a theoretical foundation for the argument that there are too many high-type workers from a welfare perspective. We identify circumstances under which pooling signal reaches a saturation point such that it becomes sufficiently ineffective in generating the investment ratio above a certain equilibrium level. In this case, it is socially preferred to reduce signaling costs than to increase high-type workers, and separating equilibrium generates too many high-type workers while still having to use the incentive-compatible signal to treat high-type workers differently in the market. The inefficiency of overinvestment can be reduced only in pooling equilibrium where workers use the same signal without having to be treated differently. We also identify circumstances under which pooling signal remains sufficiently effective in generating the investment ratio further. In this case, pooling equilibrium generates too few high-type workers, and the inefficiency of underinvestment can be reduced in separating equilibrium.

Our findings are based on a model that has fairly standard features. Thus, the main theme of our model can be generally extended for the setting in which an investment decision endogenously generates asymmetric information about the quality of products in the market, and this asymmetric information in the market in return causes a moral hazard problem in the investment stage.

# 8 Appendix

**Proof of Lemma 3.** We just need to show the case with a fixed $\lambda \in (0,1)$; specifically, (i) the ban on signaling generates a strictly higher welfare than any pooling equilibrium with $e > 0$; and (ii) if $u_L$ is concave in $w$, then the ban on signaling generates a strictly higher welfare than any separating equilibrium. A separating equilibrium $(e_L, e_H)$ generates the social welfare:

$$\lambda u_H\left(y_H, e_H\right) + (1-\lambda) u_L\left(y_L, 0\right).$$

Since $u_H\left(y_H, e_H\right)$ is strictly decreasing in $e_H \in [\underline{e}_H, \overline{e}_H]$, the least costly signaling for type $H$, $\underline{e}_H$, generates the highest social welfare in the separating equilibrium:

$$U_s = \lambda u_H\left(y_H, \underline{e}_H\right) + (1-\lambda) u_L\left(y_L, 0\right).$$

A pooling equilibrium, $e_H = e_L = e$, generates the social welfare:

$$U_p = \lambda u_H\left(\mathbb{E}^{\lambda}\left[y\right], e\right) + (1-\lambda) u_L\left(\mathbb{E}^{\lambda}\left[y\right], e\right).$$

In comparison, the ban on signaling leads to the same wage $\mathbb{E}^{\lambda}[y]$ and generates the social welfare:

$$U_0 = \lambda u_H\left(\mathbb{E}^{\lambda}[y], 0\right) + (1-\lambda)u_L\left(\mathbb{E}^{\lambda}[y], 0\right).$$

For a separating equilibrium, since $u_H(y_H, 0) > u_H(y_H, \underline{e}_H)$, we have

$$\lambda u_H(y_H, 0) + (1-\lambda)u_L(y_L, 0) > U_s.$$

Thus, to verify the result $U_0 > U_s$, it suffices to show that

$$\lambda u_H\left(\mathbb{E}^{\lambda}[y], 0\right) + (1-\lambda)u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - [\lambda u_H(y_H, 0) + (1-\lambda)u_L(y_L, 0)] \geq 0.$$

The LHS of this inequality becomes

$$\begin{aligned}
&\lambda[u_H\left(\mathbb{E}^{\lambda}[y], 0\right) - u_H(y_H, 0)] + (1-\lambda)[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_L, 0)] \\
&= \lambda[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_H, 0)] + (1-\lambda)[u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - u_L(y_L, 0)] \\
&= u_L\left(\mathbb{E}^{\lambda}[y], 0\right) - [\lambda u_L(y_H, 0) + (1-\lambda)u_L(y_L, 0)] \geq 0.
\end{aligned}$$

The first equality follows from the assumption that the utility gain from any wage increase is type-irrelevant, and the last inequality is given by concavity of $u_L$ in $w$. For a pooling equilibrium, for any $e > 0$, it is immediate that $U_0 > U_p$. ■

**Proof of Proposition 1.** Suppose first that there exists $\lambda \in (0,1)$ such that $D(\overline{e}(\lambda)) \geq \lambda$. Define a correspondence $\Psi : [0,1] \rightrightarrows [0,1]$ using (9) such that

$$\Psi(\lambda) \equiv \{x \in [0,1] : x = D(e) \text{ for } e \in [0, \overline{e}(\lambda)]\}.$$

Thus, an equilibrium fraction of type $H$, $\lambda^*$, is a fixed point of $\Psi$, $\lambda^* \in \Psi(\lambda^*)$. Since $D(e) \in (0,1)$ is an increasing function of $e$, the correspondence can be rewritten as $\Psi(\lambda) = [0, D(\overline{e}(\lambda))]$, and the condition implies the existence of $\lambda^* \in (0,1)$ such that $\lambda^* \in \Psi(\lambda^*)$ and $\left(k_p^*, e^*\right)$ is derived from $G\left(k_p^*\right) = D(e^*) = \lambda^*$. Suppose next that there exists a pooling equilibrium and $D(\overline{e}(\lambda)) < \lambda$ for all $\lambda \in (0,1)$. Then only a boundary pooling equilibrium with $\lambda = 0$ or $\lambda = 1$ exists, which causes a contradiction. ■

**Proof of Proposition 2.** Given $D(\overline{e}(1)) \in (0,1)$, suppose that there exists a sufficiently small $\lambda' > 0$ such that $dD(\overline{e}(\lambda))/d\lambda > 0$ is sufficiently small on $[\lambda', 1]$. Then there exists $\lambda^* \in [\lambda', 1)$ such that $D(\overline{e}(\lambda^*)) = \lambda^*$ with $\lambda^*$ sufficiently close to $D(\overline{e}(1))$. If

$D(\overline{e}(1)) = 1$, $dD(\overline{e}(\lambda))/d\lambda < 1$ at $\lambda = 1$ is sufficient to have a pooling equilibrium with $\lambda$ sufficiently close to 1. ■

**Proof of Lemma 4.** The result is immediate for a sufficiently small $\Delta'(e) > 0$. We thus focus on the condition on $g(k_p)$. Let $g(k_p) = 0$. Then, given $\Delta'(e) = \partial u_H/\partial e - \partial u_L/\partial e$,

$$U_p'(k_p) = G(k_p) + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)} < 1 + \frac{\partial u_L}{\partial e} \cdot \frac{1}{\Delta'(e)} = \frac{1}{\Delta'(e)}\left(\Delta'(e) + \frac{\partial u_L}{\partial e}\right) = \frac{1}{\Delta'(e)}\frac{\partial u_H}{\partial e} < 0.$$

Hence, for a sufficiently small $g(k_p) > 0$, $U_p'(k_p) < 0$. ■

**Proof of Lemma 6.** Suppose that the government uses the regulation $P$ such that a pair $(u_H(0, \overline{E}) - u_L(0, \overline{E}), \overline{E})$ satisfies (9) and is a pooling equilibrium. Let $\widehat{E}$ satisfying $u_L(w, \widehat{E}) = u_L(y_H, \overline{E})$, where $w \equiv \theta y_H + (1-\theta) y_L$ is the pooling's wage. Then, for each $e \in [\widehat{E}, \overline{E}]$, we have $u_L(w, e) \leq u_L(w, \widehat{E}) = u_L(y_H, \overline{E}) \leq u_L(y_H, e')$ for all $e' \leq \overline{E}$. Hence, such $e$ satisfies the criterion, since there is no $e' \leq \overline{E}$ such that $u_L(w, e) > u_L(y_H, e')$. Now, we show that there exists $\overline{E}$ such that any $e \in [\underline{E}, \widehat{E})$ does not satisfy the criterion. Choose $\underline{E}$ satisfying $u_H(w, \underline{E}) < u_H(y_H, \overline{E})$. Suppose that there is a pooling equilibrium with such $e$. Then, $u_L(w, e) > u_L(w, \widehat{E}) = u_L(y_H, \overline{E})$, and $u_H(w, e) \leq u_H(w, \underline{E}) < u_H(y_H, \overline{E})$. A type $H$ worker can attain a higher payoff by deviating from the pooling equilibrium to $\overline{E}$, and a type $L$ worker cannot imitate the action of the type $H$ worker. Hence, $[\underline{E}, \widehat{E}) \cup \{\overline{E}\}$ yields a unique equilibrium. ■

**Proof of Proposition 3.** From Proposition 1, there exists a separating equilibrium with $\underline{e}_H$, which has a threshold $\overline{k}_s = \Delta(\overline{e}(1))$. Denote the separating's capital accumulation by $\overline{\lambda} \equiv G(\overline{k}_s)$. Now, if $D'(e)$ is sufficiently small on $\{e : D(e) = \lambda, \lambda \in [\lambda', 1]\}$, from Proposition 2 (ii), there exists a pooling equilibrium. In particular, choose a fixed point $\lambda^*$ sufficiently close to $\overline{\lambda}$ such that $D(\overline{e}(\lambda^*)) = \lambda^*$. Denote $k_p^*$ satisfying $G(k_p^*) = \lambda^*$. It follows from $D(\overline{e}(\lambda^*)) = \lambda^* = D(e)$ that $\overline{e}(\lambda^*) = e$, and $u_L(\mathbb{E}^{\lambda^*}[y], e) = u_L(\mathbb{E}^{\lambda^*}[y], \overline{e}(\lambda^*)) = u_L(y_L, 0)$. Then,

$$\begin{aligned} U_p(k_p^*) - U_s(\overline{k}_s) &= u_L(\mathbb{E}^{\lambda^*}[y], e) - u_L(y_L, 0) + \int_{\overline{k}_s}^{k_p^*} G(c)dc \\ &= \int_{\overline{k}_s}^{k_p^*} G(c)dc < 0. \end{aligned}$$

However, for a fixed $\overline{k}_s$, as $D'(e) \to 0$ for all $e$ satisfying $D(e) \geq \lambda^*$, so $k_p^* \to \overline{k}_s$, which leads to $U_p(k_p^*) - U_s(\overline{k}_s) \to 0$. In addition, for a fixed $\overline{k}_s$, as $D'(e) \to 0$ for all $e$ satisfying $D(e) \geq \lambda^*$, from Lemma 4, $U_p(k_p)$ is strictly decreasing at $k_p^*$. Hence, there exists $k_p$ sufficiently close to $k_p^*$ such that $U_p(k_p) > U_s(\overline{k}_s)$. The remaining proof follows from Lemma 6. ■

**Proof of Proposition 5.** Consider first separating equilibrium. For a designated high type's signal $e_H^*$, choose the following subsidy $s > 0$ scheme to encourage more signaling such that

$$\begin{cases} u_q(w, e) & \text{if} \quad e < e_H^*, \\ u_q(w + s, e) & \text{if} \quad e \geq e_H^*, \end{cases}$$

which admits a discontinuity. In addition, choose the amount of subsidy that satisfies $u_H(y_H, \underline{e}_H) < u_H(y_H + s, e_H^*)$. Show that the high type's signal $e_H^*$ with the low type's zero signal is a separating equilibrium satisfying the intuitive criterion. From the single-crossing and no-cross effect (1)-(2),

$$\begin{aligned} & u_H(y_H, e_H^*) - u_H(y_H, \underline{e}_H) > u_L(y_H, e_H^*) - u_L(y_H, \underline{e}_H), \\ \Leftrightarrow\ & 0 > u_H(y_H + s, e_H^*) - u_H(y_H, \underline{e}_H) > u_L(y_H + s, e_H^*) - u_L(y_H, \underline{e}_H). \end{aligned} \tag{18}$$

Hence,

$$u_H(y_H + s, e_H^*) > u_H(y_L, 0),\ u_L(y_L, 0) > u_L(y_H + s, e_H^*),$$

where the first inequality follows from $u_H(y_H + s, e_H^*) > u_H(y_H, \underline{e}_H) > u_L(y_H, \underline{e}) = u_L(y_L, 0) = u_H(y_L, 0)$, and the second from (18) and $u_L(y_L, 0) = u_L(y_H, \underline{e})$. By the construction, for each $e \in (\underline{e}, e_H^*)$, the high type's payoff as well as the low type's strictly decreases, leaving the criterion intact. It remains to show that no other high type's signal constitutes a separating equilibrium satisfying the criterion, given the subsidy scheme. Suppose that $e$ is a high type's equilibrium signal. For each $e \geq \underline{e}$ with $e \neq e_H^*$, $e$ does not satisfy the criterion since $u_H(y_H, \underline{e}) > u_H(y_H, e)$ for $e \in [\underline{e}, e_H^*)$, and $u_H(y_H + s, e_H^*) > u_H(y_H + s, e)$ for $e > e_H^*$. Then, there exists $e' \in (\underline{e}, e)$ such that the high type's payoff strictly increases, whereas the low type's payoff strictly decreases.

Now, consider pooling equilibrium. For a designated pooling's signal $e^*$, choose the following tax $t > 0$ scheme to encourage less signaling such that $\widehat{e} > e^*$, and

$$\begin{cases} u_q(w, e) & \text{if} \quad e < \widehat{e}, \\ u_q(w - t, e) & \text{if} \quad e \geq \widehat{e}. \end{cases}$$

In addition, choose the amount of tax that satisfies $u_H(\mathbb{E}[y], e^*) = u_H(y_H - t, \widehat{e})$. By the construction, clearly, the pooling's signal constitute a pooling equilibrium satisfying the criterion. It remains to show that no other signal is a pooling equilibrium satisfying the criterion, given the tax scheme. Suppose that $e$ is a pooling's equilibrium signal. For each $e < e^*$, there is an interval, $(e', \widehat{e})$ with $u_L(\mathbb{E}[y], e^*) = u_L(y_H, e')$, such that by deviating to any signal in the interval, the high type's payoff strictly increases, whereas the low type's payoff strictly decreases. For each $e > e^*$, there is an interval, $(\widehat{e}, e')$ with $u_H(\mathbb{E}[y], e^*) = u_H(y_H - t, e')$, such that by deviating to any signal in the interval, the high type's payoff strictly increases, whereas the low type's payoff strictly decreases. ■

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