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Inflation, Credit, and Indexed Unit of Account

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Inflation, Credit, and Indexed Unit of Account

Hyung Sun Choi[*], Ohik Kwon[†], Manjong Lee[‡]

Abstract

A simple monetary model is constructed to study the implications of an indexed unit of account (Indexed-UoA). In an economy with an Indexed-UoA, credit trade friction attributed to inflation is resolved and there is no redistributional effect from unexpected inflation between debtors and creditors. However, in an economy without an Indexed-UoA, credit trades occur only if inflation is not too high and unexpected inflation renders debtors better off but creditors worse off. Adopting a medium of exchange as a unit of account is most apposite for a low-inflation economy, whereas introducing an alternative Indexed-UoA enhances welfare in an economy where inflation undermines credit trades.

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1. Introduction

An indexed unit of account (Indexed-UoA) has a long history dating back to Mill (1848, p.349) who pointed out the problem of an unstable monetary standard as follows:

> All variations in the value of the circulating medium are mischievous: they disturb existing contracts and expectations, and the liability to such changes renders every pecuniary engagement of long date entirely precarious.

In the same spirit as Mill, Jevons (1875, chapter XXV) emphasized the necessity of an Indexed-UoA, called the "Tabular Standard of Value," by referring to the proposals made by Lowe and Scorope:

> He [Joseph Lowe] proposes that persons should be appointed to collect authentic information concerning the prices at which the staple articles of household consumption were sold. ... . Having regard to the comparative quantities of commodities consumed in a household, he would then frame a table of reference, showing in what degree a money contract must be varied so as to make the purchasing power uniform. ... Mr. Scrope suggests ... that a standard might be formed by taking an average of the mass of commodities which, ... , might serve to determine and correct the variations of the legal standard. ... Such schemes for a tabular or average standard of value appear to be perfectly sound ... and the practical difficulties are not of a serious character.

In a similar vein, Friedman (1974), Fischer (1986), Tobin (1987), and Shiller (1999, 2002, 2003) advocate indexing payments to inflation. From a real-world perspective, indexed units of account have indeed been introduced in some Latin-American countries such as Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay.

This paper attempts to delve deeper into understanding the role of an Indexed-UoA using a microfounded monetary model that is studied extensively these days. In particular, we try to elaborate on the mechanism by which an Indexed-UoA can affect real allocations and the circumstances that render an Indexed-UoA essential.

In order to do that, considering the concern raised by Mill (1848), we first introduce a deferred-payment trade intermediated by a benevolent government into the model of

Berentsen, Camera, and Waller (2005). In particular, we take notice of the key features of the deferred-payment trade like a credit card payment as follows: (i) the point of delivering a good does not coincide with the point of clearing the trade and (ii) the trade is typically made at the price of the point of delivering a good. We then make comparisons between an economy where deferred-payment trades are denominated in money (No-Indexed-UoA economy) and an economy where deferred-payment trades are denominated in Indexed-UoA (Indexed-UoA economy) to spotlight the kernel of an Indexed-UoA. Here, our Indexed-UoA is a mimic of the Chilean CPI-Indexed-UoA (*Unidad de Fomento*) in the sense that the monetary conversion rate is adjusted according to a realized price level.

Our main results are as follows. We first show that in a No-Indexed-UoA economy, a deferred-payment trade (hereinafter referred to as the "credit trade") occurs only if inflation is not too high but it takes place regardless of inflation in an Indexed-UoA economy. This discrepancy mainly originates in the difference in nominal stickiness of the terms of a credit trade. In an Indexed-UoA economy, a balance-of-credit trade is denominated in Indexed-UoA at the point of delivering the good and is adjusted flexibly at the point of settlement according to realized inflation. However, in a No-Indexed-UoA economy, a balance-of-credit trade is denominated in money at the point of delivering good and is settled without any adjustment in response to realized inflation. This implies that if inflation is too high, the balance-of-credit trade repaid at the point of settlement will not be sufficient to compensate for the cost borne by a seller at the point of delivering the good. This makes sellers reluctant to accept credit trades in a high-inflation economy.

In addition, in a No-Indexed-UoA economy, as suggested in most standard models, unexpected inflation renders debtors better off and creditors worse off. However, there is no such redistributional effect in an Indexed-UoA economy because the balance-of-credit trade is denominated in Indexed-UoA, the value of which is adjusted instantly to a change in the money supply.

The results above conform with the claim of Shiller (1999, 2002, 2003) that introducing an Indexed-UoA could resolve some problems caused by inflation. They are also somewhat in line with the view of Keynes (1923) that the role of money as a unit of account would deteriorate if its value were unstable and then an alternative unit of account would emerge. As mentioned, this prediction has indeed come to pass in some Latin-American countries—that is, Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay introduced indexed units of account during episodes of high inflation (Shiller 2002).

We then check whether the existence result of an equilibrium with credit trades is affected if the government cannot force a debtor to repay her credit balance. Our result suggests that if the government can refuse to intermediate a credit trade for a defaulter and exclude her permanently from a credit-trade market, a sufficiently patient agent will never default and voluntarily repay a credit balance. Hence, even if we assume no repayment enforcement, a credit trade is still made in an Indexed-UoA economy regardless of inflation, whereas it takes place only if inflation is not too high in a No-Indexed-UoA economy.

Finally, if there is a fixed maintenance cost for an Indexed-UoA that is not too high, adopting a medium of exchange as a unit of account is most apposite for a low-inflation economy, whereas introducing an alternative Indexed-UoA enhances welfare in an economy where inflation undermines credit trades. Once the government introduces an Indexed-UoA in a high-inflation economy, buyers willingly bear the maintenance cost of an Indexed-UoA and sellers accept the credit trades denominated in Indexed-UoA only. Therefore, as an inflation-proof unit of account, an Indexed-UoA eventually facilitates credit trades. This result accounts for why some Latin-American countries adopted indexed units of account during periods of high inflation.

The paper proceeds as follows. Section 2 describes the model economy, followed by an equilibrium characterization in Section 3. Section 4 explores the credit friction associated with inflation and the implications of an Indexed-UoA. Section 5 discusses the robustness of our results by relaxing some simplification assumptions. And Section 6 summarizes the paper with a few concluding remarks, followed by the Appendix which contains proofs for our main results.

2. Model

The background environment is that of Berentsen, Camera, and Waller (2005) with competitive markets.[1] Time is discrete and continues forever. There is a $[0, 1]$ continuum of infinitely lived agents with one perishable and divisible good that can be produced and consumed by all agents. There is also an intrinsically useless, divisible, and durable object called money. Each agent is endowed with $M_0 > 0$ units of money at the beginning of the initial period. In each period, agents trade in three Walrasian markets, called market 1, 2, and 3, which open and close sequentially. Agents discount across periods with factor $\beta \in (0, 1)$.

At the beginning of market 1, each agent receives one of two equally probable preference shocks such that an agent can consume but cannot produce (a buyer) with probability a half, while an agent can produce but cannot consume (a seller) with the remaining probability a half. A seller suffers disutility q from producing $q \in \mathbb{R}_+$ units of a good and can trade with anonymous buyers and an onymous government. Trades with anonymous buyers cannot be recorded and hence they should be *quid pro quo*.[2] The buyer obtains utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of the good where $u'' < 0 < u'$, $u'(\infty) = 0$, $u(0) = 0$, and $u'(0) = \infty$.

The government can record trades associated with her only in this market and plays an intermediary role of a deferred-payment trade (credit trade) at no cost. Specifically, the government purchases goods from sellers who are willing to make a credit trade such that goods are delivered on the spot at the current market price and the relevant balance is cleared

[1] Among the related models of competitive pricing in the framework of Lagos and Wright (2005) are Rocheteau and Wright (2005), Lagos and Rocheteau (2005), and Berentsen, Camera, and Waller (2007).

[2] See, for example, Kocherlakota (1998), Wallace (2001), Corbae, Temzelides, and Wright (2003), and Aliprantis, Camera, and Puzzello (2007).

at the end the period (i.e., market 3). The government then transforms one unit of good purchased into one unit of so-called deferred-payment good at no cost and resells it at the purchasing price to buyers who wish to consume it with a delayed payment in market 3. A balance-of-credit trade is denominated in money in an economy without an Indexed-UoA, whereas it is denominated in Indexed-UoA in an economy with an Indexed-UoA, the value of which is indexed to a realized price level. (In Section 5, we relax this assumption and discuss the choice problem of a unit of account.) The buyer obtains utility $\upsilon(q)$ from consuming $q \in \mathbb{R}_+$ units of a deferred-payment good where $\upsilon'' < 0 < \upsilon'$, $\upsilon(0) = 0$, and $\upsilon'(0)$ is sufficiently large.

It is worth mentioning here that a deferred-payment trade at the current price is motivated by Shiller (1999, p.1): "The general public appears to have sufficient difficulty with indexation, ..., that they will do so only in rare or extreme situations. Even in times of moderate to high inflation, most people will not purchase inflation-indexed debt, will not borrow with an indexed mortgage, will not agree to indexed alimony or child support payments and will not push hard for indexed rent or wage contracts."

After closing market 1 but before opening market 2, the government injects new money in a lump-sum manner. That is, the money stock evolves according to $M_t = \mu_t M_{t-1}$ over the period where M_t denotes the money supply at the end of period t and μ_t is a random variable such that $\mu_t^h = \bar{\mu}(1+\varepsilon)$ with probability ρ and $\mu_t^l = \bar{\mu}(1-\varepsilon)$ with probability $1-\rho$. We here assume $\varepsilon \in (0,1)$ and $\rho = 1/2$ so that $\mathbb{E}(\mu_t) = \bar{\mu}$. Except for intermediating credit trades and injecting money, the government engages in neither consumption nor production.

With a money balance after the trades in market 1 and the lump-sum transfer, each agent moves on to market 2 where she again receives an idiosyncratic preference shock such that she becomes either a buyer or a seller with equal probability. As in market 1, a buyer obtains utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of a good and a seller suffers disutility q from producing $q \in \mathbb{R}_+$ units of the good. Since in this market the government cannot access a record-keeping technology and agents cannot commit to future actions, all trades should be on the spot. It is also worthwhile to note that unavailability of a record-keeping technology implies that the balance-of-credit trades made in market 1 cannot be used to purchase the market-2 good. This feature distinguishes money acquired in market 1 from the balances of credit—that is, the former is liquid in market 2, whereas the latter is illiquid.

In market 3, all agents can consume, produce, and obtain utility $U(q)$ from consuming $q \in \mathbb{R}_+$ units of a good and suffer disutility q from producing $q \in \mathbb{R}_+$ units of the good where $U'' < 0 < U'$, $U'(\infty) = 0$, $U(0) = 0$, and $U'(0) = \infty$.[3] In addition, all the credit trades made in market 1 are cleared in this market. We assume that the government can force repayment at no cost. Hence according to the record, the government collects the credit balances from debtors who consume via credit trades in market 1 and transfers them to creditors who produce for credit trades. As pointed out properly by Berentsen, Camera, and Waller (2007), for instance, default is a critical issue for models dealing with credit. We here simplify such an issue by assuming full enforcement which has the advantage of focusing exclusively on credit trade friction attributed to inflation. (In Section 5, we relax this assumption and derive the conditions that ensure voluntary repayment in an environment of no enforcement.)

3. Stationary Equilibrium

We will consider a stationary monetary equilibrium in which the end-of-period real money balance is constant over time: i.e., $\phi_{t-1}M_{t-1} = \phi_t^h \mu_t^h M_{t-1} = \phi_t^l \mu_t^l M_{t-1}$ where ϕ^i for $i \in \{h, l\}$ is the real price of money in market 3 when the realized money growth shock is μ^i. Hereinafter we drop the time subscript t and index the next-period (previous period) variable by $+1$ (-1) if there is no risk of confusion.

[3]As discussed in Berentsen, Camera, and Waller (2005), the different preference in market 3 is simply a technical device to ensure a degenerate distribution at the beginning of each period. In particular, scaling of $U(q)$ so that $q_3^* \geq 2q^* + q_d^*$ is required to guarantee the result where $q_3^* = \arg\max[U(q_3) - q_3]$, $q^* = \arg\max[u(q) - q]$, and $q_d^* = \arg\max[v(q_d) - q_d]$.

3.1. Without an Indexed Unit of Account

We first study an economy that does not introduce an INDEXED-UOA. Let $V_j(m_j, d)$ denote the expected value for an agent entering market $j \in \{2, 3\}$ with m_j and d amount of money and credit balance, respectively, and let $V_1(m_1)$ denote the expected value for an agent entering market 1 with m_1 amount of money. Then the lifetime utility of an agent entering market 3 with $m_3 \in \mathbb{R}_+$ and $d \in \mathbb{R}$ is given by

$$V_3(m_3, d) = \max_{(q_3^b, q_3^s, m_{1,+1})} \left[U(q_3^b) - q_3^s + \beta V_{1,+1}(m_{1,+1})\right] \tag{1}$$
$$\text{s.t. } q_3^b + \phi m_{1,+1} = q_3^s + \phi(m_3 + d)$$

where q_3^b (q_3^s) is consumption (production) in market 3, $\phi = 1/p_3$ with p_3 denoting the nominal price of the market-3 good, and d is positive (negative) for a creditor (debtor). Substituting q_3^s from the constraint, we have

$$V_3(m_3, d) = \phi(m_3 + d) + \max_{(q_3^b, m_{1,+1})} \left[U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_{1,+1}(m_{1,+1})\right].$$

The first order conditions for $(q_3^b, m_{1,+1}) \in \mathbb{R}^2_{++}$ are

$$U'(q_3^b) = 1 \tag{2}$$

$$\beta V'_{1,+1}(m_{1,+1}) = \phi \tag{3}$$

where $V'_{1,+1}$ is the marginal value of an additional unit of money taken into market 1 in the next period. The envelope condition is

$$V'_{3,i}(m_3, d) = \phi \tag{4}$$

where $V'_{3,i}$ for $i \in \{m, d\}$ is the marginal value of an additional unit of i taken into market 3. As in Lagos and Wright (2005), regardless of (m_3, d), all agents consume $q_3^b = q_3^* = \arg\max[U(q_3^b) - q_3^b]$ and exit market 3 with an identical balance of money. This conveniently allows us to restrict our attention to the case where the distribution of money holdings is degenerate at the beginning of each period.

We next turn to market 2. The lifetime utility of an agent entering market 2 with $m_2 \in \mathbb{R}_+$ amount of money and $d \in \mathbb{R}$ amount of credit balance is given by

$$V_2(m_2, d) = \frac{1}{2}\left\{\max_{q_2^b}\left[u(q_2^b) + V_3(m_2 - p_2 q_2^b, d)\right]\right\} + \frac{1}{2}\left\{\max_{q_2^s}\left[V_3(m_2 + p_2 q_2^s, d) - q_2^s\right]\right\} \quad (5)$$

where q_2^b (q_2^s) is consumption (production) in market 2 and p_2 is the nominal price of the market-2 good. Taking $p_2 \in \mathbb{R}_{++}$ as given, a seller chooses $q_2^s \in \mathbb{R}_{++}$ that solves the second term of the right-hand side in (5), which yields an optimality condition

$$V'_{3,m}(m_2 + p_2 q_2^s, d) = \frac{1}{p_2}. \quad (6)$$

Then (4) immediately gives

$$p_2 = p_3 = \phi^{-1}. \quad (7)$$

Similarly, a buyer chooses $q_2^b \in \mathbb{R}_{++}$ that solves the first term of the right-hand side in (5), which yields an optimality condition

$$u'(q_2^b) = p_2\left[V'_{3,m}(m_2 - p_2 q_2^s, d) + \lambda_2\right] \quad (8)$$

where $\lambda_2 \in \mathbb{R}_+$ is the Lagrangian multiplier on the buyer's budget constraint ($p_2 q_2^b \leq m_2$). Using (4), (6), and (7), (8) reduces to

$$u'(q_2^b) = 1 + \frac{\lambda_2}{\phi}. \quad (9)$$

Notice that $q_2^b = q^* = \arg\max[u(q) - q]$ if $\lambda_2 = 0$. In addition, (6) and (7), together with $(\partial q_2^b / \partial m_2) = (1/p_2)$ for $\lambda_2 \neq 0$ and $u'(q_2^b) = 1$ for $\lambda_2 = 0$, imply that the marginal value of an additional unit of money at the beginning of market 2 is given by

$$V'_{2,m}(m_2, d) = \begin{cases} \phi & \text{for } \lambda_2 = 0 \\ \frac{\phi}{2}\left[u'(q_2^b) + 1\right] & \text{otherwise.} \end{cases} \tag{10}$$

The marginal value of an additional unit of credit balance at the beginning of market 2 is given by

$$V'_{2,d}(m_2, d) = V'_{3,d} = \phi. \tag{11}$$

We now move on to market 1. The lifetime utility of an agent entering market 1 with $m_1 \in \mathbb{R}_+$ amount of money is given by

$$\begin{aligned} V_1(m_1) \quad = \quad & \frac{1}{2}\left\{ \max_{(q_{1,m}^b, q_{1,d}^b)} \left[u(q_{1,m}^b) + \upsilon(q_{1,d}^b) + \mathbb{E}V_2\left(m_1 - p_1 q_{1,m}^b + T, -p_1 q_{1,d}^b\right)\right] \right\} + \\ & \frac{1}{2}\left\{ \max_{(q_{1,m}^s, q_{1,d}^s)} \left[\mathbb{E}V_2(m_1 + p_1 q_{1,m}^s + T, p_1 q_{1,d}^s) - (q_{1,m}^s + q_{1,d}^s)\right] \right\} \end{aligned} \tag{12}$$

where p_1 is the nominal price of the market-1 good, $q_{1,m}$ $(q_{1,d})$ is the quantity of a good traded for money (deferred-payment trade), and T denotes the lump-sum transfer from the government after the market-1 trade, $T = (\mu^h - 1)M_{-1}$ with probability a half and $T = (\mu^l - 1)M_{-1}$ with the remaining probability a half. Taking $p_1 \in \mathbb{R}_{++}$ as given, a seller chooses $(q_{1,m}^s, q_{1,d}^s)$ and a buyer chooses $(q_{1,m}^b, q_{1,d}^b)$. The choice problems of $(q_{1,m}^s, q_{1,m}^b) \in \mathbb{R}_{++}^2$ yield optimality conditions

$$p_1 \mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, p_1 q_{1,d}^s) = 1 \tag{13}$$

$$p_1 \mathbb{E}V'_{2,m}(m_1 - p_1 q_{1,m}^b + T, -p_1 q_{1,d}^b) = u'(q_{1,m}^b) \tag{14}$$

where we use the result in Lemma 2 of Berentsen, Camera, and Waller (2005) that $p_1 q_{1,m}^b =$

m_1 cannot happen because $u'(0) = \infty$. Using (13), (14), and market clearing condition ($q^b_{1,m} = q^s_{1,m} = q_{1,m}$), we then have

$$u'(q_{1,m}) = \frac{\mathbb{E}V'_{2,m}(m_1 - p_1 q_{1,m} + T, -p_1 q^b_{1,d})}{\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, p_1 q^s_{1,d})}. \tag{15}$$

Furthermore, (13), (14), and $p_1 q^b_{1,m} < m_1$ imply the marginal valuation of an additional unit of money at the beginning of market 1 as follows:

$$V'_1(m_1) = \frac{1}{2}\left[\frac{u'(q^b_{1,m}) + 1}{p_1}\right]. \tag{16}$$

Finally, a seller and a buyer choose $q^s_{1,d} \in \mathbb{R}_+$ and $q^b_{1,d} \in \mathbb{R}_+$, respectively, which satisfy

$$\begin{cases} p_1 \mathbb{E}V'_{2,d}(m_1 + p_1 q_{1,m} + T, p_1 q^s_{1,d}) \leq 1 & \text{“=” if } q^s_{1,d} > 0 \\ q^b_{1,d} = 0 & \text{if } q^s_{1,d} = 0 \\ p_1 \mathbb{E}V'_{2,d}(m_1 - p_1 q_{1,m} + T, -p_1 q^b_{1,d}) = \upsilon'(q^b_{1,d}) & \text{if } q^s_{1,d} > 0 \end{cases} \tag{17}$$

and market clearing condition ($q^b_{1,d} = q^s_{1,d} = q_{1,d}$). Noting that $\mathbb{E}V'_{2,d} = \phi$ from (11), a seller is not willing to produce for a credit trade ($q^s_{1,d} = 0$) if $p_1 \mathbb{E}\phi < 1$ because its expected gain is not sufficient to compensate for her disutility cost incurred from production. If $p_1 \mathbb{E}\phi = 1$, (17) implies that $q_{1,d} = q^*_d = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$. Now a stationary monetary equilibrium for an economy without an Indexed-UoA can be defined as follows.

Definition 1 *A stationary monetary equilibrium for a* No-Indexed-UoA *economy is a list of* $[(p_j)^3_{j=1}, (q_{1,m}, q_{1,d}, q_2, q_3), \lambda_2, m_{1,+1}]$ *that satisfies (2)-(3), (7)-(9), (13)-(14), and (17).*

3.2. With an Indexed Unit of Account

We now suppose that the government introduces an Indexed-UoA and like the *Unidad de Fomento* (CPI indexed unit of account in Chile), its exchange rate with a unit of money is indexed to a realized price level. (In Section 5, we discuss the government problem of whether to introduce an INDEXED-UOA.) Specifically, one unit of an INDEXED-UOA in the market $j \in \{1,2,3\}$ is converted into p_j units of money. Noting that trades using money are on the spot, there is no reason for traders to use an alternative unit of account in place of money (medium of exchange). However, since money-supply shock is realized between the point of delivering the good in a credit trade and the point of its settlement, a credit trade is exposed to inflation risk. This suggests that if there is no extra cost incurred due to an INDEXED-UOA, there is no reason for credit traders not to use an INDEXED-UOA as a unit of account. Hence, we assume that a balance-of-credit trade is denominated in INDEXED-UOA. (This assumption is also relaxed in Section 5.)

Now the lifetime utility of an agent entering market 3 with $m_3 \in \mathbb{R}_+$ amount of money and $d^u \in \mathbb{R}$ amount of credit balance denominated in INDEXED-UOA is given by

$$V_3(m_3, d^u) = \phi(m_3 + p_3 d^u) + \max_{(q_3^b, m_{1,+1})} \left[U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_{1,+1}(m_{1,+1}) \right]. \tag{1'}$$

The first order conditions for $(q_3^b, m_{1,+1}) \in \mathbb{R}^2_{++}$ are identical to those for a NO-INDEXED-UOA economy, (2)-(3). Noting that $\phi p_3 = 1$, the envelope conditions are given by

$$V'_{3,m}(m_3, d^u) = \phi, \quad V'_{3,d^u}(m_3, d^u) = 1. \tag{4'}$$

The lifetime utility of an agent entering market 2 with $m_2 \in \mathbb{R}_+$ and $d^u \in \mathbb{R}$ amount of credit balance denominated in INDEXED-UOA is given by

$$V_2(m_2, d^u) = \frac{1}{2} \left\{ \max_{q_2^b} \left[u(q_2^b) + V_3(m_2 - p_2 q_2^b, d^u) \right] \right\} + \frac{1}{2} \left\{ \max_{q_2^s} \left[V_3(m_2 + p_2 q_2^s, d^u) - q_2^s \right] \right\} \tag{5'}$$

and the relevant optimality conditions are identical to (6)-(8).

The lifetime utility of an agent entering market 1 with m_1 is given by

$$
\begin{aligned}
V_1(m_1) = \frac{1}{2}\Bigg\{ & \max_{(q^b_{1,m}, q^b_{1,d})} \left[u(q^b_{1,m}) + \upsilon(q^b_{1,d}) + \mathbb{E}V_2\left(m_1 - p_1 q^b_{1,m} + T, -q^b_{1,d}\right)\right] \Bigg\} + \\
\frac{1}{2}\Bigg\{ & \max_{(q^s_{1,m}, q^s_{1,d})} \left[\mathbb{E}V_2\left(m_1 + p_1 q^s_{1,m} + T, q^s_{1,d}\right) - (q^s_{1,m} + q^s_{1,d})\right] \Bigg\} \qquad (12')
\end{aligned}
$$

where the credit balance, the second argument of V_2 on the right-hand side in (12′), is expressed in terms of Indexed-UoA, $q_{1,d} = (p_1 q_{1,d})/p_1$. The first order conditions for ($q^s_{1,m}$, $q^b_{1,m}$) are the same as (13)-(14) which together with market clearing condition ($q^b_{1,m} = q^s_{1,m} = q_{1,m}$) imply that

$$
u'(q_{1,m}) = \frac{\mathbb{E}V'_{2,m}(m_1 - p_1 q_{1,m} + T, -q^b_{1,d})}{\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, q^s_{1,d})}. \qquad (15')
$$

Finally, a seller and a buyer choose $q^s_{1,d} \in \mathbb{R}_+$ and $q^b_{1,d} \in \mathbb{R}_+$, respectively, which satisfy

$$
\begin{cases}
\mathbb{E}V'_{2,d^u}(m_1 + p_1 q_{1,m} + T, q^s_{1,d}) \le 1 & \text{“=” if } q^s_{1,d} > 0 \\
q^b_{1,d} = 0 & \text{if } q^s_{1,d} = 0 \\
\mathbb{E}V'_{2,d^u}(m_1 - p_1 q_{1,m} + T, -q^b_{1,d}) = \upsilon'(q^b_{1,d}) & \text{if } q^s_{1,d} > 0
\end{cases} \qquad (17')
$$

and market clearing condition ($q^b_{1,d} = q^s_{1,d} = q_{1,d}$). Now a stationary monetary equilibrium for an Indexed-UoA economy can be defined as follows.

Definition 2 *A stationary monetary equilibrium for an* Indexed-UoA *economy is a list of* $[(p_j)^3_{j=1}, (q_{1,m}, q_{1,d}, q_2, q_3), \lambda_2, m_{1,+1}]$ *that satisfies (2)-(3), (7)-(9), (13)-(14), and (17′).*

4. Inflation, Credit Trade, and Welfare

We are now ready to explore the implications of an Indexed-UoA. Notice that, as mentioned, sellers are not willing to make credit trades if $p_1 \mathbb{E}\phi < 1$ because in such a case it is cheaper for sellers to acquire money in market 3 than in market 1. The following proposition implies that such friction in credit trades can be resolved in an Indexed-UoA economy.

Proposition 1 *Suppose* $\bar{\mu} \geq \beta^* = [\beta/(1-\varepsilon^2)]$.

1. *In a* No-Indexed-UoA *economy, there is a stationary monetary equilibrium with* $q_{1,d} = q_d^*$ *if* $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, *whereas* $q_{1,d} = 0$ *if* $\bar{\mu} > \bar{\mu}^*$.

2. *In an* Indexed-UoA *economy, there is a stationary monetary equilibrium with* $q_{1,d} = q_d^*$ *regardless of* $\bar{\mu}$.

Proof. See Appendix. ■

Proposition 1 suggests that an essential role of an Indexed-UoA lies in the facilitation of credit trades against inflation. An Indexed-UoA essentially enables the terms of credit trade to be flexible, making credit trades irrelevant to inflation. More specifically, in an Indexed-UoA economy, a balance-of-credit trade is denominated in Indexed-UoA and repaid in market 3 after adjustment according to realized inflation. Hence, the friction of a credit trade attributed to inflation is effectively resolved. However, in a No-Indexed-UoA economy, a balance-of-credit trade is denominated in money and repaid in market 3 without any adjustment. This implies that if inflation is too high, the credit balance repaid in market 3 will fall short of the cost borne by a seller in market 1. Hence, sellers are not willing to accept credit trades in a high-inflation economy. The other side of this argument is that there is no point to discussing an Indexed-UoA if agents in a No-Indexed-UoA economy tend to make an inflation-contingent credit trade. However, as properly pointed out by Shiller (1997, 1999, 2002), people in the real world typically have not tended to do this.

Proposition 1 also supports the claim of Shiller (1999, 2002, 2003) that introducing an Indexed-UoA could resolve the problem caused by inflation. Also, it somewhat conforms to the view of Keynes (1923) that the role of money as a unit of account would deteriorate if its value were unstable and then an alternative unit of account would emerge. Indeed, the prediction by Keynes came to pass in the real world: for instance, during the episode of German hyperinflation in the early 1920s, prices were typically posted in terms of goldmarks

(1 goldmark=358 mg of pure gold), not circulated currency (see Wolf 2002); Brazil introduced a kind of an indexed unit of account following hyperinflation in the 1980s and early 1990s and other Latin American countries such as Chile, Colombia, Ecuador, Mexico, and Uruguay also introduced it during episodes of high inflation (Shiller 2002).

We now compare the welfare between the two types of economies where welfare is defined as the expected lifetime utility of a representative agent at the beginning of a period: i.e., welfare $\mathbb{W}$ can be expressed as

$$\mathbb{W} = \frac{1}{1-\beta}\left\{\frac{1}{2}\left[u(q_{1,m}) + \upsilon(q_{1,d}) - (q_{1,m} + q_{1,d})\right] + \frac{1}{4}\sum_{i=\{p,r\}}\left[u(q_2^i) - \bar{q}_2\right] + \left[U(q_3) - q_3\right]\right\}$$

where q_2^p (q_2^r) is the consumption of a poor (rich) buyer in market 2 and $\bar{q}_2 = (1/2)(q_2^p + q_2^r)$. The following proposition shows that as it can be inferred from Proposition 1, a No-Indexed-UoA economy is indifferent to an Indexed-UoA economy when average inflation is sufficiently low, but a No-Indexed-UoA economy is dominated by an Indexed-UoA economy when average inflation is sufficiently high.

Corollary 1 *Let $\mathbb{W}$ and $\tilde{\mathbb{W}}$ denote the welfare for a* No-Indexed-UoA *economy and an* Indexed-UoA *economy, respectively. $\mathbb{W} = \tilde{\mathbb{W}}$ for $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, whereas $\mathbb{W} < \tilde{\mathbb{W}}$ for $\bar{\mu} > \bar{\mu}^*$.*

Proof. See Appendix. ■

The intuition for the result above is straightforward. Other than $q_{1,d}$, allocations are the same between economies with and without an Indexed-UoA. Now from Proposition 1, $q_{1,d} = q_d^*$ in both a No-Indexed-UoA and an Indexed-UoA economy if average inflation lies in the range of $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$. However, if average inflation increases beyond $\bar{\mu}^*$, credit trades disappear in a No-Indexed-UoA economy, whereas $q_{1,d}$ still remains at q_d^* in an Indexed-UoA economy.

Finally, the following proposition shows that in an economy where average inflation ($\bar{\mu}$) is not overly high, an unexpected injection of money can boost aggregate market-2 consumption temporarily.

Proposition 2 *Suppose $\mu = \mu^l = \bar{\mu}(1-\varepsilon)$ is realized but the government unexpectedly injects money $\Delta = 2\varepsilon\bar{\mu}M_{-1}$ so that $M = \bar{\mu}(1+\varepsilon)M_{-1}$.*

1. *If $\bar{\mu} \in (\beta^*, \bar{\mu}^*]$, market-2 aggregate consumption strictly increases both in an* Indexed-UoA *and a* No-Indexed-UoA *economy.*

2. *An unanticipated money injection renders debtors better off but creditors worse off in a* No-Indexed-UoA *economy, whereas there no such redistributional effect in an* Indexed-UoA *economy.*

Proof. See Appendix ■

As discussed in Berentsen, Camera, and Waller (2005), Craig and Rocheteau (2006) and Molico (2006), this real effect of unexpected inflation comes from its asymmetric effect on the real balances of the poor and the rich in market 2. Notice that inflation is basically a proportional tax on money holdings. Then for the rich, the extra money received because of an unexpected lump-sum injection is not sufficient to offset the inflation-tax burden triggered by the unexpected money injection and hence, their real balances decline. However, for the poor, it is more than enough to offset the inflation-tax burden and hence their real balances increase. Now under the Friedman rule ($\bar{\mu} = \beta^*$), both the rich ant the poor are not binding in market 2 and hence unexpected inflation has no effect on market-2 consumption. If inflation is sufficiently low so that the poor are always binding, unexpected inflation increases the consumption of the poor. If inflation is too high so that even the rich are always binding, the negative effect of unexpected inflation on the rich offsets its positive effect on the poor. The second result in Proposition 2 shows that in a No-Indexed-UoA economy, unexpected inflation causes wealth redistribution at the point of clearing credit balances (i.e., market 3). This conforms with the claim of Shiller (1999) that if prices stay fixed in money terms across periods, wealth redistribution occurs in times of economic change.

5. Discussion

In the model, some assumptions for simplification such as full enforcement and no maintenance cost of an Indexed-UoA were made. We here relax such assumptions and observe what effects this has on our main result.

5.1. No Enforcement

We first remove the assumption of full enforcement on repayment. Although the government cannot force repayment of a credit balance, she can still penalize defaulters through use of record-keeping technology. That is, the government can refuse to intermediate a credit trade for a defaulter and exclude her from a credit-trade market permanently. Then the following proposition shows that if agents are sufficiently patient, the existence result of an equilibrium with credit trades in Proposition 1 will not be affected. Furthermore, more stringent condition is required to ensure voluntary repayment in a No-Indexed UoA economy as the volatility of inflation (ε) increases, whereas such condition turns out to be irrelevant to the volatility of inflation in an Indexed-UoA economy.

Proposition 3 *Suppose the government cannot force repayment.*

1. *If $\beta \in (\bar{\beta}_N, 1)$ and $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, $q_{1,d} > 0$ in a stationary monetary equilibrium of a* No-Indexed UoA *economy. The critical value $\bar{\beta}_N$ increases with ε.*

2. *If $\beta \in (\bar{\beta}_I, 1)$, $q_{1,d} > 0$ in a stationary monetary equilibrium of an* Indexed-UoA *economy regardless of $\bar{\mu}$. The critical value $\bar{\beta}_I$ is less than $\bar{\beta}_N$ and does not rely on ε.*

Proof. See Appendix. ■

Another enforcement assumed in our model is that the government can impose lump-sum taxes: i.e., $\mu^l = \bar{\mu}(1-\varepsilon)$ can be less than 1. We here discuss this assumption in connection with the results in Proposition 3. In an equilibrium with credit trades, all agents make trades with the government and hence they can be identified by the government in market 1. This suggests that the government can virtually impose taxes in market 1 because by using record-keeping technology, she can exclude an agent who fails to pay tax from a credit-trade market permanently. In an equilibrium with no credit trade, however, no agent makes trades with the government and hence no one can be identified. From Proposition 3, this equilibrium arises in a NO-INDEXED-UOA economy with $\bar{\mu} > \bar{\mu^*}$ which then implies that whether the government can run a deflation will not be a crucial issue if $\bar{\mu^*}(1-\varepsilon) \geq 1$.

5.2. Maintenance Cost of an Indexed-UoA

Our model had also assumed away any cost incurred by an INDEXED-UOA. We now suppose that there is a fixed maintenance cost for an INDEXED-UOA and the government introduces an INDEXED-UOA if there are agents who are willing to bear the cost. The following proposition suggests that adopting a medium of exchange (money) as a unit of account is most apposite for a low-inflation economy, whereas introducing an alternative INDEXED-UOA enhances welfare in an economy where inflation undermines credit trades.

Proposition 4 *Suppose there is a fixed cost θ incurred at the point of clearing a credit balance denominated in* INDEXED-UOA.

1. *If $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, no one is willing to bear the maintenance cost of an* INDEXED-UOA *and hence money is in use as a unit of account for credit trades.*

2. *If $\bar{\mu} > \bar{\mu}^*$ and $\theta < \bar{\theta}$, buyers are willing to bear θ and sellers accept credit trades*

denominated in INDEXED-UOA *only and hence the government can improve welfare by introducing an* INDEXED-UOA*.*

Proof. See Appendix. ■

Notice that in a low-inflation economy, sellers accept credit trades even if they are not denominated in an INDEXED-UOA and buyers then do not need to use a costly INDEXED-UOA for credit trades. Hence, money emerges as a unit of account for all trades in this economy. However, if inflation is too high, sellers accept credit trades denominated in an inflation-proof INDEXED-UOA but reject them denominated in money. Since credit trades make buyers better off if the relevant cost is not too high, buyers are willing to bear the cost of making credit trades. Therefore, once the government introduces an Indexed-UoA in a high-inflation economy, it will be in use as a unit of account for credit trades which eventually will improve welfare by facilitating credit trades.

Proposition 4 accounts for why some Latin-American countries introduced an INDEXED-UOA during episodes of high inflation. In addition, it seems to be the rationale for the recent discontinuation assertion of *Unidad de Fomento* in Chile based on a stabilized price. (See, for instance, Shiller 2002, pp.7-8.) That is, a continuation of an INDEXED-UOA might lead to only a deadweight loss unless inflation is so high that it discourages people from making credit trades. Proposition 4 is also somewhat consistent with Kim and Lee (2013) in which sellers are willing to post a price in terms of commodities rather than money in an inflationary economy.

6. Concluding Remarks

We have set out a simple monetary model suitable for studying an INDEXED-UOA. The results suggest that the presence of an INDEXED-UOA facilitates credit trades against inflation and hence adopting an inflation-proof INDEXED-UOA eventually could improve welfare in a high-inflation economy.

With our model as a base, different complications relevant to credit trades could be added. For example, as in Berentsen, Camera, and Waller (2007), credit trades could be introduced not in the form of a deferred-payment contract but in the form of lending and borrowing. In such a variant, due to interest payments determined endogenously in a financial market, the transaction cost of credit would be proportional to the transaction amount. But it is not believed that such a change would alter the main results qualitatively. The framework could also be extended to study the relationship between default risk and inflation by incorporating financial market frictions such as limited commitment or private information.

7. Appendix: Proofs

Proof of Proposition 1: In a No-Indexed-UoA economy, we have $p_1 = [1/\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot)] \leq (1/\mathbb{E}\phi)$ from (10) and (13). Then, if $p_1\mathbb{E}\phi = 1$, a seller's expected return from making a credit trade is just enough to compensate for the cost incurred from producing a unit of good in market 1. However, if $p_1\mathbb{E}\phi < 1$, it is cheaper for a seller to acquire money in market 3 and then she is not willing to produce goods in market 1 for a credit trade. Hence, only in the equilibrium with $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$, sellers are willing to make credit trades in market 1. Notice that from (10), $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$ if $(\lambda^r_2)_{\mu^h} = (\lambda^r_2)_{\mu^l} = 0$ where $(\lambda^r_2)_{\mu^i}$ for $i \in \{h, l\}$ is the Lagrangian multiplier for a rich buyer in market 2. Since $\phi^h\mu^h = \phi^l\mu^l$ and $\phi^l > \phi^h = \phi^l(\mu^l/\mu^h)$ in a steady state, $\phi^l(\mu^l M_{-1} - p_1 q_{1,m}) < \phi^h(\mu^h M_{-1} - p_1 q_{1,m}) < \phi^h(\mu^h M_{-1} + p_1 q_{1,m}) < \phi^l(\mu^l M_{-1} + p_1 q_{1,m})$. This implies that the candidate equilibrium with credit trades would then be the following cases: (1) $[(\lambda^r_2)_{\mu^l}, (\lambda^r_2)_{\mu^h}, (\lambda^p_2)_{\mu^h}, (\lambda^p_2)_{\mu^l}] = [0, 0, 0, 0]$, (2) $[(\lambda^r_2)_{\mu^l}, (\lambda^r_2)_{\mu^h}, (\lambda^p_2)_{\mu^h}, (\lambda^p_2)_{\mu^l}] = [0, 0, 0, +]$, and (3) $[(\lambda^r_2)_{\mu^l}, (\lambda^r_2)_{\mu^h}, (\lambda^p_2)_{\mu^h}, (\lambda^p_2)_{\mu^l}] = [0, 0, +, +]$. Now we will check each case in sequence in compliance with Berentsen, Camera, and Waller (2005).

We first consider case (1). From (13), $p_1\mathbb{E}\phi = 1$ and from (16), $V'_1(m_1) = \mathbb{E}\phi$. Then from

(3), we have $\phi_{-1}[(\beta/\bar{\pi})-1] \leq 0$ where $1/\bar{\pi} = (\mathbb{E}\phi/\phi_{-1}) = [1/\bar{\mu}(1-\varepsilon^2)]$. Now if $(\beta/\bar{\pi}) > 1$ or if $(\beta/\bar{\pi}) < 1$, there is no monetary equilibrium because $m_{1,+1} = \infty$ for the former and $m_{1,+1} = 0$ for the latter. If $(\beta/\bar{\pi}) = 1$ (i.e., $\bar{\mu} = [\beta/(1-\varepsilon^2)] = \beta^*$), there are an infinite number of monetary equilibria with $q_{1,d} = q_d^*$ and $q_{1,m} = q^*$.

We next consider case (2). Since $(\lambda_2^r)_{\mu^h} = (\lambda_2^r)_{\mu^l} = (\lambda_2^p)_{\mu^h} = 0$ in this equilibrium, $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = (q_{2,p}^b)_{\mu^h} = q^*$ and $V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) = \phi$ from (10). We then have $(1/p_1) = \mathbb{E}\phi$ from (13). As regards $q_{1,m}$, since $q_{1,m} = q_{1,m}^s = q_{1,m}^b$ in equilibrium, $2V'_1(m_1) = \mathbb{E}\phi[u'(q_{1,m}) + 1]$ from (16) and $V'_1(m_1) = (\phi_{-1}/\beta)$ from (3). These together with $(\phi_{-1}/\mathbb{E}\phi) = \bar{\pi} = \bar{\mu}(1-\varepsilon^2)$ imply that

$$u'(q_{1,m}) = \frac{2}{\beta}\left(\bar{\pi} - \beta\right) + 1 \tag{18}$$

which gives a unique value $q_{1,m}$. In addition, since $1 = p_1\mathbb{E}\phi$, (17) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$. Now, a stationary condition, $\phi^l\mu^l = \phi^h\mu^h = \phi_{-1}$, gives

$$(q_{2,p}^b)_{\mu^l} = \Phi - (\phi^l/\mathbb{E}\phi)q_{1,m} = \Phi - (1+\varepsilon)q_{1,m} \tag{19}$$

where Φ is the real balance of money ($\Phi = \phi M = \phi_{+1}M_{+1}$), $(q_{2,p}^b)_{\mu^l}$ is consumption of a poor buyer in market 2 with the realized money-supply shock μ^l. Then, from (15), we have

$$4u'(q_{1,m}) = 2(1-\varepsilon) + (1+\varepsilon)\left[u'\left(\Phi - (1+\varepsilon)q_{1,m}\right) + 1\right] \tag{20}$$

which determines a unique value of Φ for a given $q_{1,m}$ from (18). Using the solution for $(q_{1,m}, \Phi)$, we can obtain $(q_{2,p}^b)_{\mu^l}$ from (19), $\phi^l\mu^l = \Phi/M_{-1}$, $\phi^h\mu^h = \Phi/M_{-1}$, and $(1/p_1) = (1/2)(\phi^h + \phi^l)$. Finally, for this equilibrium to exist, it must be the case that $(q_{2,p}^b)_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$, $(q_{2,p}^b)_{\mu^h} = q^* \leq \Phi - (1-\varepsilon)q_{1,m}$, $(q_{2,r}^b)_{\mu^l} = q^* \leq \Phi + (1+\varepsilon)q_{1,m}$, and $(q_{2,r}^b)_{\mu^h} = q^* \leq \Phi + (1-\varepsilon)q_{1,m}$. Combining all inequalities, the sufficient condition for this equilibrium is

$$q^* + (1-\varepsilon)q_{1,m} \le \Phi < q^* + (1+\varepsilon)q_{1,m}. \tag{21}$$

Notice that as $\bar{\pi} \to \beta$, $q_{1,m} \to q^*$ from (18) and $\Phi \to q^* + (1+\varepsilon)q^*$ from (20). In addition, $(\partial\Phi/\partial q_{1,m}) > (1+\varepsilon)$ from (20). Therefore for $\bar{\pi}$ that is sufficiently close to β, the inequalities in (21) hold. However, as $\bar{\pi}$ is far away from β, the left-hand inequality in (21) will be violated, although the right-hand inequality is preserved. That is, there exists $\bar{\pi}_1 > \beta$ such that $q^* + (1-\varepsilon)q_{1,m} = \Phi$ at $\bar{\pi} = \bar{\pi}_1$ and for $\bar{\pi} > \bar{\pi}_1$, $q^* + (1-\varepsilon)q_{1,m} > \Phi$. Now let $\bar{\mu}_1$ be average inflation corresponding to $\bar{\pi}_1$, $\bar{\mu}_1 = \bar{\pi}_1/(1-\varepsilon^2)$. Then, type-(2) equilibrium with $q_{1,d} = q_d^*$ exists if $\bar{\mu} \in (\beta^*, \bar{\mu}_1]$.

We next consider case (3). Since $(\lambda_2^r)_{\mu^h} = (\lambda_2^r)_{\mu^l} = 0$ in this equilibrium, $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = q^*$ and $V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) = \phi$ from (10). We then have $(1/p_1) = \mathbb{E}\phi$ from (13). Since $2V_1'(m_1) = \mathbb{E}\phi[u'(q_{1,m}) + 1]$ from (16) and $V_1'(m_1) = (\phi_{-1}/\beta)$ from (3), the solution for $q_{1,m}$ is again given by (18). In addition, since $1 = p_1\mathbb{E}\phi$, (17) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$. Next, from (15), we have

$$4u'(q_{1,m}) = (1-\varepsilon)\left\{u'\left[\Phi - (1-\varepsilon)q_{1,m}\right] + 1\right\} + (1+\varepsilon)\left\{u'\left[\Phi - (1+\varepsilon)q_{1,m}\right] + 1\right\} \tag{22}$$

where we use $(q_{2,p}^b)_{\mu^h} = \Phi - (\phi^h/\mathbb{E}\phi)q_{1,m} = \Phi - (1-\varepsilon)q_{1,m}$ and $(q_{2,p}^b)_{\mu^l} = \Phi - (\phi^l/\mathbb{E}\phi)q_{1,m} = \Phi - (1+\varepsilon)q_{1,m}$. Then (22) determines a unique value of Φ for a given $q_{1,m}$ from (18). Using the solutions for $(q_{1,m}, \Phi)$, we can obtain $[(q_{2,p}^b)_{\mu^h}, (q_{2,p}^b)_{\mu^l}]$, $\phi^l\mu^l = \Phi/M_{-1}$, $\phi^h\mu^h = \Phi/M_{-1}$, and $(1/p_1) = (1/2)(\phi^h + \phi^l)$. Finally, for this equilibrium to exist, it must be the case that $(q_{2,p}^b)_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$, $(q_{2,p}^b)_{\mu^h} = \Phi - (1-\varepsilon)q_{1,m} < q^*$, $(q_{2,r}^b)_{\mu^l} = q^* \le \Phi + (1+\varepsilon)q_{1,m}$, and $(q_{2,r}^b)_{\mu^h} = q^* \le \Phi + (1-\varepsilon)q_{1,m}$. Combining all inequalities, the sufficient condition for this equilibrium is

$$q^* - (1-\varepsilon)q_{1,m} \le \Phi < q^* + (1-\varepsilon)q_{1,m}. \tag{23}$$

Notice that at $\bar{\pi} = \bar{\pi}_1$, $q^* - (1-\varepsilon)q_{1,m} < \Phi = q^* + (1-\varepsilon)q_{1,m}$. As $\bar{\pi}$ increases above $\bar{\pi}_1$, $q_{1,m}$

decreases from (18) and $(\partial\Phi/\partial q_{1,m}) > (1-\varepsilon)$ from (22). Hence, the right-hand inequality is preserved for $\bar{\pi} > \bar{\pi}_1$ but the left-hand inequality binds. That is, there exists $\bar{\pi}^* > \bar{\pi}_1$ such that $q^* - (1-\varepsilon)q_{1,m} = \Phi$ at $\bar{\pi} = \bar{\pi}^*$ and for $\bar{\pi} > \bar{\pi}^*$, $q^* - (1-\varepsilon)q_{1,m} > \Phi$. Let $\bar{\mu}^*$ be average inflation corresponding to $\bar{\pi}^*$, $\bar{\mu}^* = \bar{\pi}^*/(1-\varepsilon^2)$. Then, type-(3) equilibrium with $q_{1,d} = q_d^*$ exists if $\bar{\mu} \in (\bar{\mu}_1, \bar{\mu}^*]$.

Now, if there exists a monetary equilibrium for $\bar{\mu} > \bar{\mu}^*$, it will be either (4) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,+,+,+]$ or (5) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [+,+,+,+]$. In any case, $\mathbb{E}\phi p_1 < 1$ because $\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) > \mathbb{E}\phi$. Then it is cheaper for a seller to acquire money in market 3 and hence she is not willing to accept a credit trade.

Finally, in order to characterize an equilibrium for an Indexed-UoA economy, it suffices to identify $q_{1,d}$ with an Indexed-UoA because except for it, an Indexed-UoA and a No-Indexed-UoA economy are identical. Noting that $\mathbb{E}V'_{2,d^u} = 1$ from (11) and (4′), (17′) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$ regardless of $\bar{\mu}$ which completes the proof.

Proof of Corollary 1: As mentioned above, other than $q_{1,d}$, consumption in markets 2 and 3 is the same between economies with and without an Indexed-UoA. Then $q_{1,d} = q_d^*$ both in a No-Indexed-UoA and an Indexed-UoA economy for $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$ implies the first claim, $\mathbb{W} = \tilde{\mathbb{W}}$ if $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$. In addition, for $\bar{\mu} > \bar{\mu}^*$, $q_{1,d} = 0$ in a No-Indexed-UoA economy but $q_{1,d} = q_d^*$ in an Indexed-UoA economy implies the second claim, $\mathbb{W} < \tilde{\mathbb{W}}$, if $\bar{\mu} > \bar{\mu}^*$.

Proof of Proposition 2.1: Conditional on the realization of μ^l, suppose there is a surprise injection of money Δ so that M increases to $\bar{\mu}(1+\varepsilon)M_{-1}$ from $\bar{\mu}(1-\varepsilon)M_{-1}$: i.e., $\Delta = 2\varepsilon\bar{\mu}M_{-1}$. Since it is an unanticipated change at the beginning of market 2, $p_1 = (1/\mathbb{E}\phi) = \bar{\mu}(1-\varepsilon^2)/\phi_{-1}$ and (18) still determines $q_{1,m}$. Notice that there is no difference in market-2 consumption between economies with and without an Indexed-UoA. Then from Proposition 1, we can consider the following five types of monetary equilibria: (1) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,0,0,0]$, (2) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,0,0,+]$, (3) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] =$

$[0, 0, +, +]$, (4) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0, +, +, +]$, and (5) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [+, +, +, +]$.

In case (1), $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = (q_{2,p}^b)_{\mu^h} = (q_{2,p}^b)_{\mu^l} = q^*$. Then $[(q_{2,r}^b)_{\mu^h} + (q_{2,p}^b)_{\mu^h}] = 2q^* = [(q_{2,r}^b)_{\mu^l} + (q_{2,p}^b)_{\mu^l}]$ implies that an aggregate consumption in market 2 is unaffected by a surprise injection Δ. In case (2), $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = (q_{2,p}^b)_{\mu^h} = q^*$. In addition, since $\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) = \mathbb{E}\phi$, $(q_{2,p}^b)_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$. Then $[(q_{2,r}^b)_{\mu^h} + (q_{2,p}^b)_{\mu^h}] = 2q^* > [(q_{2,r}^b)_{\mu^l} + (q_{2,p}^b)_{\mu^l}]$ implies that a surprise injection Δ increases aggregate consumption in market 2. In case (3), $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = q^*$. In addition, since $\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) = \mathbb{E}\phi$, $(q_{2,p}^b)_{\mu^h} = \Phi - (1-\varepsilon)q_{1,m}$ and $(q_{2,p}^b)_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m}$. Then $[(q_{2,r}^b)_{\mu^h} + (q_{2,p}^b)_{\mu^h}] = q^* + \Phi - (1-\varepsilon)q_{1,m} > q^* + \Phi - (1+\varepsilon)q_{1,m} = [(q_{2,r}^b)_{\mu^l} + (q_{2,p}^b)_{\mu^l}]$ implies again that a surprise injection Δ boosts aggregate consumption in market 2. In case (4), $(q_{2,r}^b)_{\mu^l} = q^*$, $(q_{2,r}^b)_{\mu^h} = \Phi + \phi^h p_1 q_{1,m}$, $(q_{2,p}^b)_{\mu^l} = \Phi - \phi^l p_1 q_{1,m}$, and $(q_{2,p}^b)_{\mu^h} = \Phi - \phi^h p_1 q_{1,m}$. Notice that $(q_{2,r}^b)_{\mu^l} = q^* \leq \Phi + \phi^l p_1 q_{1,m}$ and hence $[(q_{2,r}^b)_{\mu^h} + (q_{2,p}^b)_{\mu^h}] = 2\Phi = [\Phi + \phi^l p_1 q_{1,m} + \Phi - \phi^l p_1 q_{1,m}] \geq q^* + \Phi - \phi^l p_1 q_{1,m} = [(q_{2,r}^b)_{\mu^l} + (q_{2,p}^b)_{\mu^l}]$. Therefore, in this type of an equilibrium, aggregate consumption in market 2 is weakly increasing in response to a surprise injection Δ. Finally, in case (5), $(q_{2,r}^b)_{\mu^l} = \Phi + \phi^l p_1 q_{1,m}$, $(q_{2,r}^b)_{\mu^h} = \Phi + \phi^h p_1 q_{1,m}$, $(q_{2,p}^b)_{\mu^l} = \Phi - \phi^l p_1 q_{1,m}$, and $(q_{2,p}^b)_{\mu^h} = \Phi - \phi^h p_1 q_{1,m}$. Then $[(q_{2,r}^b)_{\mu^h} + (q_{2,p}^b)_{\mu^h}] = 2\Phi = [(q_{2,r}^b)_{\mu^l} + (q_{2,p}^b)_{\mu^l}]$ implies that an aggregate consumption in market 2 is unaffected by a surprise injection Δ. In summary, in cases of (2)-(3) with the realized money growth shock is μ^l, market-2 aggregate consumption is strictly increasing in response to a surprise injection of money and such equilibria exist if $(\beta^*, \bar{\mu}^*]$.

Proof of Proposition 2.2: In a No-Indexed-UoA economy, the nominal balance-of-credit trade is fixed at $p_1 q_{1,d} = q_{1,d}/[\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, \cdot)]$ and is not adjusted in response to a change in the money stock. Hence, the real value of the credit-trade balance redeemed to a creditor when $\mu^i = \mu^l$ is $p_1 q_{1,d}\phi^l$, whereas that for $\mu^i = \mu^h$ is $p_1 q_{1,d}\phi^h$. Then $\phi^l > \phi^h$ immediately implies that a surprise injection Δ in a No-Indexed-UoA economy renders creditors worse off but debtors better off. However, in an INDEXED-UOA economy, the balance-of-credit trade in market 1 is recorded as $q_{1,d}$ units of INDEXED-UOA which is redeemed in market 3 by $q_{1,d}(1/\phi^i)\phi^i = q_{1,d}$ units of a market-3 good where $p_3 = 1/\phi^i$ for $i \in \{h, l\}$. Therefore, the real value of a credit-trade balance is irrelevant to a surprise injection Δ.

Proof of Proposition 3: We first consider a NO-INDEXED-UOA economy. Notice that for $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, $p_1\mathbb{E}\phi = 1$, as shown in Proposition 1. In order to be $q_{1,d} > 0$ without enforcement, debtors are willing to repay voluntarily rather than default. The benefit of defaulting is

$$p_1 q_{1,d}\phi^i = \begin{cases} (1+\varepsilon)q_d^* & \text{if } i = l \\ (1-\varepsilon)q_d^* & \text{if } i = h \end{cases}$$

where we use $p_1\mathbb{E}\phi = 1$, and for the case $q_{1,d} = q_d^*$. The cost is that she cannot participate in a credit-trade market for the rest of her life which can be expressed as

$$\left[\frac{\beta}{2(1-\beta)}\right]\left[\frac{\upsilon(q_d^*)}{q_d^*} - 1\right] q_d^*. \tag{24}$$

Since the cost of defaulting does not rely on the currently realized ϕ^i and the benefit of it with $\phi^i = \phi^l$ is greater than that with $\phi^i = \phi^h$, the sufficiency condition for voluntary repayment is

$$\left[\frac{\upsilon(q_d^*)}{q_d^*} - 1\right] > \frac{2(1+\varepsilon)(1-\beta)}{\beta}. \tag{25}$$

Since $\upsilon(q_d^*) > q_d^*$ due to $\upsilon(0) = 0$ and a sufficiently large $\upsilon'(0)$, $[(\upsilon(q_d^*)/q_d^*) - 1] > 0$. Now let $g(\beta, \varepsilon) = [2(1+\varepsilon)(1-\beta)/\beta]$. Then $g(1, \varepsilon) = 0$ and there is a unique $\bar{\beta}_N \in (0, 1)$ such that

$$\left[\frac{\upsilon(q_d^*)}{q_d^*} - 1\right] - \left[\frac{2(1+\varepsilon)(1-\bar{\beta}_N)}{\bar{\beta}_N}\right] = 0 \tag{26}$$

because $[\partial g(\beta, \varepsilon)/\partial\beta] < 0$. Therefore, (25) is satisfied for $\beta \in (\bar{\beta}_N, 1)$ which implies that in a No-Indexed-UoA economy, a debtor is willing to repay her credit balance rather than default if $\beta \in (\bar{\beta}_N, 1)$. In addition, it is straightforward to show $[\partial\bar{\beta}_N/\partial\varepsilon] > 0$ from (26).

In an Indexed-UoA economy, the benefit of defaulting is q_d^*, whereas the cost of it is the same as that in a No-Indexed-UoA economy represented by (24). Hence, the condition for voluntary repayment is

$$\left[\frac{\upsilon(q_d^*)}{q_d^*} - 1\right] > \frac{2(1-\beta)}{\beta}. \tag{27}$$

Let $h(\beta) = [2(1-\beta)/\beta]$. Then $h(1) = 0$ and there is a unique $\bar{\beta}_I \in (0,1)$ such that

$$\left[\frac{\upsilon(q_d^*)}{q_d^*} - 1\right] - \left[\frac{2(1-\bar{\beta}_I)}{\bar{\beta}_I}\right] = 0 \tag{28}$$

because $[\partial h(\beta)/\partial\beta] < 0$. Therefore, (27) is satisfied for $\beta \in (\bar{\beta}_I, 1)$ which implies that in an Indexed-UoA economy, a debtor is willing to repay her credit balance rather than default if $\beta \in (\bar{\beta}_N, 1)$. Notice that from (28), $\bar{\beta}_I$ does not rely on ε. Finally, from (26) and (28), we have $[(1+\varepsilon)(1-\bar{\beta}_N)/\bar{\beta}_N] = [(1-\bar{\beta}_I)/\bar{\beta}_I]$ which immediately implies $\bar{\beta}_N > \bar{\beta}_I$ for $\varepsilon > 0$.

Proof of Proposition 4: Notice that the first claim is an obvious consequence of Proposition 1. As regards the second claim, as shown in Proposition 1, a seller in an economy with $\bar{\mu} > \bar{\mu}^*$ is not willing to accept a credit trade denominated in money. But if it is denominated in Indexed-UoA, for a seller, an expected gain from a credit trade is just enough to compensate for the relevant cost. Hence, if there is no extra burden incurred from accepting a credit trade denominated in Indexed-UoA, a seller will make a credit trade. For a buyer, the gain from a credit trade is $\upsilon(q_d^*) - q_d^*$ which is strictly positive due to $\upsilon(0) = 0$ and a sufficiently large $\upsilon'(0)$. Hence, if the fixed maintenance cost θ is less than $\bar{\theta} \equiv \upsilon(q_d^*) - q_d^*$, she is always willing to bear θ to make a credit trade.

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