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Abstract

This paper offers a model in which there is direct competition between different groups. We deliberately endow an environment with many employers and workers in which opportunities are limited such that each employer is randomly matched with two workers from the entire worker population, which consists of two *ex ante* identical sub-groups, and selects at most one of them. We show that with the competition, a set of feasible equilibria has a conflict structure unlike the conflict-free structure found in typical statistical discrimination models, and that we can find employers' strategy such that employers benefit from discrimination, and this strategy can be sustained as a collusion between employers and an advantaged group in a repeated game.

Keywords and Phrases: Statistical discrimination, Group inequality, Asymmetric information

JEL Classification Numbers: D63, D82, J71

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1 Introduction

The literature on discrimination, a subject first formally studied by Becker (1971) in the field of economics, suggests three causes for economic discrimination (see Cain (1986), and for a broader survey, England (1992)).[1] First, economic discrimination is driven by demand-side traits such as employers' or co-workers' tastes. Second, economic discrimination stems from supply-side traits such as different turnover rates for men and women. Third, economic discrimination arises from self-fulfilling beliefs, called statistical discrimination.[2] What is surprising regarding statistical discrimination following the influential works by Arrow (1972) and Phelps (1972) is that even after *controlling* for all the exogenous variables, for example, those listed in the first two explanations, discrimination may still emerge.

In the typical statistical discrimination models, given two *ex ante* identical groups, multiple equilibria are derived from the relationship between an employer and one group of workers such as a "good" equilibrium **G** with more qualified workers in the group and a "bad" equilibrium **B** with fewer qualified workers. Since the two identical groups have no interaction, this generates a *conflict-free structure*; the set of feasible equilibria is given as {(**B**, **B**), (**B**, **G**), (**G**, **B**), (**G**, **G**)}. It follows that the discrimination allocation (**B**, **G**) is not Pareto optimal because with (**G**, **G**), workers in the disadvantaged group will be better off, and employers will obtain higher payoffs. However, in reality, different groups often compete to be employed in a fixed number of positions, so most conflicts between them occur under this type of environments.[3]

[1] Our classification is not exactly the same as either Cain (1986) or England (1992).

[2] Statistical discrimination itself can be divided into two groups: one classification originates from Arrow (1972) and Arrow (1973), and the other Phelps (1972).

[3] The most notable recent case is the lawsuit against the law school of the University of Michigan (*New York Times*, May 11, 1999).

We also re-quote Ross (1948) from Cain (1986), which excellently illustrates this type of case:

"The depression of 1921 put many Negro and white workers on the street. There was violent competition to keep or grab places on any pay rolls. In 1921 there began a series of shootings

This paper offers a model in which there is direct competition between different groups. We deliberately endow an environment with many employers and workers in which opportunities are limited. Each employer is randomly matched with two workers from the entire worker population, which consists of two *ex ante* identical sub-groups, and selects at most one of them. If a worker becomes qualified, he signals stronger (discrete) test results. After observing the test results, the employer decides to choose *at most* one worker for a position. In particular, each employer chooses a worker from group i when (i) the worker's signal is stronger than a standard, and (ii) group i's probability of being qualified is greater than the other's.[4] This leads to strategic interaction between the two group members.

The main results of this paper fall into two categories. In a one-stage game, we provide a sharp characterization of relationships between symmetric and asymmetric equilibria. First, with the competition, a set of feasible equilibria has an *inter-group conflict structure* with a low (high) symmetric equilibrium **LS** (**HS**) and modest (extreme) asymmetric equilibria **MA** (**EA**). In a symmetric equilibrium, both groups have the same qualification level, and **HS**'s level is higher than **LS's**, whereas in an asymmetric equilibrium, the two groups acquire different qualification levels, and **EA** is the one with a "wider" gap than **MA**. Comparing asymmetric equilibria with symmetric equilibria shows that in order for one group's qualification level to increase, the other group's level must decrease in equilibrium, which is not required in the typical models, and thus a move from an asymmetric equilibrium to a symmetric equilibrium is *not* a Pareto improvement. Geometrically, a set of feasible equilibria

from ambush at Negro firemen on Southern trains. Five were killed and eight wounded.... [In] the depression year of 1931... a Negro fireman, Clive Sims, was wounded on duty by a shot fired out of the dark beyond the track, the first of fourteen such attacks which stretched out over the next twelve months. This was not a racial outbreak in hot blood. It was a cold calculated effort to create vacancies for white firemen in the surest way possible, death, and, by stretching out the period of uncertainty and horror, to frighten away the others" (pp. 119-120).

[4] In the typical statistical discrimination models, only the former condition (i) is imposed.

changes from the square form to a skewed triangle form (see Figure 2).[5] In addition, we show that depending on different *output/wage ratio* values, either **HS** and **MA** can be a unique set of equilibria or **LS** and **EA** can be a *unique* set of equilibria. The intuition for these results is simple: if one group is more qualified, a worker from that group can send stronger signals, and given the same signal, he can have a higher chance to be employed, which negatively affects the other group.

Second, because of this feasible set with the conflict structure, we can find employers' strategy such that employers benefit from discrimination. Hence, this strategy can be sustained as a collusion between employers and an advantaged group in a repeated game framework. To show this, we consider two extreme stationary strategies of employers, which are related to *adverse selection* and *uncertainty* problems they face, respectively. Solving both adverse selection and uncertainty problems provides an incentive for employers to choose a certain level of discriminatory action between two extremes. Therefore, discrimination can arise even after controlling for all the exogenous variables.[6] This could explain why employers may prefer "one good group and one bad" to "two equally mediocre groups" when the output/wage ratio is low, and a certain degree of *specialization* to equalization in order to diminish the uncertainty problem under asymmetric information between them and workers.

Mailath, Samuelson and Shaked (2000) and Moro and Norman (2004) introduce interaction between groups through *externality*. In the former, with a search ap-

[5] Each number in the figure indicates the number of the Proposition that derives the relevant relationship.

[6] If the output/wage ratio is interpreted as a proxy for a country's *economic development*, the above findings from the repeated game have an interesting implication for the relationship between group inequality and economic development. When a country is in the stage of underdevelopment (low output/wage ratios), employers and an advantaged group have an incentive to collude, and the resulting inequality can benefit firms, so it can work as a driving force for development. After the country enters the developed stage (high output/wage ratios), however, group inequality is no longer optimal for firms, but when the advantaged group has more power, the inequality will be continued, and thus will negatively affect development (see Galor and Moav (2004) for a unified view on the dynamic relationship between *income* inequality and development).

proach, one group's search benefit depends on the other group's qualification level, and in the latter, with a general equilibrium model, one group's marginal product depends on the other group's. Neither study addresses the strategic interplay between workers from different groups under direct competition. Furthermore, no previous paper provides a characterization of the set of equilibria and the possibility that employers benefit from discrimination, so the collusion between them and one group can lead to discrimination. Lang, Manove and Dickens (2005) feature multiple job applicants, but no "competing procedure" for the hiring.[7]

The information structure of this paper borrows from Coate and Loury (1993), and the collusion in the repeated game has the same *flavor* as the one in Fudenberg, Kreps and Maskin (1990) and in Kreps (1990). Recent papers on discrimination in economics (Blume (2005), Fryer (2007) and Chaudhauri and Sethi (2008)) study the dynamic effect or peer effect on statistical discrimination.[8]

We start by introducing a one-stage game and analyzing it in Section 2. Section 3 contains the main analysis in a repeated game. Concluding remarks are in Section 4, and all proofs are collected in the appendix.

2 Model: a one-stage game

Consider a market in which there are many identical employers and workers.[9] Workers belong to one of two distinct groups, A and B, and the population share of group A is $\lambda_A \in (0,1)$. Each worker from group $i \in \{A, B\}$ decides whether to make a human capital investment to become qualified. The group identity is publicly ob-

[7] In their paper, if the employer receives more than one application, he chooses to hire one applicant at random.

[8] The latest works in sociology (see Pager and Shepherd (2008) for a survey) focus on how to measure discrimination.

[9] The employers can be managers, judges, or admissions officers, and the workers candidates, competitors, or applicants, respectively. Hence, the selection decision can be broadly interpreted as a decision on employment, competition, or admission.

servable with zero cost, but each worker's qualifications are known only to that worker.[10] The investment cost c_i of each worker is drawn from a continuous CDF F which has a density $f > 0$ and a support $[\underline{c}, \overline{c}]$ with $0 \leq \underline{c} < \overline{c}$ for both groups.[11]

If a worker becomes qualified, he signals his qualifications through a test with two signs $\{H, M\}$, H-excellent with probability $(1 - q)$, and M-mediocre with q, whereas if a worker becomes *un*qualified, he signals with two signs $\{M, L\}$, M-mediocre with u, and L-poor with $(1 - u)$.[12] By assuming $q > u$ and $q, u \in (0, 1)$, the signaling distribution when a worker becomes qualified not only stochastic dominates the one when unqualified but also has a greater probability mass than the other given signal M.[13]

Each employer is randomly matched with two workers from the whole population, and after observing their test results, the employer decides to select at most one worker for a position. Each employer gains a return $x > 0$ if a worker is qualified, 0 otherwise, and pays a reward $v \in (0, x)$, which is fixed as in Coate and Loury (1993) and Blume (2005), for a selected worker.[14] We call x/v the *output/wage*

[10] Some observable group memberships are consciously chosen, but some are given; each worker may choose a university, an education level, and a religion, but *nature* dictates race, sex, region, or country of birth. In addition, different countries have different major issues related to group inequality, for example, race in the United States, and region in South Korea.

[11] We need $f > 0$ for Lemma 2 in the repeated game.

[12] Various types of simplified signaling structures like this one are adopted by Blume (2005), Fryer (2007) and Chaudhauri and Sethi (2008). Ours is especially similar to the one in Fryer (2007). Even when test scores are continuous, for evaluation, we often classify them into discrete measures; for example, typical grades at universities, and qualifying examinations in doctoral programs. The more discrete signals, the more "layers" we have. Hence, with 3 signals, there are two distinct sets of asymmetric equilibria as in subsection 2.4, but with more signals, there will be more distinct sets of asymmetric equilibria. Technically, with a continuous signaling, it is hard to identify *asymmetric* equilibria with this type of interaction although it is easy to find symmetric ones.

[13] This assumption should not be seen as strong, because otherwise we cannot derive the first result in Lemma 4, which is quite intuitive: β is an increasing function of k_i.

[14] According to Petersen and Saporta (2004), within-job *wage* discrimination is least prevalent

ratio, denoted by $r \equiv x/v$. Hence, the selected obtains the gross benefit v, and the non-selected obtains the normalized gross benefit 0.

Let $\Theta \equiv \{H, M, L\}^2$ and $\theta \in \Theta$. Each worker's strategy is a mapping $Q_i : [\underline{c}, \overline{c}] \to \{0, 1\}$ where 1 denotes qualified, and each employer's strategy is a mapping $E : \Theta \to \{i, j, \phi\}$. The payoff of each worker $i \in \{A, B\}$ when selected is given as

$$u_i \equiv v - c_i q_i,$$

The payoff of each employer from hiring a worker from group i is

$$u_E \equiv x q_i - v.$$

2.1 Two types of beliefs

Since each worker's *type* c_i is not included in the benefit part, and his decision is binary, the optimal strategy of each worker is a "cutoff strategy." That is, there exists $k \in [\underline{c}, \overline{c}]$ such that a worker becomes qualified if $c_i < k$ but unqualified if $c_i > k$. From the specified signaling structure, it is clear that given signal H, a worker is qualified (with probability 1), and given signal L, a worker is unqualified. Then, we can focus on the case with signal M, the mediocre sign. We denote by $\mu : [\underline{c}, \overline{c}] \to [0, 1]$ each employer's posterior probability that a worker from group i is qualified given signal M and the employer's belief about group i's cutoff:

$$\mu(k_i) \equiv \begin{cases} 1/\left(1 + (u/q)\pi(k_i)\right) & \text{if} \quad k_i \in (\underline{c}, \overline{c}], \\ 0 & \text{if} \quad k_i = \underline{c}, \end{cases}$$

where $\pi : (\underline{c}, \overline{c}] \to \mathbb{R}_+$ is given as

$$\pi(k_i) \equiv \frac{1 - F(k_i)}{F(k_i)}.$$

Define a *group standard* k_s such that $\mu(k_s)x - v = 0$, which implies $k_s \in (\underline{c}, \overline{c})$. Since μ is strictly increasing, for $k_i \geq k_s$, an employer's expected net benefit from

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and least important since it is illegal and easy to document.

choosing one from group i with signal M is positive, so given signal M, it is optimal for each employer to select a worker from group i only when $k_i \geq k_s$.

Suppose that an employer believes that $k_i \geq k_s$ and $k_j \geq k_s$. Then, if the employer is matched with one from group i and the other from group j, the sequentially rational strategy is to choose a worker from the group that is more likely to have qualified workers. Thus, the probability that an employer chooses a worker from group i given his belief about two groups' cutoffs, (k_i, k_j), can be expressed as the function $\varphi : [\underline{c}, \overline{c}]^2 \rightarrow [0, 1]$,

$$\varphi(k_i, k_j) = \begin{cases} 1 & \text{if} \quad k_i > k_j, \\ 1/2 & \text{if} \quad k_i = k_j, \\ 0 & \text{if} \quad k_i < k_j. \end{cases}$$

This model is different from those in papers without direct competition between groups in that it must also examine *one group's beliefs about the other's qualifications*. $P_S(k_j)$ denotes the probability that a worker of group j emits signal S given group i's belief about group j's cutoff k_j. For each S, $P_S(k_j)$ can be derived as follows:

$$\begin{aligned} P_H(k_j) &\equiv F(k_j)(1-q), \\ P_M(k_j) &\equiv [F(k_j)q + (1-F(k_j))u], \\ P_L(k_j) &\equiv (1-F(k_j))(1-u). \end{aligned}$$

2.2 Equilibrium

Suppose that an employer is matched with members from two *different* groups. Then, he receives 9 possible combinations of signals, $\{H, M, L\} \times \{H, M, L\}$, from two workers before making the selection decision. Hence, if a worker of group i becomes qualified, the *increase* in the probability that group i's worker is selected when qualified given the belief (k_i, k_j) can be written as the function $\beta : [\underline{c}, \overline{c}]^2 \rightarrow$

$[0,1]$,

$$\beta(k_i,k_j) = (1-q)\left[P_H(k_j)\frac{1}{2}+P_M(k_j)+P_L(k_j)\right]+(q-u)\mathbf{1}_{\{k_i\geq k_s\}}\left[P_M(k_j)\varphi(k_i,k_j)+P_L(k_j)\right]. \tag{1}$$

The first term is the probability that group i's worker is selected when he emits signal H, and the second term is the increase in the probability when he emits signal M.[15] One key feature of β function is that if group i's standard k_i is below a group standard k_s, the second term will disappear, so the effect from the group comparison, $\varphi(k_i,k_j)$, does not have any role at all. Since each employer is randomly matched with two workers from the whole population, every worker of group i has λ_i chance to compete with a worker of the same group and $1-\lambda_i$ chance to compete with a worker of the other group. Hence, we define $G_i(k_i,k_j)v$ as *the group i worker's incentive to become qualified*, where

$$G_i(k_i,k_j) \equiv [\lambda_i\beta(k_i,k_i)+(1-\lambda_i)\beta(k_i,k_j)]. \tag{2}$$

We assume a class of v, $\underline{c}$ and $\bar{c}$ to focus on interior equilibria. The agent with the lowest cost, $\underline{c}$, in each group is the one whose cost is so low relative to v that it is optimal to become qualified even if the employer has the "worst belief" about the group to which he belongs; the employer believes that no one in the group is qualified and that all in the other group are qualified.

$$G_i(\underline{c},\bar{c})v > \underline{c}. \tag{3}$$

If $\underline{c}=0$, this condition, obviously, is always satisfied.[16] On the other hand, the agent with the highest cost, $\bar{c}$, in each group is the one whose cost is so high relative to v that it is optimal to become *un*qualified even if the employer has the "best belief"

[15] When qualified (unqualified), group i's worker emits M with the probability q (u), and given $P_M(k_j)$ the probability that group j's worker emits M, group i's worker is selected if $k_i \geq k_s$ and $k_i > k_j$ (with probability 1/2 if $k_i = k_j$). β is derived as the difference between the probability that a qualified member of group i is selected and the probability that an unqualified member of group i is selected, which can be found from the proof of Proposition 5.

[16] One can check $G_i(\underline{c},\bar{c}) = \lambda_i(1-q)+\frac{(1-\lambda_i)}{2}\left(1-q^2\right)$.

for the group to which he belongs; the employer believes that all in the group are qualified and that none in the other group is qualified.[17]

$$G_i(\overline{c}, \underline{c})v < \overline{c}. \tag{4}$$

Then, an equilibrium[18] is defined as a combination $(k_A^*, k_B^*) \in [\underline{c}, \overline{c}]^2$ such that for each $i \in \{A, B\}$,

$$G_i(k_i^*, k_j^*)v = k_i^*.$$

We examine the existence of a symmetric equilibrium, defined as (k_A^*, k_B^*) with $k_A^* = k_B^*$, and that of an asymmetric equilibrium, defined as (k_A^*, k_B^*) with $k_A^* \neq k_B^*$, in the following subsections.

2.3 Symmetric equilibrium

A symmetric equilibrium is defined as $k^* \in [\underline{c}, \overline{c}]$ such that

$$G_i\left(k^*, k^*\right)v = \beta\left(k^*, k^*\right)v = k^* \text{ for } i \in \{A, B\}.$$

We define functions $\beta_l : [\underline{c}, \overline{c}] \to [0, 1]$ and $\beta_h : [\underline{c}, \overline{c}] \to [0, 1]$ such that

$$\beta_l\left(k\right) \equiv (1 - q)\left[P_H\left(k\right)\frac{1}{2} + P_M(k) + P_L\left(k\right)\right], \tag{5}$$

[17] This restriction does not necessarily imply that for any $(k_i, k_j) \in [\underline{c}, \overline{c}]^2$,

$$\underline{c} < G_i\left(k_i, k_j\right)v < \overline{c},$$

since $\beta\left(k_i, k_i\right)$ is strictly decreasing for $k_i < k_s$, so $\beta(\underline{c}, \overline{c})$ is not the minimum of $\beta\left(k_i, k_i\right)$, and similarly, $\beta(\overline{c}, \underline{c})$ is not the maximum of $\beta\left(k_i, k_i\right)$.

[18] If an investment cost c is interpreted as a *type*, it is the same as a perfect *Bayesian* equilibrium. Formally, Q_A^*, Q_B^* and E^* with the belief μ is a perfect Bayesian equilibrium if for each $c_i \in [\underline{c}, \overline{c}]$ of every group $i \in \{A, B\}$,

$$Q_i^*\left(c_i\right) = \arg\max_{q_i \in \{0,1\}} U_i(q_i, E^*, Q_j^*, c_i)$$

and for each $\theta \in \Theta$,

$$E^*\left(\theta\right) = \arg\max_{e \in \{i,j,\phi\}} U_E(e, Q_A^*, Q_B^*, \theta),$$

where U_i and U_E are the expected payoff for the worker from group i and the payoff for the firm, respectively.

and

$$\beta_h(k) \equiv (1-q)\left[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)\right] + (q-u)[P_M(k)\frac{1}{2} + P_L(k)]. \quad (6)$$

β_l is β when k is below the standard k_s and β_h is β when k is above the standard k_s with $\varphi(k_i, k_j) = 1/2$. Note that for each k, $\beta_h(k) > \beta_l(k)$, and both are continuous and strictly decreasing functions of k. Then, $\beta(k,k)$ can be rewritten as

$$\beta(k,k) \equiv \begin{cases} \beta_l(k) & \text{if } k < k_s, \\ \beta_h(k) & \text{if } k \geq k_s. \end{cases} \quad (7)$$

There are three possible equilibrium scenarios depending on the value of k_s, which is determined by the employer's output/wage ratio r.

Proposition 1 *There exist k_l, $k_h \in (\underline{c}, \overline{c})$ with $k_h > k_l$ such that*

(i) if $k_s \leq k_l$, there is a unique symmetric equilibrium, **HS** $= (k_h, k_h)$*,*

(ii) if $k_l < k_s \leq k_h$, there are multiple symmetric equilibria **LS** $= (k_l, k_l)$ *and* **HS***,*

(iii) if $k_s > k_h$, there is a unique symmetric equilibria **LS***.*

If the return x from selecting a qualified worker is high enough relative to the value v, so that the standard k_s is sufficiently low, **HS** is the only symmetric equilibrium; if the return x is low enough relative to the value v, so that the standard k_s is sufficiently high, **LS** is the only symmetric equilibrium; and if the return x is in a middle range relative to the value v, then there are multiple equilibria. When F is uniform, this is described in Figure 1.

Although **HS** has a higher qualification level for both groups compared with **LS**, unlike the typical statistical discrimination models, each group's higher qualification level does *not* necessarily mean that group's higher welfare, as a result of competition between it and the other group that also has a higher qualification level. In addition, it will be shown in Proposition 4 that a move from **LS** to **HS** is not feasible given three regimes of output/wage ratios.

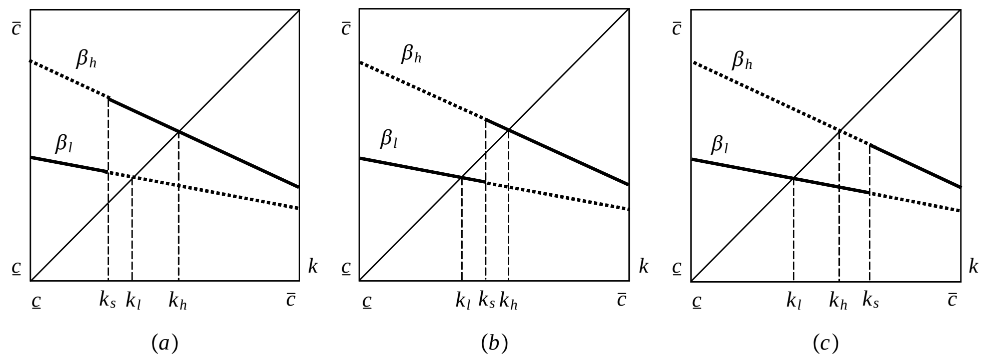

Figure 1: Symmetric equilibria

2.4 Asymmetric equilibrium

Characterizing an asymmetric equilibrium might be a little bit more complicated but can have a richer structure. For an asymmetric equilibrium, without loss of generality, we examine the case where group i's equilibrium cutoff is greater than group j's, that is, $k_i^* > k_j^*$. We define functions $G_{id} : [\underline{c}, \bar{c}]^2 \rightarrow [0,1]$ and $G_{iu} : [\underline{c}, \bar{c}]^2 \rightarrow [0,1]$ such that

$$G_{id}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i)\beta_d(k_j), \tag{8}$$

where

$$\beta_d(k) \equiv (1-q)\left[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)\right] + (q-u)P_L(k),$$

and

$$G_{iu}(k_i, k_j) \equiv \lambda_i \beta_h(k_i) + (1-\lambda_i)\beta_u(k_j), \tag{9}$$

where

$$\beta_u(k) \equiv (1-q)\left[P_H(k)\frac{1}{2} + P_M(k) + P_L(k)\right] + (q-u)[P_M(k) + P_L(k)].$$

β_d is β when k_i is above the standard k_s and $\varphi(\cdot) = 0$ and β_u is β when k_i is above the standard k_s and $\varphi(\cdot) = 1$. Hence, $G_{iu}(k_i, k_j)v$ is the group i worker's incentive to become qualified when k_i is above the standard k_s and $\varphi(\cdot) = 1$ and $G_{id}(k_i, k_j)v$

is the group i worker's incentive when k_i is above the standard k_s and $\varphi(\cdot) = 0$. Note that both β_d and β_u are continuous and strictly decreasing functions of k. Compared with β_l and β_h in the previous subsection, the following relationship is useful to prove some of the results in this subsection. For each $k \in [\underline{c}, \overline{c}]$,

$$\beta_u(k) > \beta_h(k) > \beta_d(k) > \beta_l(k). \tag{10}$$

Lastly, we denote

$$G_{il}(k_i, k_j) \equiv \lambda_i \beta_l(k_i) + (1 - \lambda_i)\beta_l(k_j), \tag{11}$$

and $G_{il}(k_i, k_j)\, v$ is the group i worker's incentive when k_i is below the standard k_s. It follows that for (k_i, k_j) satisfying $k_i > k_j$, $G_i(k_i, k_j)$ can be rewritten as

$$G_i(k_i, k_j) \equiv \begin{cases} G_{il}(k_i, k_j) & \text{if } k_i < k_s, \\ G_{iu}(k_i, k_j) & \text{if } k_i \geq k_s. \end{cases} \tag{12}$$

and for (k_j, k_i) satisfying $k_i > k_j$, $G_j(k_j, k_i)$ can be rewritten as

$$G_j(k_j, k_i) \equiv \begin{cases} G_{jl}(k_j, k_i) & \text{if } k_j < k_s, \\ G_{jd}(k_j, k_i) & \text{if } k_j \geq k_s. \end{cases} \tag{13}$$

We show that there exists an implicit function for each $\gamma \in \{d, u, l\}$.

Lemma 1 *For each $\gamma \in \{d, u, l\}$, there exists a unique continuous and strictly decreasing function $g_{i\gamma} : [\underline{c}, \overline{c}] \to (\underline{c}, \overline{c})$ such that*

$$G_{i\gamma}(g_{i\gamma}(k_j), k_j)\, v = g_{i\gamma}(k_j).$$

Intersections of these implicit functions will form different sets of asymmetric equilibria. However, there can be multiple asymmetric equilibria, unlike the symmetric equilibrium case in subsection 2.3, so we introduce a notation for comparison between them. We may call (k_i, k_j) an allocation in terms of their qualifications and define a *partially ordered* binary relation $>_D$ such that if for $x, y \in \mathbb{R}^2$, $x_i > y_i$ and $x_j < y_j$, then we say that an allocation x *i-dominates* y, denoted by $x >_D y$. Proposition 2 shows that there are two distinct sets of asymmetric equilibria with three ranges of k_s such that extreme asymmetric equilibria, **EA** can be a unique set of asymmetric equilibria, modest asymmetric equilibria **MA** can be a unique set of asymmetric equilibria or both.

Proposition 2 *There are two sets of asymmetric equilibria,* **EA** *and* **MA***, such that for each* $x \in \mathbf{EA}$ *and for every* $y \in \mathbf{MA}$*,* $x >_D y$*, and there exist* $(k^e_{i\max}, k^e_{j\min})$, $(k^m_{i\min}, k^m_{j\max}) \in (\underline{c}, \overline{c})^2$ *such that*

$$\begin{aligned}(k^e_{i\max}, k^e_{j\min}) &\equiv \{x \in \mathbf{EA} \mid x >_D y \text{ for all } y \in \mathbf{EA}\};\\ (k^m_{i\min}, k^m_{j\max}) &\equiv \{x \in \mathbf{MA} \mid x <_D y \text{ for all } y \in \mathbf{MA}\},\end{aligned}$$

and

(i) if $k_s \leq k^e_{j\min}$, **MA** *is a unique set of asymmetric equilibria,*

(ii) if $k^e_{j\min} < k_s \leq k^m_{j\max}$*, there are asymmetric equilibria* $x \in \mathbf{EA}$ *and* $y \in \mathbf{MA}$*,*

(iii) if $k^m_{j\max} < k_s \leq k^e_{i\max}$, **EA** *is a unique set of asymmetric equilibria.*

(iv) if $k_s > k^e_{i\max}$*, there exist no asymmetric equilibrium.*

If the output/wage ratio r from selecting a qualified worker is high enough that the standard k_s is sufficiently low, **MA** is a unique set of equilibria; if the output/wage ratio r is low enough that the standard k_s is sufficiently high, **EA** is a unique set of equilibria; and if the output/wage ratio r is in a middle range, then two types of asymmetric equilibria exist. The output/wage ratio has a one-to-one relationship with the standard given $r = 1/\mu(k_s)$, and it has a better interpretation, so we use the output/wage ratio instead of the standard in what follows.

One can see that these two asymmetric equilibrium sets are "partially ordered" such that any switch from one set to the other involves a "trade-off" between the two group's qualification levels; one group's increased level must accompany the other's decrease.

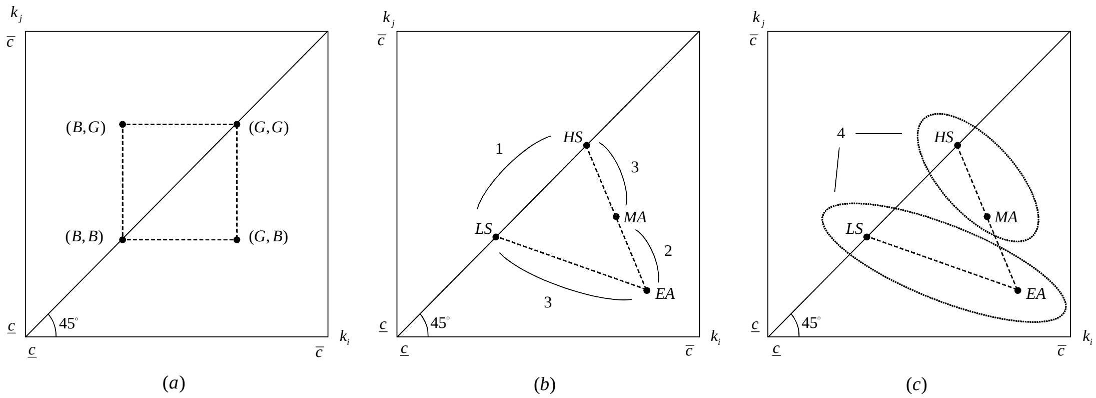

Figure 2: Feasible equilibrium sets

2.5 Synthesis and welfare analysis in the one-stage game

We have multiple equilibria such that symmetric or asymmetric equilibria are generated in a self-fulfilling manner as in typical statistical discrimination models. However, introducing competition between workers restricts the *feasible set* of those equilibria. The first main result in the one-stage game shows important relationships between the former and the latter.

Proposition 3 *Symmetric and asymmetric equilibria have the following relationships:*

(i) for each $x \in \mathbf{MA}$*,* $x >_D \mathbf{HS}$*,*

(ii) for each $x \in \mathbf{EA}$*,* $x >_D \mathbf{LS}$*.*

Proposition 3 shows that for each $x \in \mathbf{MA}$, x i-dominates the symmetric equilibrium **HS**, and for each $x \in \mathbf{EA}$, x i-dominates the symmetric equilibrium **LS**. Combining Proposition 2 and 3, for each $x \in \mathbf{EA}$, x i-dominates the symmetric equilibrium **HS**.

This entails that one group sacrifices its qualification level for a move from a discriminatory allocation, an asymmetric equilibrium, to a symmetric equilibrium, and furthermore that a policy maker often faces this type of conflict between Pareto optimality and fair allocation.

The second main result in the one-stage game establishes three distinct output/wage ratio levels that generate certain relationships between symmetric and asymmetric equilibria.

Proposition 4 *There are three regimes that show the relationships between symmetric and asymmetric equilibria depending on output/wage ratio levels such that*

(i) there exists a unique output/wage ratio level $r_l > 0$ such that if $r < r_l$, symmetric equilibrium **LS** *is a unique equilibrium,*

(ii) there exists a unique output/wage ratio level $r_m > r_l$ such that if $r_l \leq r < r_m$, $\mathbf{EA} \cup \{\mathbf{LS}\}$ *is a unique set of equilibria*

(iii) there exists a unique output/wage ratio level $r_h > r_m$ such that if $r \geq r_h$, $\mathbf{MA} \cup \{\mathbf{HS}\}$ *is a unique set of equilibria*

There exist three *regimes* such that, depending on different values of output/wage ratio, **LS** is a unique equilibrium; $\mathbf{EA} \cup \{\mathbf{LS}\}$ is a unique set of equilibria; or $\mathbf{MA} \cup \{\mathbf{HS}\}$ is a unique set of equilibria. A feasible allocation set restricts certain moves between symmetric and asymmetric equilibria. In particular, it is not plausible to shift from **LS** to **HS**.

Next, we show that discrimination cannot be considered a coordination problem; resolving a coordination failure cannot be a solution to discrimination since there is no way to make one group better off without making the other worse off.[19]

Proposition 5 *Let $r_l \leq r < r_m$. The move from (k_i^e, k_j^e) to (k_l, k_l) makes each worker type $c \geq k_l$ in group i worse off, and if $q \in [\sqrt{2} - 1, 1)$, the move from (k_i^e, k_j^e) to (k_l, k_l) makes each worker type $c < k_l$ in group i worse off.*

[19] For $c < k_l$, we need the condition since as k_i decreases, the intensity of the "own group competition" diminishes.

Hence, in this model, discrimination can be an allocation that is Pareto optimal, whereas in the typical statistical discrimination models, discrimination is always an allocation that is not Pareto optimal.

3 Model: a repeated game

Consider an infinitely repeated game in which there is a sequence of two groups, and in each period, two groups and employers play the one-stage game described in section 2. In the repeated game, the employers remain the same, whereas in each period, the workers of the two groups change. Hence, workers in each group can be called "short-run players," and employers "long-run players."

The main explanation in the one-stage game for group inequality, an asymmetric equilibrium, was self-fulfilling beliefs. In this section, we provide a different story: group inequality can instead be a result of collusion between a dominant group and the employers through a repeated interaction. The mechanics that the model works in the repeated game is different from the mechanics in the one-stage game. In the latter, employers' beliefs about two groups' qualifications play a critical role, but in the former, employers' strategies for how to choose a worker upon observing the signals, without considering those beliefs, are crucial.[20] Still, those equilibrium points in the one-stage game can be replicated and work as reference points for the analysis in the repeated game.

It is clear that by Proposition 5, workers of an *advantaged* group can obtain higher payoffs in an asymmetric equilibrium than in a symmetric equilibrium. How-

[20] As an illustration of how this equilibrium strategy works, consider a variant of the prisoners' dilemma game in the introduction of Fudenberg, Kreps and Maskin (1990). A long-run player meets with a sequence of short-run players, in which each short-run player moves first, and the long-run player moves later in each period. There exists a "cooperative" equilibrium such that all players choose to cooperate; given that the long-run player will choose to cooperate, the best response for each short-run player is to cooperate, and the long-run player cooperates given a grim strategy that all the short-run players will punish him by choosing to "cheat" afterward if there is a defection.

ever, it is not always the case that each employer gains a higher payoff from unequal qualifications between two groups, since although he can enjoy greater average qualifications from the advantaged group, he will be affected negatively by lower average qualifications from the "disadvantaged" group. Thus, the critical step is to examine whether there exist employers' strategies that make it possible to have each employer obtain a higher payoff in the repeated game than the payoffs in the one-stage game. We restrict our attention to *stationary* strategies and allow employers to choose mixed strategies in the repeated game,[21] which are *observable* in the spirit of chapter 2 in Fudenberg, Kreps and Maskin (1990).[22]

Suppose that there exists each employer's stationary strategy with the same "one-period action" that makes the employer obtain a higher payoff than the one-stage game payoff. We construct the following *grim* strategy for the repeated game. Under collusion, each employer chooses the collusive stationary strategy, and anticipating this, workers of an advantaged group choose their best response, as analyzed in the one-stage game. If and when workers of the advantaged group learn that an employer's defection has taken place in period t, they choose the equilibrium strategy in the one-stage game afterward. Then, if a common discount factor is sufficiently close to 1, as usual, there exists a collusive equilibrium.

Each employer's set of strategies when matched with one worker from group A and another from group B is $\Delta(\{A, B, \phi\})$ in which $\Delta(\{A, B, \phi\})$ is the set of probability distributions over $\{A, B, \phi\}$. We consider two *extreme* selection rules: a *most biased rule* (MBR) and a *most unbiased rule* (MUR) to show that it is optimal for the employer to choose one between two extremes.[23] Without loss of generality, let A be a group under collusion with employers in the repeated game, and therefore

[21] In the one-stage game, even with mixed strategies, employers will choose pure strategies in equilibrium; each employer's sequentially rational strategy must be pure strategies in section 2.

[22] Otherwise, as noted by Fudenberg, Kreps and Maskin (1990), especially with short-run players, a feasible equilibrium set in the repeated game is quite limited.

[23] The reason is that one cannot simply set up the problem as finding out strategies that maximize each employer's payoff. See footnote 25 for the general form of the employer's payoff.

the advantaged group. We say that an employer exercises MBR *for A* if when the employer is matched with workers from two *different* groups, he always chooses a worker of A *regardless of* signals from A and B. By (1), we derive

$$\beta\left(k_A, k_B\right) = P_H\left(k_B\right) \cdot 0 + P_M\left(k_B\right) \cdot 0 + P_L\left(k_B\right) \cdot 0 = 0.$$

Contrary to the intention of MBR, fewer workers in group A may become qualified; that is, MBR incurs a *moral hazard* problem for group A. In addition, given the specified selection rule by MBR–the employer always hires a worker of the less qualified group A–MBR can result in *adverse selection.* This will be analyzed in depth in subsection 3.2.

When an employer is matched with workers from two different groups, we say that the employer exercises MUR under the condition: he chooses a worker from A if the worker's signal is "stronger"; and chooses a worker from A with one-half chance if the worker's signal is the same as that of the other worker. It follows from (1) that

$$\beta\left(k_A, k_B\right) = (1-q)\left[P_H\left(k_B\right)\frac{1}{2} + P_M(k_B) + P_L\left(k_B\right)\right] + (q-u)\mathbf{1}_{\{k_A \geq k_s\}}\left[P_M(k_B)\frac{1}{2} + P_L\left(k_B\right)\right].$$

MUR induces an allocation that is the same as a symmetric equilibrium in subsection 2.3, but this symmetric allocation under asymmetric information could raise *levels of uncertainty* regarding the qualifications of future selected workers. Subsection 3.3 provides a detailed argument in relation to it.

3.1 Low output/wage ratio case

When $r_l \leq r < r_m$, **LS** is a unique symmetric equilibrium in the one-stage game. We focus on this low output/wage ratio case in the next two subsections in order to obtain a clear characterization of adverse selection and uncertainty problems, which relies on the *curvature* of the indifference curve of the employer's expected payoff.[24]

[24] Generally, the curvature of the indifference curve of each employer's expected payoff is not determinant. For example, even with $\lambda_A = 1/2$, it is given as $[P_H\left(k_A\right) + P_H\left(k_B\right) -$

Then, given the signal M from a worker, each employer's expected payoff of taking him is negative, so it is optimal for each employer not to choose a worker signaling M. If (k_A, k_B) is an equilibrium in the repeated game, and both cutoffs are below k_s,[25] the employer's *ex ante* expected payoff U_E can be derived as follows:[26]

$$U_E(k_A, k_B) = \{2\lambda_A P_H(k_A) + 2(1-\lambda_A) P_H(k_B) - [\lambda_A P_H(k_A) + (1-\lambda_A) P_H(k_B)]^2\}(x-v).$$

As a function of the two groups' qualifications, U_E has the following properties.

Lemma 2 *U_E has the following properties:*

(i) there exists an implicit function $e(k_A)$ such that $U_E(k_A, e(k_A)) = U_E(k_l, k_l)$ satisfying $e'(k_A) < 0$ for all $k_A \in [\underline{c}, \overline{c}]$ and $|e'(k_l)| = \frac{\lambda_A}{(1-\lambda_A)}$,

(ii) If F is linear, U_E is linear; if F is strictly concave, U_E is strictly quasi-concave; and if F is strictly convex, U_E is strictly quasi-convex .

We examine the case in which F is a concave function in what follows, since if F is a strictly convex function, by Lemma 2, U_E is a strictly quasi-convex function, so it is *not* surprising that each employer prefers an asymmetric allocation to a symmetric one, which will be discussed in subsection 3.3. The following Lemma examines the shape of the $G_{Bl}(k_B, k_A) = k_B$ graph when F is concave, where $G_{Bl}(k_B, k_A)$ and $g_{Bl}(k_A)$ are defined in (13) and Lemma 1, respectively.

$\frac{1}{4}(P_H(k_A) + P_H(k_B))^2](x-v) + \frac{1}{2} P_M(k_A) P_M(k_B)[\max\{\mu(k_A), \mu(k_B)\}x - v] + \frac{1}{4}[P_M(k_A) + 2P_L(k_A) + 2P_L(k_B)] P_M(k_A)[\mu(k_A)x - v] + \frac{1}{4}[P_M(k_B) + 2P_L(k_A) + 2P_L(k_B)] P_M(k_B)[\mu(k_B)x - v]$.

[25] This means that if it moves to an allocation in which one of the cutoffs becomes greater than the standard, the expected payoff's form should change accordingly.

[26] It is derived from $\lambda_A[P_H(k_A) + P_H(k_A) - P_H(k_A) P_H(k_A)](x-v) + (1-\lambda_A)^2 [P_H(k_B) + P_H(k_B) - P_H(k_B) P_H(k_B)](x-v) + 2\lambda_A(1-\lambda_A)[P_H(k_A) + P_H(k_B) - P_H(k_A) P_H(k_B)](x-v)$. With λ_A probability, each employer is matched with two workers of group i, and in that case, will select a worker only if he observes signal H, and with signal H, the probability of a worker being qualified is 1. The second line is derived similarly. The third is the case where the employer is matched with workers of two different groups.

Lemma 3 *If F is concave, G_{Bl} is a quasi-convex function, and* $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$.

If F is concave, we have a nice combination of the shapes of each employer's objective function U_E and G_{Bl}: U_E is quasi-concave, and G_{Bl} is quasi-convex. In addition to this property of two functions, the two different slopes of implicit functions, $|e'(k_l)| = \frac{\lambda_A}{(1-\lambda_A)}$ and $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$, play an important role to show the two main results in the following subsections. From Figure 3, one implicit function is *not tangent* to the other: they must cross each other.[27]

3.2 MBR: adverse selection and moral hazard

It follows from MBR for A that in an equilibrium,

$$\begin{aligned} \lambda_A \beta_l(k_A^*) &= k_A^*, \\ G_{Bl}(k_B^*, k_A^*) &= k_B^*. \end{aligned}$$

Hence, $k_A^* < k_l$. Proposition 6 proves a quite intuitive result: each employer's payoff from MBR is lower than that from **LS**.

Proposition 6 *If F is concave, MBR incurs adverse selection problem:* $U_E(k_A^*, k_B^*) < U_E(k_l, k_l)$.

Thus, MBR for A brings a "bad" outcome to each employer adopting it. Figure 3 (a) describes the adverse selection problem by MBR, where k_A^* is lower than k_l, when F is uniform.

3.3 MUR: uncertainty problem

In this subsection, we show that MUR is not an optimal collusion strategy for employers by exploring the asymmetric strategies available to them that make each

[27] The convex shape (quasi-concave function) represents e, whereas the linear one represents g_{Bl}. In general, as shown by Lemma 3, g_{Bl} can have a concave shape (quasi-convex function).

employer obtain higher payoff than that by MUR. This result is somewhat surprising since each employer is assumed to have a quasi-concave payoff function, so a symmetric allocation is better than or as good as an asymmetric one.

Such an outcome is possible mainly because (i) Lemmas 2 and 3 show that U_E is quasi-concave with $|e'(k_l)| = \frac{\lambda_A}{(1-\lambda_A)}$, and G_{Bl} is quasi-convex with $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$, which entails that there exists an asymmetric selection rule better for each employer than MUR and (ii) the asymmetric selection rule provides a greater incentive for the advantaged group to become qualified more while leaving the incentive for the disadvantaged group *unchanged*. Note that it is not the case that a group's welfare depends exclusively on that group's incentive to become qualified in the model because of competition between the two groups, as discussed in the one-stage game, whereas it is true in the typical statistical discrimination models. Even if the disadvantaged group's incentive remains the same, the group's qualification level and welfare can still decrease given the other's greater incentive to become qualified. In addition, we study a collusion between firms and the advantaged group, so the disadvantaged group is simply not a part of the collusion.

To find alternative asymmetric equilibria, we define the function $\alpha_{xy} : [\underline{c}, \overline{c}] \times [0,1]^2 \to [0,1]$ such that

$$\alpha_{xy}(k_B, x, y) \equiv P_H(k_B)(1-q)\frac{1}{2} + P_M(k_B)\{(1-q) + (q-u)x\} \qquad (14)$$
$$+ P_L(k_B)\{(1-q) + (q-u)y\}.$$

x denotes the probability of choosing a worker of group A when workers from two different groups emit the same signal M, and y denotes the probability of choosing a worker of group A when the worker from group A emits M, but one from group B emits L. Following the analysis in section 2, we introduce similar notations:

$$G_{xy}(k_A, k_B, x, y) \equiv \lambda_A \beta_l(k_A) + (1-\lambda_A)\alpha_{xy}(k_B, x, y),$$

and

$$G_{xy}(k_A, g_{xy}(k_A, x, y), x, y) = k_A.$$

The existence of an implicit function g_{xy} can be easily derived. In Proposition 7, we find a combination of x and y that makes each employer's payoff greater than the one by MUR.[28]

Proposition 7 *If F is concave, MUR incurs an uncertainty problem; there exists (x, y) generating an equilibrium (k_A^*, k_B^*) such that $U_E(k_A^*, k_B^*) > U_E(k_l, k_l)$.*

Figure 3 (b) describes the uncertainty problem by MUR when F is uniform, which shows the shift of the advantaged group's line to the right hand side while the disadvantaged group's line remains the same. While we are successful in providing

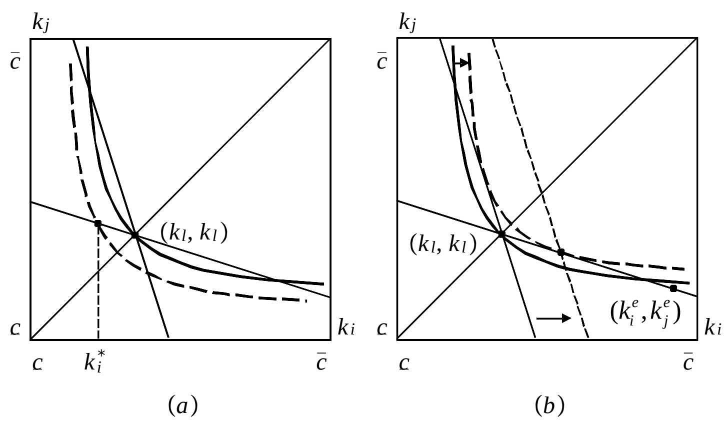

Figure 3: MBR and MUR

a clear conjecture regarding a possibility of collusion in the repeated game between employers and the advantaged group in the low output/wage ratio case, prediction

[28] Given (x, y), each employer chooses a worker from group A even with signal M, so each employer's total payoff will increase if the positive effect from this result is greater than a negative effect from M if $k_A^* < k_s$. Of course, if $k_A^* \geq k_s$, the employer will not have a negative effect from it. A sufficient condition to ensure $k_A^* \geq k_s$ can also be derived: $U_E\left(k_h, g_{Al}^{-1}(k_h)\right) > U_E(k_l, k_l)$. Recall $r_l \equiv 1/\mu(k_{A\max}^e)$ and $r_m \equiv 1/\mu(k_h)$ in (18). Hence, $r_l \leq r < r_m$ is equivalent to $k_h < k_s \leq k_{A\max}^e$. Since $U_E\left(k_h, g_{Al}^{-1}(k_h)\right) > U_E(k_l, k_l)$, there exists $k_s \in (k_h, k_{A\max}^e]$ sufficiently close to k_h such that $U_E\left(k_s, g_{Al}^{-1}(k_s)\right) > U_E(k_l, k_l)$.

is subtle in the high output/wage ratio case: it all depends on the curvature of each employer's payoff function.

3.4 High output/wage ratio case

When $r > r_h$, **HS** is a unique symmetric equilibrium in the one-stage game. Then, given MUR, a group i worker's incentive to become qualified is derived as below.

$$[\lambda_A \beta_h (k_A) + (1 - \lambda_A)\beta_h(k_B)]v.$$

Given signal H from the disadvantaged group, if an employer chooses a worker of the advantaged group with a probability greater than 1/2 when his signal is H, with a probability greater than 0 when his signal is M, or with a probability greater than 0 when his signal is L, the disadvantaged group's benefit will decrease.

Similarly, given signal M from the disadvantaged group, if an employer chooses a worker of the advantaged group with a probability greater than 1/2 when his signal is M, or with a probability greater than 0 when his signal is L, the disadvantaged group's benefit will decrease. Given signal L from the disadvantaged group, there is no way to increase the advantaged group's benefit. Proposition 8 summarizes the above.

Proposition 8 *Given* **HS***, each employer has no way to provide a greater incentive for the advantaged group without leaving that for the disadvantaged group unchanged.*

Hence, if each employer's payoff has a "strong" quasi-concavity, we may not find any asymmetric allocation that provides the employer with a higher payoff than **HS**, whereas if each employer's payoff has a "weak" quasi-concavity, we could find an asymmetric allocation that provides him with a higher payoff than **HS** as in the low output/wage ratio case analyzed in the previous subsection. Figure 4 shows that the decrease in the disadvantaged group's benefit leads to the shift of the related curve to the down side when F is uniform.

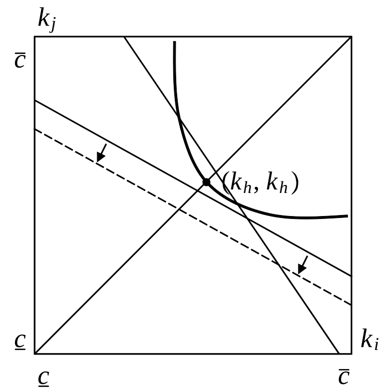

Figure 4: High return-per-reward

3.5 Synthesis in the repeated game

There are two distinct output/wage ratio levels such that if $r_l \leq r < r_m$, a collusion in the repeated game can be an equilibrium, and if $r > r_h$ and U_E has a strong quasi-concavity, a unique symmetric **HS** is better than any asymmetric equilibrium, so no collusion arises in the repeated game. Thus, we show that there exists a regime with high output/wage ratios in which, given a strong concavity of the population distribution, any discriminatory action is not optimal to employers.

4 Concluding Remarks

We provide a model with competition between two groups and show that a set of feasible equilibria has an inter-group conflict structure: in order for one group's qualification level to increase, the other group's level must decrease in equilibrium, and furthermore, two ex ante identical groups can have two different average-productivity levels because of collusion between employers and the advantaged group.

We hope that this paper will serve not as a justification for the existence of discrimination but as a justification for policies to support disadvantaged groups since development of one group may be closely related to *underdevelopment* of the other group from the nature of competition between them.

Appendix: Proofs

4.1 Symmetric equilibria

We first show that β is an increasing function of k_i and a strictly decreasing function of k_j.

Lemma 4 *β is an increasing function of k_i given a fixed k_j and a strictly decreasing function of k_j given a fixed k_i.*

Proof. Consider (1). For any pair $k_i' > k_i$, $(q-u)\mathbf{1}_{\{k_i' \geq k_s\}} \geq (q-u)\mathbf{1}_{\{k_i \geq k_s\}}$ since $q > u$, and $\varphi(k_i', k_j) \geq \varphi(k_i, k_j)$ given k_j, which shows the former. The first term is a strictly decreasing function of k_j:

$$P_H(k_j)\frac{1}{2} + P_M(k_j) + P_L(k_j) = -\frac{1}{2}F(k_j)(1-q) + 1.$$

For any pair $k_j' > k_j$, we have $\varphi(k_i, k_j') \leq \varphi(k_i, k_j)$ given k_i, and

$$\begin{aligned} & P_M(k_j)\varphi(k_i, k_j) + P_L(k_j) \\ & = -F(k_j)\left[(1-u) - (q-u)\varphi(k_i, k_j)\right] + u\varphi(k_i, k_j) + (1-u), \end{aligned}$$

where $(1-u) - (q-u)\varphi(k_i, k_j) > 0$ for all (k_i, k_j), which shows the latter. ■

Proof of Proposition 1. Since by Lemma 4, (3) and (4), $\beta(\underline{c}, \underline{c})v > \underline{c}$ and $\beta(\bar{c}, \bar{c})v < \bar{c}$, for β_l, we have $\beta_l(\underline{c})v = \beta(\underline{c}, \underline{c})v > \underline{c}$ and $\beta_l(\bar{c})v < \beta_h(\bar{c})v = \beta(\bar{c}, \bar{c})v < \bar{c}$, so there exists $k_l \in (\underline{c}, \bar{c})$ such that $\beta_l(k_l)v = k_l$. Similarly, for β_h, we have $\beta_h(\underline{c})v > \beta_l(\underline{c})v = \beta(\underline{c}, \underline{c})v > \underline{c}$ and $\beta_h(\bar{c})v = \beta(\bar{c}, \bar{c})v < \bar{c}$, so there exists $k_h \in (\underline{c}, \bar{c})$ such that $\beta_h(k_h)v = k_h$. Furthermore, k_h and k_l are unique, and $k_h > k_l$ because $\beta_h(k) > \beta_l(k)$, and β_l and β_h are decreasing functions of k.

Consider (7). If $k_s \leq k_l$, the unique fixed point of β_l, k_l, cannot be attained, and since $k_s \leq k_l < k_h$, k_h can be attained. If $k_s > k_h$, the unique fixed point of β_h, k_h, cannot be attained, and since $k_s > k_h > k_l$, k_l can be attained. If $k_l < k_s \leq k_h$, since $k_l < k_s$ and $k_s \leq k_h$, both can be attained. ■

4.2 Asymmetric equilibria

Proof of Lemma 1. We only show for g_{id} since the others can be obtained in a similar way. From (3),

$$\begin{aligned} \underline{c} &< G_i(\underline{c},\overline{c})\,v = [\lambda_i\beta_l(\underline{c}) + (1-\lambda_i)\,\beta_l(\overline{c})]v \\ &< [\lambda_i\beta_h(\underline{c}) + (1-\lambda_i)\,\beta_d(\overline{c})]v = G_{id}(\underline{c},\overline{c})\,v. \end{aligned}$$

Since β_d is decreasing, for each $c \in [\underline{c},\overline{c}]$,

$$G_{id}(\underline{c},c)\,v > \underline{c}. \tag{15}$$

From (4),

$$\begin{aligned} \overline{c} &> G_i(\overline{c},\underline{c})\,v = [\lambda_i\beta_h(\overline{c}) + (1-\lambda_i)\,\beta_u(\underline{c})]v \\ &> [\lambda_i\beta_h(\overline{c}) + (1-\lambda_i)\,\beta_d(\underline{c})]v = G_{id}(\overline{c},\underline{c})\,v. \end{aligned}$$

Since β_d is decreasing, for each $c \in [\underline{c},\overline{c}]$,

$$G_{id}(\overline{c},c)\,v < \overline{c}. \tag{16}$$

Note that $G_{id}(k_i,k_j)\,v = k_i$ can be rewritten as

$$\beta_d(k_j) = \frac{k_i}{(1-\lambda_i)\,v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(k_i)\,,$$

and (15) and (16) imply that for each $c \in [\underline{c},\overline{c}]$,

$$\beta_d(c) > \frac{\underline{c}}{(1-\lambda_i)\,v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(\underline{c}) \text{ and } \beta_d(c) < \frac{\overline{c}}{(1-\lambda_i)\,v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(\overline{c})\,.$$

Since $\frac{k_i}{(1-\lambda_i)v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(k_i)$ is a continuous and strictly increasing function of k_i, there exists a unique continuous functions $g_{id} : [\underline{c},\overline{c}] \to (\underline{c},\overline{c})$ such that

$$\beta_d(k_j) = \frac{g_{id}(k_j)}{(1-\lambda_i)\,v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(g_{id}(k_j))\,. \tag{17}$$

Moreover, β_d is decreasing, and $\frac{k_i}{(1-\lambda_i)v} - \frac{\lambda_i}{(1-\lambda_i)}\beta_h(k_i)$ is strictly increasing, so g_{id} is strictly decreasing. ■

Denote by k_{iu} and k_{id} the fixed points of $G_{iu}(k,k)\,v = k$ and $G_{id}(k,k)\,v = k$, respectively. As functions of k, g_{iu}, g_{id} and g_{il} have the following properties.

Lemma 5 *g_{iu}, g_{id} and g_{il} satisfy the following properties.*

(i) $k_{iu} > k_{jd} > k_{jl}$.

(ii) For each $k \in [\underline{c}, \overline{c}]$, $g_{jd}(k) > g_{jl}(k)$.

Proof. *(i)* Suppose $k_{jd} \geq k_{iu}$. This implies

$$\begin{aligned} & \lambda_j \beta_h(k_{jd}) + (1-\lambda_j)\,\beta_d\,(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\,\beta_u\,(k_{iu}) \\ \Leftrightarrow \quad & (1-\lambda_i)\,\beta_h(k_{jd}) + \lambda_i \beta_d\,(k_{jd}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\,\beta_u\,(k_{iu}), \end{aligned}$$

and since β_h and β_d are decreasing,

$$\begin{aligned} & (1-\lambda_i)\,\beta_h(k_{iu}) + \lambda_i \beta_d\,(k_{iu}) \geq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\,\beta_u\,(k_{iu}) \\ \Leftrightarrow \quad & (1-\lambda_i)\,[\beta_h(k_{iu}) - \beta_u\,(k_{iu})] + \lambda_i[\beta_d\,(k_{iu}) - \beta_h(k_{iu})] \geq 0, \end{aligned}$$

which contradicts $\beta_h\,(k) < \beta_u\,(k)$ and $\beta_d\,(k) < \beta_h\,(k)$. The proof for $k_{jd} > k_{jl}$ is similar.

(ii) Since for each $k \in [\underline{c}, \overline{c}]$, $\beta_h\,(k) > \beta_d\,(k) > \beta_l\,(k)$,

$$\beta_d\,(k_i) = \frac{g_{jd}\,(k_i)}{(1-\lambda_j)\,v} - \frac{\lambda_j}{(1-\lambda_j)}\beta_h\,(g_{jd}\,(k_i))$$

implies

$$\beta_l\,(k_i) < \frac{g_{jd}\,(k_i)}{(1-\lambda_j)\,v} - \frac{\lambda_j}{(1-\lambda_j)}\beta_l\,(g_{jd}\,(k_i))\,.$$

Since $\frac{k_j}{(1-\lambda_j)v} - \frac{\lambda_j}{(1-\lambda_j)}\beta_h\,(k_j)$ is a strictly increasing function of k_j, given each k_i, we must have $g_{jl}(k_i) < g_{jd}(k_i)$. ■

These properties enable us to identify asymmetric equilibria. We define functions $\mathcal{G}_{ul} : [\underline{c}, \overline{c}]^2 \to [\underline{c}, \overline{c}]^2$ and $\mathcal{G}_{ud} : [\underline{c}, \overline{c}]^2 \to [\underline{c}, \overline{c}]^2$ such that

$$\begin{aligned} \mathcal{G}_{ul}(k_i, k_j) &\equiv (G_{iu}(k_i, k_j), G_{jl}(k_j, k_i)), \\ \mathcal{G}_{ud}(k_i, k_j) &\equiv (G_{iu}(k_i, k_j), G_{jd}(k_j, k_i)), \end{aligned}$$

and denote by **EA** and **MA** sets of all the fixed points of $\mathcal{G}_{ul}$ and $\mathcal{G}_{ud}$, respectively. Let (k_i^e, k_j^e) be an element of the set **EA** and (k_i^m, k_j^m) an element of the set **MA**.

Proof of Proposition 2. **Step 1**. Show that there exist (k_i^e, k_j^e) and (k_i^m, k_j^m).

It follows from Lemma 1 and Lemma 5 that $k_{iu} > k_{jd} = g_{jd}(k_{jd}) > g_{jd}(k_{iu})$, which in turn implies

$$g_{iu}^{-1}(k_{iu}) - g_{jd}(k_{iu}) > 0.$$

On the other hand, since g_{iu} is strictly decreasing, $g_{iu}(\underline{c})$ is the maximum of g_{iu}, and $g_{iu}(\underline{c}) < \overline{c}$. By Lemma 1,

$$g_{iu}^{-1}(g_{iu}(\underline{c})) - g_{jd}(g_{iu}(\underline{c})) < \underline{c} - \underline{c} = 0.$$

The continuity of g_{iu} and g_{jd} entails that there exists $k_i^m \in (k_{iu}, \overline{c})$ such that

$$g_{iu}^{-1}(k_i^m) - g_{jd}(k_i^m) = 0.$$

Given k_i^m, the value of $g_{iu}^{-1}(k_i^m)$ is k_j^m, which must be in $(\underline{c}, \overline{c})$. Hence, given (k_i^m, k_j^m),

$$G_{iu}(k_i^m, k_j^m)v = k_i^m \text{ and } G_{jd}(k_j^m, k_i^m)v = k_j^m.$$

Since g_{iu}^{-1} is strictly decreasing, and $k_{iu} = g_{iu}^{-1}(k_{iu})$, given $k_i^m > k_{iu}$, we have $k_i^m > k_j^m$.

By Lemma 5,

$$0 = g_{iu}^{-1}(k_i^m) - g_{jd}(k_i^m) < g_{iu}^{-1}(k_i^m) - g_{jl}(k_i^m).$$

On the other hand, $g_{iu}^{-1}(g_{iu}(\underline{c})) - g_{jl}(g_{iu}(\underline{c})) < 0$. The continuity of g_{iu} and g_{jl} implies that there exists $k_i^e \in (k_i^e, \overline{c})$ such that

$$g_{iu}^{-1}(k_i^e) - g_{jl}(k_i^e) = 0.$$

Hence, given (k_i^e, k_j^e),

$$G_{iu}(k_i^e, k_j^e)v = k_i^e \text{ and } G_{jl}(k_j^e, k_i^e)v = k_j^e.$$

Since g_{iu}^{-1} is strictly decreasing, and $k_{iu} = g_{iu}^{-1}(k_{iu})$, given $k_i^e > k_{iu}$, we have $k_i^i > k_j^i$.

Step 2. Show the characterization.

Note that both (k_i^m, k_j^m) and (k_i^e, k_j^e) are on the graph $k_i = g_{iu}(k_j)$ where g_{iu} is strictly decreasing, so $k_i^e > k_i^m$ implies $k_j^m > k_j^e$. Thus, we have

$$\underline{c} < k_j^e < k_j^m < k_i^m < k_i^e < \bar{c}.$$

Consider (12) and (13). If $k_s \leq k_j^e$, the fixed point (k_i^e, k_j^e) cannot be attained, and since $k_s \leq k_j^m < k_i^m < k_i^e$, (k_i^m, k_j^m) can be attained. If $k_j^m < k_s \leq k_i^e$, the fixed point (k_i^m, k_j^m) cannot be attained, and since $k_j^e < k_j^m < k_s \leq k_i^e$, (k_i^e, k_j^e) can be attained. If $k_j^e < k_s \leq k_j^m$, since $k_j^e < k_s < k_i^e$ and $k_s \leq k_j^m < k_i^m$, both can be attained. Lastly, if $k_s > k_i^e$, neither can be attained. ■

Figure 5 describes (i) and (ii) when F is uniform.

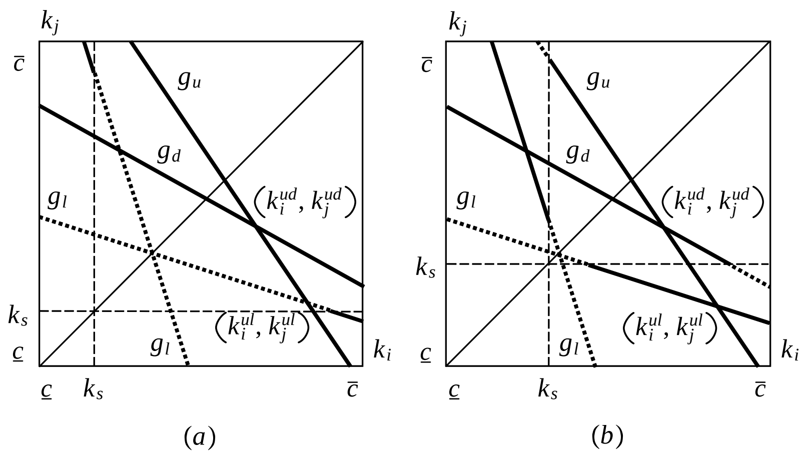

Figure 5: Asymmetric equilibria

Proof of Proposition 3. *(i)* First, we show $k_{iu} > k_h$. Suppose $k_{iu} \leq k_h$. This implies $\lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\beta_u(k_{iu}) \leq \lambda_i \beta_h(k_h) + (1-\lambda_i)\beta_h(k_h)$, and since β_h is decreasing, we have $\lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\beta_u(k_{iu}) \leq \lambda_i \beta_h(k_{iu}) + (1-\lambda_i)\beta_h(k_{iu})$, so $\beta_u(k_{iu}) \leq \beta_h(k_{iu})$, which contradicts $\beta_u(k_{iu}) > \beta_h(k_{iu})$. It follows from the proof of Proposition 2 that $k_h < k_{iu} < k_i^m$. Similarly, we show $k_{jd} < k_h$. Suppose $k_{jd} \geq k_h$. This implies $(1-\lambda_j)\beta_h(k_{jd}) + \lambda_j \beta_d(k_{jd}) \geq (1-\lambda_j)\beta_h(k_h) + \lambda_j \beta_h(k_h)$, and since β_h is decreasing, we have $(1-\lambda_j)\beta_h(k_{jd}) + \lambda_j \beta_d(k_{jd}) \geq (1-\lambda_j)\beta_h(k_{jd}) + \lambda_j \beta_h(k_{jd})$, so $\beta_d(k_{jd}) \geq \beta_h(k_{jd})$, which contradicts $\beta_d(k_{jd}) < \beta_h(k_{jd})$. Since g_{jd} is strictly decreasing, and $g_{jd}(k_{jd}) = k_{jd}$, $k_h > k_{jd} > k_j^m$.

(ii) Similarly, the proof of Proposition 2 and the property of g_{jl} can show it. ■

Proof of Proposition 4. Proposition 3 entails that $k^e_{j\min} < k_l < k_h < k^e_{i\max}$. On the other hand, the relationship between $k^m_{j\max}$ and k_l is not determinant. Denote

$$r_l \equiv 1/\mu(k^e_{i\max}),\ r_m \equiv 1/\mu(k_h) \text{ and } r_h \equiv 1/\mu(k^e_{j\min}). \tag{18}$$

Proposition 1 and Proposition 2 establish the results. ■

Proof of Proposition 5. Let $\beta_1(k_i, k_j)$ denote the probability that a qualified member of group i is selected, and $\beta_0(k_i, k_j)$ denote the probability that an unqualified member of group i is selected:

$$\begin{aligned}\beta_1(k_i,k_j) \equiv{}& P_H(k_j)(1-q)\frac{1}{2} + P_M(k_j)[(1-q) + q\varphi(k_i,k_j)\mathbf{1}_{\{k_i \geq k_s\}}] \\ &+ P_L(k_j)[(1-q) + q\mathbf{1}_{\{k_i \geq k_s\}}], \\ \beta_0(k_i,k_j) \equiv{}& P_M(k_j)u\varphi(k_i,k_j)\mathbf{1}_{\{k_i \geq k_s\}} + P_L(k_j)u\mathbf{1}_{\{k_i \geq k_s\}}.\end{aligned}$$

By Proposition 4, $\mathbf{EA} \cup \{(k_l, k_l)\}$ is a unique set of equilibria in which the corresponding standard k_s satisfies $k_j^e < k_l < k_s < k_i^e$ for all $(k_i, k_j) \in \mathbf{EA}$. Using β_1 and β_0 in (1), define G_{i1} and G_{i0}

$$\begin{aligned} G_{i1}(k_i,k_j) &\equiv [\lambda_i \beta_1(k_i,k_i) + (1-\lambda_i)\beta_1(k_i,k_j)]; \\ G_{i0}(k_i,k_j) &\equiv [\lambda_i \beta_0(k_i,k_i) + (1-\lambda_i)\beta_0(k_i,k_j)]. \end{aligned}$$

Part 1. Note that G_{i0} is a decreasing function of k_j. Furthermore, let

$$G_{i0}(k_i^e,k_l) - G_{i0}(k_l,k_l) = \lambda_i\beta_0(k_i^e,k_i^e) + (1-\lambda_i)\beta_0(k_i^e,k_l) - [\lambda_i\beta_0(k_l,k_l) + (1-\lambda_i)\beta_0(k_l,k_l)].$$

By the definition of G_{i0} that for any $k_l < k_s < k_i^e$,

$$\begin{aligned} \beta_0(k_i^e,k_l) - \beta_0(k_l,k_l) &= [P_M(k_l) + P_L(k_l)]u > 0; \\ \beta_0(k_i^e,k_i^e) - \beta_0(k_l,k_l) &= [P_M(k_i^e)\frac{1}{2} + P_L(k_i^e)]u > 0. \end{aligned}$$

Then, $G_{i0}\left(k_i^e, k_l\right) - G_{i0}\left(k_l, k_l\right) > 0$, which implies that for $k_j^e < k_l < k_s < k_i^e$,

$$G_{i0}(k_i^e, k_j^e) \geq G_{i0}\left(k_i^e, k_l\right) > G_{i0}\left(k_l, k_l\right).$$

It follows that for each $c \in [k_l, k_i^e]$,

$$G_{i1}(k_i^e, k_j^e) - c > G_{i0}(k_i^e, k_j^e) > G_{i0}\left(k_l, k_l\right).$$

In addition, for each $c \in [k_i^e, \overline{c}]$,

$$G_{i0}(k_i^e, k_j^e) > G_{i0}\left(k_l, k_l\right).$$

Hence, given a move from (k_i^e, k_j^e) to (k_l, k_l), each $c \geq k_l$ is worse off.

Part 2. Note that G_{i1} is a decreasing function of k_j. Let

$$G_{i1}\left(k_i^e, k_l\right) - G_{i1}\left(k_l, k_l\right) = \lambda_i \beta_1(k_i^e, k_i^e) + \left(1 - \lambda_i\right) \beta_1\left(k_i^e, k_l\right) - [\lambda_i \beta_1(k_l, k_l) + \left(1 - \lambda_i\right) \beta_1\left(k_l, k_l\right)].$$

By the definition of G_{i1} that for any $k_l < k_s < k_i^e$,

$$\begin{aligned}
& \beta_1\left(k_i^e, k_l\right) - \beta_1\left(k_l, k_l\right) = [P_M(k_l) + P_L\left(k_l\right)]q > 0; \\
& \beta_1\left(k_i^e, k_i^e\right) - \beta_1\left(k_l, k_l\right) \\
= & [P_M(k_i^e) + P_L\left(k_i^e\right)]q + (1-q)\left\{[P_H\left(k_i^e\right)\frac{1}{2} + P_M(k_i^e) + P_L\left(k_i^e\right)] - [P_H\left(k_l\right)\frac{1}{2} + P_M(k_l) + P_L\left(k_l\right)]\right\} \\
= & [F\left(k_i^e\right) - F\left(k_l\right)]\frac{1}{2}\left(q^2 - 1\right) + F\left(k_l\right)q^2 + \left(1 - F\left(k_l\right)\right)q \\
= & \frac{1}{2}\left(q-1\right)\left[F\left(k_i^e\right)\left(q+1\right) + F\left(k_l\right)\left(q-1\right)\right] + q > \frac{1}{2}\left(q^2 - 1\right) + q.
\end{aligned}$$

For each $q \in [\sqrt{2}-1, 1)$, $\frac{1}{2}\left(q^2-1\right) + q \geq 0$. Suppose $q \in [\sqrt{2}-1, 1]$. Then $G_{i1}\left(k_i^e, k_l\right) - G_{i1}\left(k_l, k_l\right) > 0$, which implies that for $k_j^e < k_l < k_s < k_i^e$,

$$G_{i1}(k_i^e, k_j^e) \geq G_{i0}\left(k_i^e, k_l\right) > G_{i1}\left(k_l, k_l\right).$$

For each $c \in [\underline{c}, k_l)$,

$$G_{i1}(k_i^e, k_j^e) - c > G_{i1}\left(k_l, k_l\right) - c.$$

Hence, given a move from (k_i^e, k_j^e) to (k_l, k_l), each $c < k_l$ is worse off. ∎

Proof of Lemma 2. *(i)* Given U_E,

$$\frac{\partial U_E}{\partial k_B} = 2\left(1-\lambda_A\right)P_H'\left(k_B\right)\left\{1 - [\lambda_A P_H\left(k_A\right) + \left(1-\lambda_A\right)P_H\left(k_B\right)]\right\}(x - v) > 0,$$

which implies that there is an implicit function $e(k_A)$ such that

$$U_E(k_A, e(k_A)) = U_E(k_l, k_l).$$

In addition,

$$\frac{dk_B}{dk_A} = -\frac{\lambda_A P'_H(k_A)}{(1-\lambda_A) P'_H(k_B)} < 0.$$

Hence, it shows that $e'(k_A) < 0$ for all $k_A \in [\underline{c}, \overline{c}]$ and $|e'(k_l)| = \frac{\lambda_A}{(1-\lambda_A)}$.

(ii) Denote

$$\overline{P}(k_A) \equiv \frac{\lambda_A}{(1-\lambda_A)} \frac{P'_H(k_A)}{P'_H(e(k_A))}. \tag{19}$$

Then, we have

$$\frac{d\overline{P}(k_A)}{dk_A} = \frac{\lambda_A}{(1-\lambda_A)} \frac{P''_H(k_A) P'_H(e(k_A)) - P''_H(e(k_A)) e'(k_A) P'_H(k_A)}{P'_H(e(k_A))^2},$$

which results in

$$\frac{d\overline{P}(k_A)}{dk_A} = \begin{cases} > 0 & \text{if } F'' > 0, \\ = 0 & \text{if } F'' = 0, \\ < 0 & \text{if } F'' < 0. \end{cases}$$

■

Proof of Lemma 3. We derive

$$\frac{dk_B}{dk_A} = -\frac{\lambda_A \beta'_l(k_A)}{(1-\lambda_A)\beta'_l(k_B) - 1}.$$

It follows from Lemma 1 that $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$. Denote

$$\overline{\beta}(k_A) \equiv \frac{\lambda_A \beta'_l(k_A)}{(1-\lambda_A)\beta'_l(g_{jl}(k_A)) - 1}. \tag{20}$$

Then, we have

$$\frac{d\overline{\beta}}{dk_A} = \frac{\frac{1}{2}\beta''_l(k_A)[\frac{1}{2}\beta'_l(g_{Bl}(k_A)) - 1] - \frac{1}{2}\beta''_l(g_{Bl}(k_A))g'_{Bl}(k_A)\frac{1}{2}\beta'_l(k_A)}{[\frac{1}{2}\beta'_l(k_A) - 1]^2} > 0.$$

■

Proof of Proposition 6. Define three upper contour sets:

$$\Pi_1 \equiv \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid U_E(k_A, k_B) \geq U_E(k_l, k_l)\},$$
$$\Pi_2 \equiv \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid \lambda_A k_A + (1-\lambda_A) k_B \geq 2k_l\},$$
$$\Pi_3 \equiv \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid G_{Bl}(k_B, k_A) = k_B\}.$$

Note $(k_l, k_l) \in \Pi_1 \cap \Pi_2 \cap \Pi_3$. Since by Lemma 2, U_E is a quasi-concave function, and $|e'(k_l)| = \frac{\lambda_A}{(1-\lambda_A)}$, we have $\Pi_1 \subseteq \Pi_2$. It follows from Lemma 3 that G_{Bl} is a quasi-convex function, and $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$, which implies that for any $k_A < k_l$, $\Pi_3 \subset [\underline{c}, \overline{c}]^2 \backslash \Pi_2$. Hence, $\Pi_3 \cap \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid k_A < k_l\} \subset [\underline{c}, \overline{c}]^2 \backslash \Pi_1$. For each $(k_A, k_B) \in \Pi_3 \cap \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid k_A < k_l\}$, $U_E(k_A, k_B) < U_E(k_l, k_l)$. ■

Proof of Proposition 7. First, since the disadvantaged group B's standard k_B is below a group standard k_s in the low output/wage ratio case, the second term will disappear in (1), so providing a greater incentive for the advantaged group does not have any effect on the other's incentive, and we have G_{Bl} for the disadvantaged group's incentive.

Step 1. Show that there exists (k_A^*, k_B^*) such that each employer's payoff strictly increases. Note $(k_l, k_l) \in \Pi_3 \cap \Pi_2$. Since by Lemma 3, G_{Bl} is a quasi-convex function, and $|g'_{Bl}(k_l)| < \frac{\lambda_A}{(1-\lambda_A)}$, for any $k_A > k_l$, $\Pi_3 \cap \Pi_1 \neq \varnothing$. Hence, there exists $(k_A^*, k_B^*) \in \Pi_3 \cap \{(k_A, k_B) \in [\underline{c}, \overline{c}]^2 \mid k_A > k_l\}$ such that $U_E(k_A^*, k_B^*) > U_E(k_l, k_l)$.

Step 2. Show how to implement (k_A^*, k_B^*) with x and y.

We re-formulate the problem in (14) as below.

$$G_z(k_A, k_B, z) \equiv \lambda_A \beta_l(k_A) + (1-\lambda_A)\alpha_z(k_B, z),$$

where

$$\alpha_z(k_B, z) \equiv (1-q)\left[P_H(k_B)\frac{1}{2} + P_M(k_B) + P_L(k_B)\right] + z(q-u)[P_M(k_B) + P_L(k_B)],$$

and

$$G_z(k_A, g_z(k_A, z), z) = k_A.$$

If $z = 0$, $g_z(k_A, 0)$ is the same as $g_{Al}^{-1}(k_A)$. We show that for any $k_A > k_l$,

$$g_z(k_A, 0) - g_{Bl}(k_A) < 0.$$

Suppose that there exists $k_A > k_l$ such that

$$g_{il}^{-1}(k_A) - g_{Bl}(k_A) \geq 0.$$

If $g_{Al}^{-1}(k_A) - g_{Bl}(k_A) = 0$, we have a contradiction since k_l is unique. Let $g_{Al}^{-1}(k_A) - g_{Bl}(k_A) > 0$. Since g_{Al} is strictly decreasing, $g_{Al}(\underline{c})$ is the maximum of g_{Al}, and $g_{Al}(\underline{c}) < \overline{c}$. By Lemma 1,

$$g_{Al}^{-1}(g_{Al}(\underline{c})) - g_{Bl}(g_{Bl}(\underline{c})) < \underline{c} - \underline{c} = 0.$$

The continuity of g_{Al} entails that there exists $k'_A \in (k_A, \overline{c})$ such that

$$g_{Al}^{-1}(k'_A) - g_{Bl}(k'_A) = 0,$$

which contradicts the uniqueness of k_l since $k'_A > k_A > k_l$.

On the other hand, if $z = 1$, $g_z(k_A, 1)$ is the same as $g_{Au}^{-1}(k_A)$. Now, we show that for any $k_A < k^e_{A\min}$,

$$g_z(k_A, 1) - g_{Bl}(k_A) > 0.$$

Suppose that there exists $k_A < k^e_{A\min}$ such that

$$g_{Au}^{-1}(k_A) - g_{Bl}(k_A) \leq 0.$$

If $g_{Au}^{-1}(k_A) - g_{Bl}(k_A) = 0$, we have a contradiction since $k^e_{A\min}$ is the minimum of such type of equilibria. Let $g_{Au}^{-1}(k_A) - g_{Bl}(k_A) < 0$. Since g_{Au} is strictly decreasing, $g_{Au}(\overline{c})$ is the minimum of g_{Au}, and $g_{Au}(\overline{c}) > \underline{c}$. By Lemma 1,

$$g_{Au}^{-1}(g_{Au}(\overline{c})) - g_{Bl}(g_{Au}(\overline{c})) > \overline{c} - \overline{c} = 0.$$

The continuity of g_{Au} and g_{Bl} entails that there exits $k'_A \in (\underline{c}, k_A)$ such that

$$g_{Au}^{-1}(k'_A) - g_{Bl}(k'_A) = 0,$$

which contradicts the property of $k^e_{A\min}$ since $k'_A < k_A < k^e_{A\min}$. Hence, given each $k^*_A \in (k_l, k^e_{A\min})$, there exists a unique $z^* \in (0,1)$ such that

$$g_z\left(k^*_A, z^*\right) - g_{Bl}\left(k^*_A\right) = 0,$$

which implies that we can find a combination (x^*, y^*) satisfying z^*. ■

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