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Optimal Allocation of Social Cost for Electronic Payment System : A Ramsey Approach

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Abstract

Using a standard Ramsey approach, we examine an optimal allocation of the social cost for electronic payment system in the context of a dynamic general equilibrium model where money is essential. The benevolent government provides electronic payment services and allocates the relevant social cost through taxation on the beneficiaries' labor and consumption. A higher tax rate on labor yields the following desirable allocations. First, it implies a lower welfare loss due to the distortionary consumption taxation. It also enhances economy of scale in the use of electronic payment technology, reducing per transaction cost of electronic payment. Finally, it saves the cost of withdrawing and carrying around cash by reducing the frequency of cash trades. All these channels together imply optimality of the unity tax rate on labor.

Keywords: cash, electronic payment, social cost, Ramsey problem
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1. Introduction

Electronic payment system requires record-keeping and information-process technologies for transaction clearing and verification purposes. However, the installment and operation of such technologies incur some resource cost from the society's point of view. Hayashi and Keeton (2012) refer to this resource cost as the "social cost" of using electronic payment methods and distinguish it from the "private cost" which includes the fees imposed by one party to another. This social cost has the distinct feature of being largely composed of fixed cost which does not rely on the frequency or the value of transactions. That is, once the electronic record-keeping and information-process devices are in place at a substantial cost, electronic payment services could be provided with trivial marginal cost.

Considering such a high social cost incurred in the installment of electronic payment technology, it is quite natural to ask how the social cost should be allocated. However, to the best of our knowledge, no study has thoroughly addressed this question. One stream of the recent studies on electronic payment system has focused on the issues related to its private cost such as surcharge and interchange fee (e.g., Schwartz and Vincent 2006, Monnet and Roberds 2008, Prager et al. 2009, Bolt, Jonker and Renselaar 2010, Rochet and Wright 2010, Verdier 2011, Shy 2012b, and Lee 2014). The other stream of the recent works has deliberated the issues related to the competition among payment instruments, in particular between fiat money and electronic means of payment (e.g., He, Huang and Wright 2008, Klee 2008, Kim and Lee 2010, Li 2011, Lotz and Vasseli 2013, Lotz and Zhang 2013, Schuh and Stavins 2013, and Telyukova 2013).

This paper examines an optimal allocation of the social cost for electronic payment system by incorporating a standard Ramsey approach into a dynamic general equilibrium model where money is essential. Specifically, we adopt a standard search theoretic monetary model augmented with distribution of wealth. We consider wealth heterogeneity across agents

to capture economy of scale in the use of electronic payment system. At the beginning of a period, each agent becomes a buyer, a seller, or a non-trader with some probability. Each buyer is then randomly relocated to match with a seller. After the realization of such a relocation shock, a buyer determines whether to carry cash or to open a checking account for a subsequent pairwise trade. Carrying cash incurs some disutility cost due to the inconvenience and the risk of loss. A buyer can also pay in cash by withdrawing it from her checking account where, for simplification, cash-withdrawing cost is assumed to be the same as its carrying cost. A buyer opening a checking account can also use electronic payment system by shouldering its fixed installment cost.

Since agents cannot commit to their future actions and trading histories are private, all trades in a pairwise meeting are *quid pro quo*. Hence, either cash or checking-account deposit should be transferred in exchange for goods produced where the terms of trade are determined by a buyer's take-it-or-leave-it offer. Electronic transactions based on checking accounts require the installment of electronic payment system which incurs a fixed cost to its service provider (hereinafter referred to as the government). The government collects the fixed cost from the people who transfer and receive money via electronic payment system. It is worth noting that per transaction cost of electronic payment declines as more transactions are made via electronic payment system. This captures economy of scale in the use of electronic payment technology. As a key source of the efficiency for electronic payment, this economy of scale is determined by the choice of means of payment by heterogenous agents with different monetary wealth.

We use the model to examine an optimal allocation of the social cost for electronic payment system. Our Ramsey problem is a standard one—that is, in order to maximize social welfare, the government chooses a policy on how to allocate the social cost for electronic payment system across buyers and sellers who use the system in pairwise trades. Under the assumed buyers-take-all trading protocol, this problem is transformed into that of choosing a

tax scheme on the beneficiaries (i.e., buyers) of electronic transactions in the form of taxation on labor or on consumption. Since the endogeneity of a nondegenerate wealth distribution rules out closed-form solutions, the Ramsey problem is solved numerically. The main results are as follows.

The choice of taxation on a buyer's labor or consumption has not only an intensive-margin effect on the terms of pairwise trade for a given cost per electronic transaction, but also an extensive-margin effect on the choice of means of payment. The welfare implications of these two effects depend crucially on a nondegenerate distribution of wealth and economy of scale in the use of electronic payment system.

With regard to the intensive-margin effect, the labor taxation turns out to have a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred electronically, whereas the consumption taxation turns out to be distortionary in the sense that it affects quantity consumed in the electronic transactions. Hence, labor taxation is preferred by relatively poor agents whose consumption is sufficiently small, whereas consumption taxation is preferred by relatively rich agents whose consumption is sufficiently large. As the tax rate on labor increases (or the tax rate on consumption decreases), the difference between average quantity of good consumed by buyers and that produced by sellers decreases.

Further, the labor taxation has an extensive-margin effect on the choice between electronic payment and cash payment, which then affects the cost per electronic transaction via economy-of-scale channel. In particular, the frequency of electronic transactions is shown to increase with the tax rate on labor. When the tax rate on labor is zero for a given cost per electronic transaction, a poor buyer with sufficiently high marginal utility of consumption is willing to pay in cash rather than using electronic payment even if the cash-withdrawing cost exceeds the tax burden associated with using electronic payment. However, as the government increases the tax rate on labor, a buyer with a given wealth increasingly prefers

electronic payment than cash payment. This enhances economy of scale in the use of electronic payment system, which in turn decreases per transaction cost of electronic payment further and hence amplifies an extensive-margin effect of the labor taxation. This extensive-margin effect also decreases the fraction of buyers who prefer consumption taxation.

These altogether imply that overall transaction cost, including welfare loss due to the social cost for electronic payment system (which is raised in the form of labor taxation and distortionary consumption taxation) and cash-withdrawing cost, decreases with the tax rate on labor. As a result, the unity tax rate on labor (or zero tax rate on consumption) yields an optimal allocation of the social cost for electronic payment system.

The paper is organized as follows. Section 2 describes the model economy. Section 3 defines a symmetric stationary equilibrium. Section 4 formulates the Ramsey problem for the benevolent government and examines an optimal allocation of the social cost for electronic payment system. Section 5 summarizes the paper with a few concluding remarks.

2. Model

The background environment is in line with a standard random matching model of money (e.g., Shi 1995 and Trejos and Wright 1995) augmented with distribution of money holdings such as Camera and Corbae (1999), Zhu (2003), Berentsen, Camera and Waller (2005), and Molico (2006). A nondegenerate distribution of monetary wealth is considered here to capture economy of scale in the use of electronic payment system. Time is discrete. There are $N \geq 3$ number of "islands" with N types of divisible and perishable goods. In each island, there is a $[0, 1]$ continuum of infinitely lived agents. We will refer to an agent whose home-island is $n \in \{1, 2, ..., N\}$ as a type-n agent. A type-n agent potentially produces good n only which incurs a disutility cost of $c(q) = q$ for producing q units of good, whereas a type-n agent potentially consumes good $(n + 1)$ only (modulo N) which gives an utility of

$u(q)$ for consuming q units of good where $u'' < 0 < u'$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0, 1)$.

There are three exogenous nominal quantities that describe the stock of money: upper bound on individual money holdings, size of the smallest unit of money, and average money holdings per each type of agents. We normalize the smallest unit to be unity so that the set of possible individual money holdings consists of integer numbers, namely, $\mathbb{M} = \{0, 1, 2, ..., M\}$ where $M > 0$ denotes the upper bound required for compactness. We denote the average money holdings per each type of agents by $\bar{m} > 0$.

At the beginning of each period, each island is randomly connected to another island, which can be interpreted as a preference shock in the sense that an agent becomes a buyer or a seller with an equal probability $1/N$ or an agent is neither a buyer nor a seller with the remaining probability $1-(2/N)$. Now, a seller just stays on her home island, whereas a type-n buyer faces a relocation shock so that she should move to a "village" $j \in \{0, 1, 2, ..., M\}$ of $(n + 1)$ island where the village j consists of agents holding j amount of money. For a subsequent pairwise trade in a relocated island, each buyer determines how much to trade and whether to carry it in cash or deposit it into a checking account. An idle money that is not necessary for an immediate subsequent trade can be kept at home without any cost. The government has an intra-temporal record-keeping technology on checking accounts, but not on agents' trading histories. Other than the account-related tasks, the government does not engage in any other economic activities, including consumption or production of any goods.

In a newly migrated island, each buyer is matched with a seller for a pairwise trade. Trading histories are private and agents cannot commit to future actions, which rules out any possibility of credit trades and hence a medium of exchange is essential (see, for instance, Kocherlakota 1998, Wallace 2001, Corbae, Temzelides and Wright 2003, and Aliprantis, Camera and Puzzello 2007). That is, either cash or checking-account deposit should be transferred in exchange for goods produced.

In a pairwise trade, a buyer makes a take-it-or-leave-it offer (q, p) to a seller where q denotes quantity of goods produced by the seller for the buyer and p denotes the amount of money transferred by the buyer to the seller.[1] A buyer can pay p in cash by carrying it to a bilateral meeting at the disutility cost of ηp or by withdrawing it from her account at the same disutility cost.[2] Then, even a buyer who is willing to purchase good in cash will deposit the relevant amount of money into a checking account and withdraw it on the spot for a pairwise trade.

Transactions via an electronic payment method require the installment and operation of electronic payment system, which incur the government (service provider) a fixed cost of Ω. In practice, the setting up of electronic payment system entails substantial fixed cost, but trivial marginal cost. In order to focus on the former, the latter is assumed to be zero. From now on, a debit card will be regarded as the representative method of electronic payment since it is one of the primary means of electronic payment for in-store purchases and typically lacks in credit function. If a buyer uses a debit card to transfer p amount of money to a seller, the government withdraws it from the buyer's account and transfers it to the seller in cash.

In order to provide debit-card service to the society, the government should raise resources Ω to pay for the fixed cost incurred in installing the debit-card payment system. First of all, unlike a representative-agent model such as Lagos and Wright (2005), our model allows for wealth heterogeneity across agents including those with no money. Hence, it is not feasible to allocate the social cost for debit-card payment system in a lump-sum fashion. Also, under the assumption that cash-withdrawing cost from a checking account is the same as its

[1] As shown in Head and Kumar (2005) and Rocheteau and Wright (2005), for instance, this trading protocol can mimic an allocation in a competitive market (or a market with competitive pricing posting). Suppose simply type-n buyers in j village is collectively matched with type-$(n+1)$ sellers in k village and then buyers and sellers trade in a competitive market (or in a market with competitive price posting). Then an equilibrium allocation in the market would be reminiscent of an allocation with a buyers-take-all bargaining where a buyer has j amount of money and a seller has k amount of money.

[2] In Section 4, we will consider the case in which proportional cost η is replaced with fixed cost. Also, we will consider the case in which cash-handling cost is borne by a seller.

carrying cost, taxation on cash traders is not feasible because they would then evade tax by carrying around cash which is not in the government information system. On the other hand, debit-card transactions are notified to the government because they are cleared through the debit-card payment system operated by the government. Therefore, the government can effectively collect the required resources from debit-card traders.

Specifically, the government raises the social cost Ω by levying ω per debit-card transaction where ω satisfies the following government's budget constraint:

$$\Omega = \mathbf{S}\left(\tau\omega + \tau^c\omega\right). \tag{1}$$

Here $\mathbf{S}$ denotes the frequency of debit-card transactions, $\tau \in [0,1]$ is the share of social cost allocated to a buyer using a debit card, and $\tau^c = (1-\tau) \in [0,1]$ is that to a seller accepting a debit card.[3] For a given (Ω, τ), $\mathbf{S}$ captures economy of scale in the use of debit-card payment technology in the sense that a higher $\mathbf{S}$ implies a lower cost per debit-card transaction (ω). We assume that debit-card traders pay the cost to the government by producing their specialized types of goods. We will show in Section 4 (Optimal Allocation of Social Cost) that, with the bargaining rule of a buyer's take-it-or-leave-it offer, the cost share $\tau \in [0,1]$ allocated to a buyer can be interpreted as a taxation on the buyer's labor required to produce output for tax payment. On the other hand, the cost share $\tau^c \in [0,1]$ allocated to a seller can be interpreted as a taxation on the buyer's consumption.

For a type-n agent, per-period utility is given by

$$u(q_{n+1}) - \tilde{q}_n - \tau\omega\mathbb{I} - \eta c(1-\mathbb{I}) \tag{2}$$

[3] As discussed in the Introduction, the installation cost in the real world is a one-time cost. In our model, however, the government raises Ω in each period. Our interpretation for this is as follows: the government first finances total installation cost Φ from the outside of model and then raises Ω (annuity value of Φ, $\Omega = (1-\beta)\Phi$) in each period to smooth out the taxes through time.

where q_{n+1} is consumption of good $n+1$, $\tilde{q}_n$ is production of good n (including the share of social cost $\tau^c\omega$ borne as a seller by accepting debit cards), $\mathbb{I}$ is an indicator function that equals 1 if a type-n agent as a buyer meets a type-$(n+1)$ agent and trades via a debit card, and c is the amount of cash withdrawn by a type-n agent for a pairwise trade.

After a pairwise trade, a buyer returns to her home island and the information on each checking account is completely wiped away. An agent goes on to the next period with the end-of-period money holdings.

3. Equilibrium

We study a symmetric (across "islands" or specialization types) and stationary equilibrium. For a given (Ω, η) and a government policy on τ, a symmetric steady state consists of functions (v, π) and ω that satisfy the conditions described below. The functions $v : \mathbb{M} \to \mathbf{R}$ and $\pi : \mathbb{M} \to [0, 1]$ pertain to the beginning of a period and prior to the realization of a preference shock such that $v(m)$ is the expected discounted value of having monetary wealth m and $\pi(m)$ is the fraction of each island with monetary wealth m.

Consider a generic pairwise meeting between a buyer with $b \in \mathbb{M}$ and a seller with $s \in \mathbb{M}$. We let

$$\Gamma(b, s) = \{p : p \in \{0, 1, ..., \min\{b, M - s\}\}\}. \tag{3}$$

That is, $\Gamma(b, s)$ is the set of feasible wealth transfers from the buyer to the seller. After the realization of a relocation shock, the buyer determines how much to offer (p) to the seller and how to pay it, cash $(\mathbb{I} = 0)$ or debit card $(\mathbb{I} = 1)$. Noting that the seller accepts all offers that leave her no worse off (tie-breaking rule), the buyer's problem can be expressed as

$$\max_{\mathbb{I} \in \{0,1\}} \left\{ \mathbb{I} \left[\max_{p \in \Gamma(b,s)} v_d(b) \right] + (1 - \mathbb{I}) \left[\max_{p \in \Gamma(b,s)} v_c(b) \right] \right\} \tag{4}$$

where

$$v_d(b) = u\left\{\beta v\left[s + p(b, s; v)\right] - \beta v(s) - \tau^c \omega\right\} + \beta v\left[b - p(b, s, v)\right] - \tau\omega \tag{5}$$

$$v_c(b) = u\left\{\beta v\left[s + p(b, s, v)\right] - \beta v(s)\right\} + \beta v\left[b - p(b, s, v)\right] - \eta p(b, s, v). \tag{6}$$

Let $p_d(b, s, v)$ and $p_c(b, s, v)$ be respectively the solution to the first bracket and the second bracket of the right-hand side in (4): that is,

$$p_d(b, s, v) = \arg\max_{p \in \Gamma(b,s)} u\left[\beta v(s + p) - \beta v(s) - \tau^c \omega\right] + \beta v(b - p) - \tau\omega \tag{7}$$

$$p_c(b, s, v) = \arg\max_{p \in \Gamma(b,s)} u\left[\beta v(s + p) - \beta v(s)\right] + \beta v(b - p) - \eta p. \tag{8}$$

Now, let $p^*(b, s, v)$ be the associated $p_i(b, s, v)$ for $i \in \{c, d\}$ such that $p^*(b, s, v) = p_d(b, s, v)$ if the buyer chooses to pay using a debit card ($\mathbb{I} = 1$) and $p^*(b, s, v) = p_c(b, s, v)$ if the buyer chooses to pay in cash ($\mathbb{I} = 0$).[4] Since $p(b, s, v)$ is discrete, $p^*(b, s, v)$ can be multi-valued, in which case we allow for all possible randomizations over them. Let $\Delta(b, s, v)$ be the set of measures that represents those randomizations. Then $\Delta(b, s, v)$ can be described as

$$\Delta(b, s, v) = \{\delta(\cdot; b, s, v) : \delta(m; b, s, v) = 0 \text{ if } m \notin \{b - p^*(b, s, v)\} \tag{9}$$

where $\delta(m; b, s, v)$ is the probability that in a pairwise trade the buyer with $b \in \mathbb{M}$ offers $b - m$ to the seller with $s \in \mathbb{M}$, ending up with m.

Now we can describe the evolution of wealth distribution induced by pairwise trades as follows:

[4] Without the loss of generality, we can disregard the case in which some fraction of p is paid in cash and the remainder using a debit card because once a debit card is used, cost per debit-transaction (ω) is imposed regardless of the amount of debit-card transaction.

$$\Pi(v) = \left\{ \pi : \pi(m) = \frac{1}{N} \sum_{(b,s)} \pi(b)\pi(s)[\delta(m) + \delta(b - m + s)] + \frac{N-2}{N}\pi(m) \text{ for } \delta \in \Delta(b, s, v) \right\}. \tag{10}$$

The first probability measure in the right-hand side of (10) corresponds to pairwise trades, whereas the second corresponds to all other cases. Noting that in (9) δ is defined over the post-trade money holdings of the buyer, the buyer's post-trade money holdings $(b - m + s)$ corresponds to the seller's post-trade wealth m $(= b - (b - m + s) + s)$. Notice also that the dependence of Π on v comes from the dependence of δ on v in (9).

Finally, let $g(b, s, v)$ be the be the maximized value of (4). Noting that the payoff as a seller with $m \in M$ is simply $\beta v(m)$, the expected value of holding m at the beginning of a period, $v(m)$, can be written as

$$v(m) = \frac{1}{N} \sum_{s} \pi(s) g(m, s, v) + \frac{N-1}{N} \beta v(m). \tag{11}$$

Definition 1 *For given (Ω, η, τ), a symmetric stationary equilibrium is a set of functions (v, π) and ω such that (i) the value function v satisfies (11); (ii) the probability measure π of wealth distribution satisfies $\pi \in \Pi(v)$ where $\Pi(v)$ is given by (10); (iii) ω satisfies the government's budget constraint, $\Omega = \omega \sum_{(b,s)} \pi(b)\pi(s)\mathbb{I}(b, s, v)$.*

The existence of a symmetric stationary equilibrium for some parameters is a straightforward extension of the existence results in Zhu (2003) and Lee, Wallace and Zhu (2005). That is, if $(\bar{m}, M/\bar{m})$ are sufficiently large respectively and (Ω, η) are not too large respectively, then there exists a monetary symmetric stationary equilibrium (v, π, ω) with v strictly increasing and strictly concave.

4. Optimal Allocation of Social Cost

We now examine an optimal allocation of the social cost for debit-card payment system using a standard Ramsey taxation approach.

4.1. The Ramsey Problem

In order to formulate a Ramsey problem for the benevolent government as a provider of debit-card payment services, we first define welfare as the lifetime expected discounted utility of a representative agent before the assignment of wealth according to a stationary distribution. Let $\mathbb{W}_\tau$ denote the welfare of a stationary equilibrium $(v_\tau, \pi_\tau, \omega_\tau)$ for a given policy τ. Then $\mathbb{W}_\tau$ can be expressed as follows:

$$\mathbb{W}_\tau = \frac{\pi_\tau \mathbf{U} \pi_\tau'}{(1-\beta)N}. \tag{12}$$

Here the element in row $b \in \mathbb{M}$ and column $s \in \mathbb{M}$ of the matrix $\mathbf{U}$ is

$$u[q(b,s,v_\tau)] - \tilde{q}(b,s,v_\tau) - \tau\omega\mathbb{I}(b,s,v_\tau) - \eta c(b,s,v_\tau)[1 - \mathbb{I}(b,s,v_\tau)]$$

with $c(b,s,v_\tau)$ denoting the amount of cash withdrawn for a pairwise trade and $\tilde{q}(b,s,v_\tau) = q(b,s,v_\tau) + \tau^c\omega\mathbb{I}(b,s,v_\tau)$ where the second term, $\tau^c\omega\mathbb{I}(b,s,v_\tau)$, captures the disutility from the production of output for taxation borne as a seller by accepting debit cards.

Under the buyer's take-it-or-leave-it trading protocol, the benefit principle is implementable in the sense that the social cost for debit-card payment system is borne by its beneficiaries (i.e., buyers) regardless of τ. That is, from (4), in a pairwise trade between a buyer with $b \in \mathbb{M}$ and a seller with $s \in \mathbb{M}$, the buyer's net payoff for $\mathbb{I} = 1$ is

$$u[q(b,s,v_\tau) - \tau^c\omega] - \tau\omega + \beta\{v_\tau[b - p_d(b,s,v_\tau)] - v_\tau(b)\} \tag{13}$$

where $q(b, s, v_\tau) = \beta v_\tau (s + p_d) - \beta v_\tau(s)$ and $\tau + \tau^c = 1$. Therefore, the choice of τ can be interpreted as allocating the social cost to the beneficiaries of debit-card transactions in the form of labor taxation ($\tau = 1, \tau^c = 0$) or consumption taxation ($\tau = 0, \tau^c = 1$) or a certain combination of the two taxations $[\tau, \tau^c \in (0, 1)]$. It is worth noting in the above equation that the labor taxation ($\tau = 1, \tau^c = 0$) has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred via a debit card. On the other hand, the consumption taxation ($\tau = 0, \tau^c = 1$) is distortionary in the sense that it affects quantity consumed with the debit-card transactions.

Now the Ramsey problem for the government is to choose the tax rate on labor (τ) and the implied tax rate on consumption ($\tau^c = 1 - \tau$) to maximize welfare (W_τ) taking into account its effect on the equilibrium reactions of buyers and sellers in pairwise trades.

Definition 2 *The Ramsey problem for the benevolent government is to choose a symmetric stationary equilibrium* $(v_\tau, \pi_\tau, \omega_\tau)$ *in Definition 1 which maximizes (12), or equivalently to choose* $\tau^* = \arg\max_{\tau \in [0,1]} \mathbb{W}_\tau$.

In order to compare the magnitude of welfare loss across different policies, we calculate the welfare cost of τ-policy relative to that of an infeasible lump-sum taxation.[5] More specifically, we first find Δ that solves

$$\bar{\mathbb{W}} = \frac{\pi_\tau \mathbf{U}_\Delta \pi_\tau'}{(1-\beta)N} \tag{14}$$

where $\bar{\mathbb{W}}$ is the welfare with lump-sum taxation and the element in row $b \in \mathbb{M}$ and column $s \in \mathbb{M}$ of $\mathbf{U}_\Delta$ is

$$u[q(b, s, v_\tau) + \Delta] - \tilde{q}(b, s, v_\tau) - \tau\omega\mathbb{I}(b, s, v_\tau) - \eta c(b, s, v_\tau)[1 - \mathbb{I}(b, s, v_\tau)].$$

[5] If the social cost for debit-card payment system is raised by imposing a lump-sum tax on each agent, all trades are made using debit cards and trading behaviors are not affected at all. However, as discussed in Section 2, lump-sum tax cannot be forced effectively in our model.

That is, Δ is an additive consumption compensation that makes the welfare with τ-policy equal to that with lump-sum taxation ($\bar{\mathbb{W}}$).[6] The welfare cost of τ-policy is then calculated as a ratio of Δ to the average consumption in the stationary equilibrium with τ-policy.

Even though the labor taxation (τ) has a lump-sum feature as in (13), it is not obvious at all whether a higher τ improves or deteriorates welfare. The marginal gain from a higher τ is $(\partial u/\partial \tau) = u'(q)\omega$, whereas the marginal labor cost of increasing τ is just ω. Hence, a higher τ is beneficial to relatively poor buyers whose consumption is small enough so that $u'(q) > 1$, whereas it is detrimental to relatively rich buyers whose consumption is large enough so that $u'(q) < 1$. This implies that the relatively poor buyers prefer labor taxation, whereas the relatively rich buyers prefer consumption taxation. Notice that for a given ω, a higher τ increases an intensive margin (output per unit of money) which then would increase the fraction of buyers with $u'(q) < 1$.

The labor taxation (τ) also has an extensive-margin effect on the choice between electronic payment and cash payment. In order to illustrate this point simply, consider a pairwise trade between a buyer and a seller who respectively have money holdings of b and s. Suppose money is divisible and the value function v is differentiable. Now, for given (η, ω, τ), let $\bar{p}(b, s)$ denote the amount of money transfer at which a buyer is indifferent between cash and debit-card payment; that is,[7]

$$u\left\{\beta\left[v(s+\bar{p}(b,s)) - v(s)\right]\right\} - u\left\{\beta\left[v(s+\bar{p}(b,s)) - v(s)\right] - (1-\tau)\omega\right\} + \tau\omega = \eta\bar{p}(b,s). \quad (15)$$

It can be shown that $\bar{p}(b, s)$ decreases with τ and debit-card payment is preferred for $p(b, s) > \bar{p}(b, s)$. This implies that as τ increases, more buyers are willing to use debit cards and hence

[6] Consumption compensation is calculated as an addition to the consumption with τ-policy rather than as a multiple of consumption with τ-policy because consumption is zero in some pairwise meetings.

[7] Note that the left-hand side of (15) is decreasing in p, whereas the right-hand side is increasing in p. Also, the left-hand side is greater than the right-hand side when p is sufficiently close to 0. Hence, $\bar{p}(b, s)$ in (15) is well defined.

the fraction of debit-card traders with $u'(q) > 1$ would increase.

Further, it is worth noting that this extensive-margin effect will lower per transaction cost (ω) of debit-card payment system via economy-of-scale channel, which in turn would enhance both intensive-margin and extensive-margin effects of a higher τ. For instance, a lower ω will decrease $\bar{p}$ in (15), implying the larger fraction of buyers using debit-card payment.

The above discussion implies that the effect of τ on welfare depends crucially on a non-degenerate distribution of wealth across agents and economy of scale in the use of debit-card payment system. The endogeneity of a nondegenerate wealth distribution rules out closed-form solutions and hence, in what follows, we tackle our question based on numerical solutions.

4.2. Parameterization

In order to solve the model numerically, we parameterize the basic environment as follows. We first assume $N = 3$, the smallest number of types of agents which is consistent with no double-coincidence meeting, and $\beta = 0.99$. We set $(\bar{m}, M) = (40, 3\bar{m})$ so that the indivisibility of money and the upper bound on money holdings are not too severe.[8]

We let $u(q) = \theta \ln(1 + q)$ where θ is chosen, together with (Ω, η), to fit the model to the data. Based on the 2011 Survey of Consumer Payment Choice, Shy (2012a) reports $\mathbf{S} = 0.511$ where $\mathbf{S}$ is the fraction of debit-card transactions out of cash and debit-card transactions. In addition, based on the 2010 Diary of Consumer Payment Choice by the Federal Reserve Bank of Boston, Stavins (2012) reports the average value of debit-card transactions relative to that of cash, denoted by $\mathbf{D}$, equals to 1.65. In our model, $(\theta, \Omega, \eta) = (2.1, 2.0 \times 10^{-3}, 8.42 \times 10^{-4})$ with $\tau = 0$ generate $\mathbf{S} = 0.512$ and $\mathbf{D} = 1.50$. We here consider $\tau = 0$ because in the U.S.,

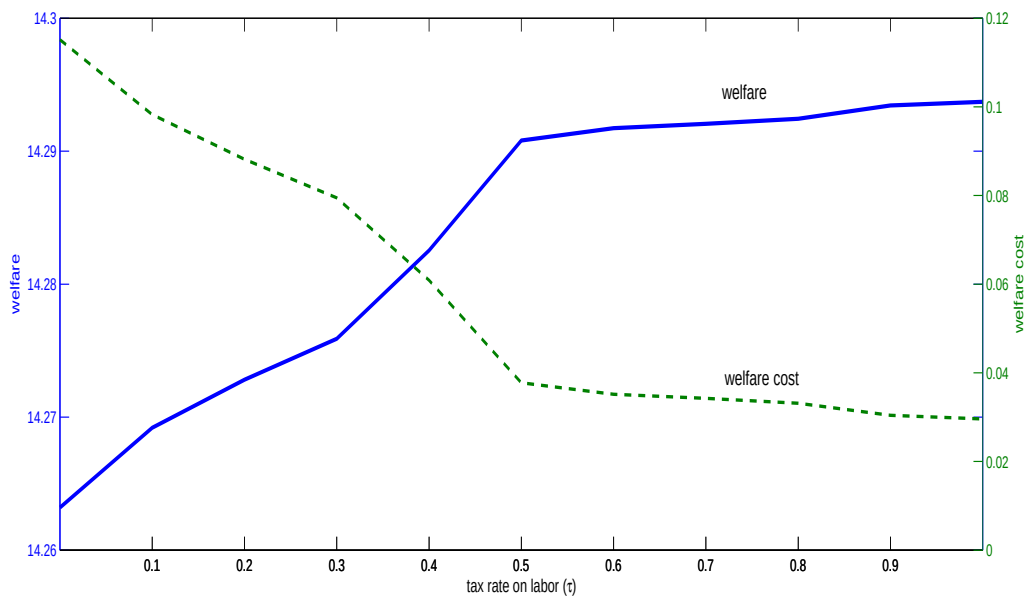
[8] In this type of model, almost all monetary offers are either 0 or 1 if the indivisibility of money is too severe. However, it is not the case in our examples. In addition, $M = 3\bar{m}$ is large enough so that almost no one is at the upper bound in a stationary equilibrium and hence the result would be hardly affected even if a larger M were assumed.

almost all the cost of debit-card transaction is imposed to sellers on the surface. Notice that $\Omega = 2.0 \times 10^{-3}$ corresponds to 0.18% of $q^* = \arg\max[u(q) - q] = 1.1$, which is close to the estimate in Aiyagari, Braun and Eckstein (1998).[9]

4.3. Labor Taxation vs Consumption Taxation

Figure 1 shows welfare level and welfare cost, respectively, as a function of the tax rate on labor (τ). The welfare increases with τ and the solution to the Ramsey problem turns out to be $\tau^* = 1$. That is, the unity tax rate on labor (or zero tax rate on consumption) attains the highest welfare. The welfare cost with the zero tax rate on consumption ($\tau = 1$) remains 0.02% only, but it increases up to 0.11% with the zero tax rate on labor ($\tau = 0$).

Figure 1: Welfare and welfare cost



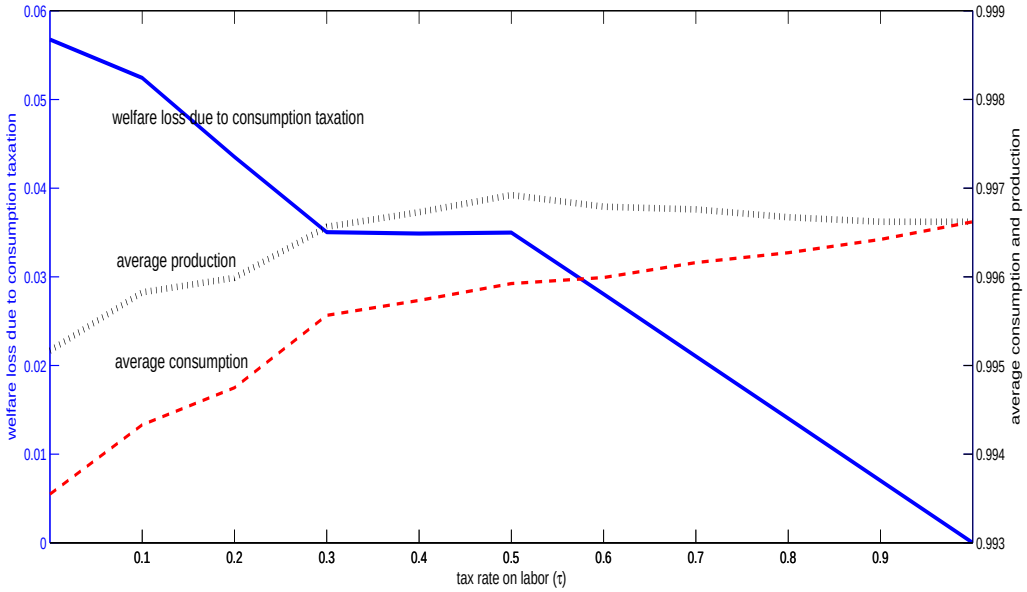
The underlying mechanism that renders the unity tax rate on labor (or zero tax rate on consumption) optimal can be explained in terms of (i) its intensive-margin effect on the

[9] Aiyagari, Braun and Eckstein (1998) estimate that the cost incurred by the U.S. banks in providing demand-deposit services is around 0.2% of GDP.

quantity consumed in exchange for money transferred via debit card for a given cost per debit transaction and (ii) its extensive-margin effect on the means-of-payment choice between debit card and cash, which affects the cost per debit transaction. The welfare implications of these two effects hinge on a nondegenerate distribution of wealth and economy of scale in the use of debit-card payment system.

First, with regard to the intensive-margin effect, a higher tax rate on labor implies a relatively lower tax rate on consumption, mitigating consumption distortion. Hence, the higher the tax rate on labor (consumption) is, the more it is preferred by the relatively poor (rich) whose consumption is sufficiently small (large). Figure 2 shows that the gap between average quantity of good consumed by buyers and that produced by sellers shrinks as the tax rate on labor increases. As a result, the welfare loss due to the distortionary consumption taxation decreases and eventually converges to zero as the tax rate on labor approaches to 1.

Figure 2: Welfare loss due to consumption taxation



Second, a higher tax rate on labor also has an extensive-margin effect on the choice of payment method, debit card or cash. Figure 3 shows that the frequency of debit-card transactions ($\mathbf{S}$) increases with τ, which eventually lowers cost per debit-card transaction ($\omega = \Omega/\mathbf{S}$). The two extreme policies, $\tau = 1$ and $\tau = 0$ for a given (ω, η), are useful to understand why the frequency of debit-card transactions ($\mathbf{S}$) increases with τ. In the case of $\tau = 1$, all transactions will be made using debit cards as long as $p > (\omega/\eta)$. However, when $\tau = 0$, transactions can be made in cash even if $p > (\omega/\eta)$. That is, p exceeding (ω/η) will be paid in cash rather than using a debit card as long as $u\{\beta v[s + p(\cdot)] - \beta v(s)\} - u\{\beta v[s + p(\cdot)] - \beta v(s) - \omega\} > \eta p(\cdot)$. The latter happens to a poor buyer in a pairwise trade when $\beta v[s+p(\cdot)]-\beta v(s) = q_c$ is sufficiently small. This higher frequency of debit-card transactions with a higher τ has the effect of enhancing economy of scale in the sense that it subsequently lowers cost per debit-card transaction, as shown in Figure 3. Also, Figure 4 shows that the extensive-margin effect decreases the fraction of buyers who prefer consumption taxation $[u'(q) < 1]$.

Third, Figure 5 shows that the total cost of withdrawing cash decreases with the tax rate on labor. This is because the frequency of cash transaction decreases as τ increases. More transactions are subject to labor taxation and hence the social cost for debit-card payment system allocated in the form of labor tax also increases with the tax rate on labor.

In sum, Figure 6 shows that overall transaction cost, including welfare loss due to the social cost for debit-card payment system (which is raised in the form of labor taxation and distortionary consumption taxation) and cash-withdrawing cost, decreases with the tax rate on labor. This immediately implies the results in Figure 1.

Finally, we check the robustness of our main results to different settings. We first consider the fixed cash-handling cost for sellers. That is, a seller accepting cash incurs a fixed handling cost κ. In computing a stationary equilibrium for this case, we set κ to fit the model to the U.S. data on the fraction of debit-card transaction ($\mathbf{S} = 0.511$) and the ratio of average

Figure 3: Frequency of debit-card transactions and cost per transaction

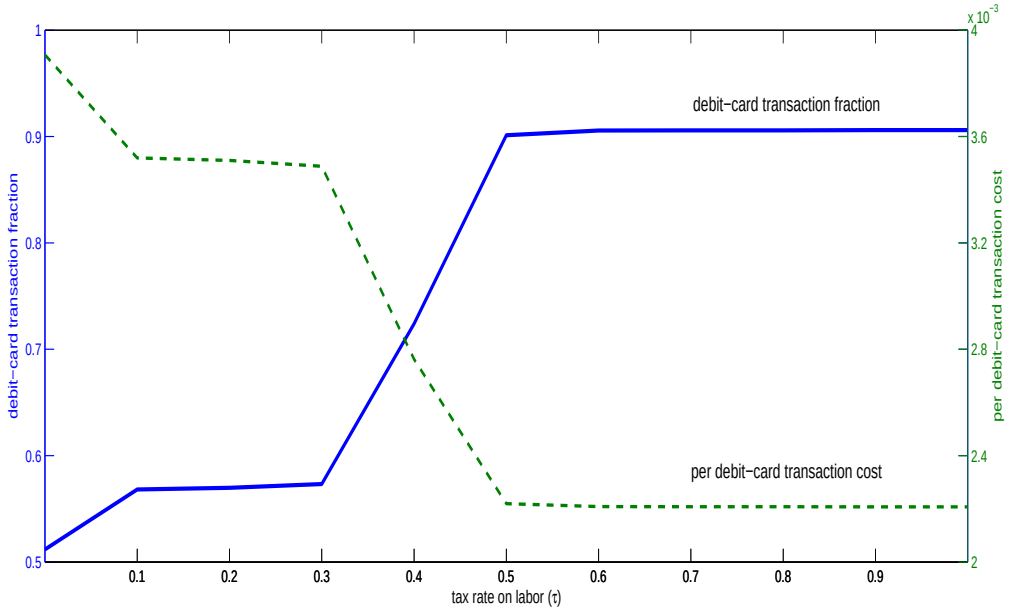


Figure 4: Fraction of buyers with $u'(q) < 1$

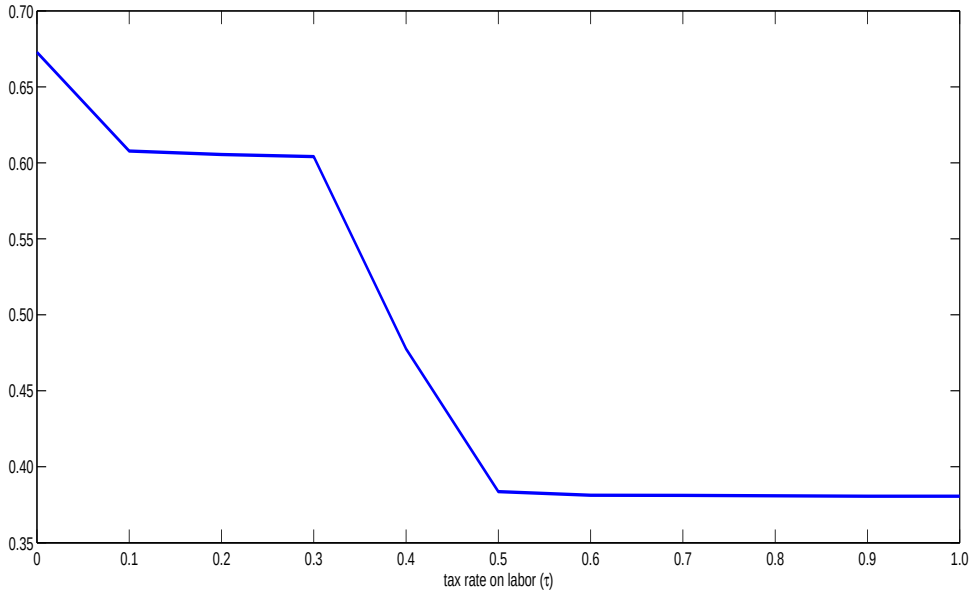


Figure 5: Cash-withdrawing cost and labor tax payment for debit-card system

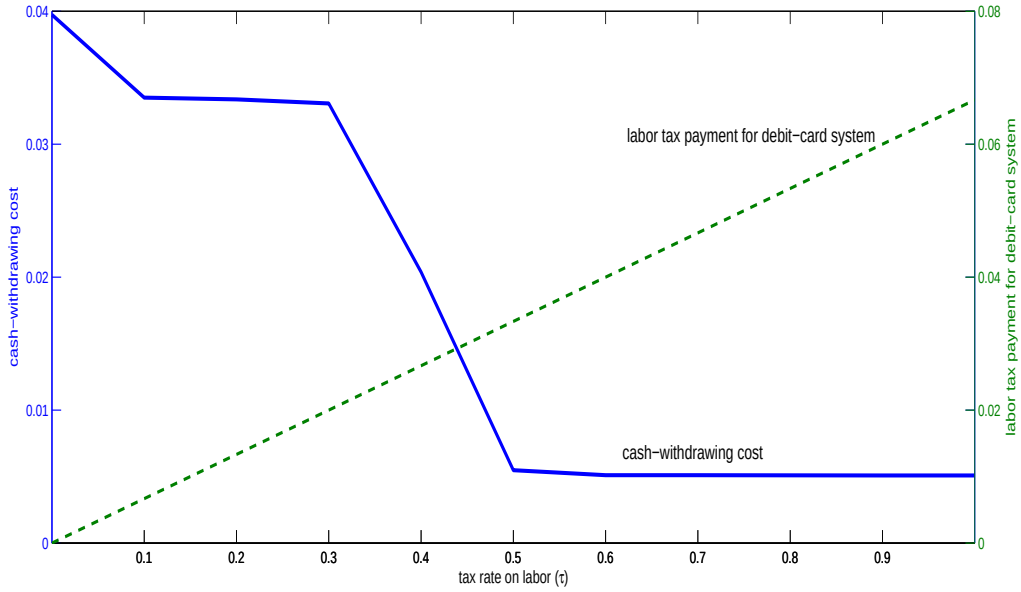
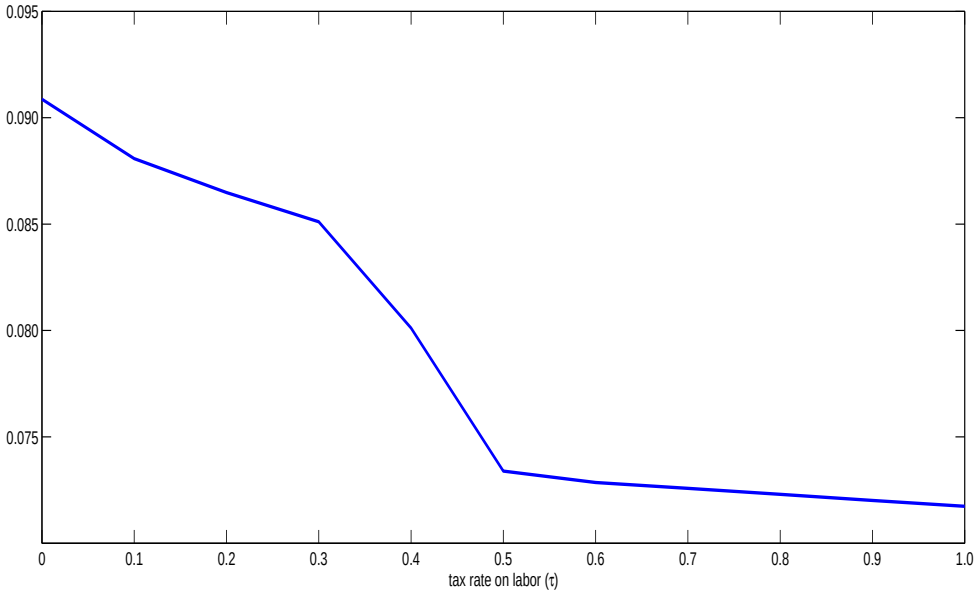
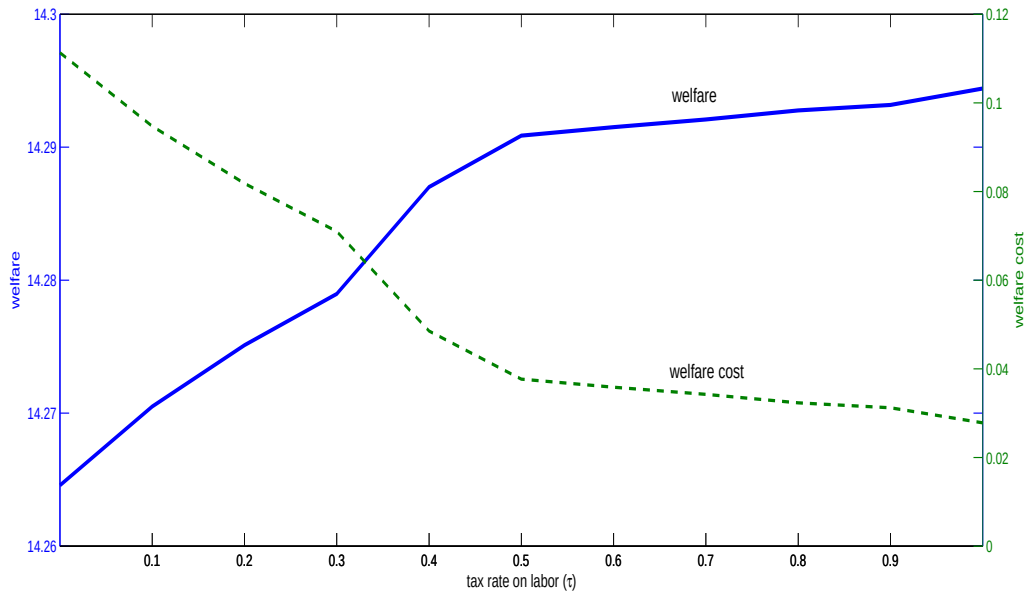


Figure 6: Overall transaction cost



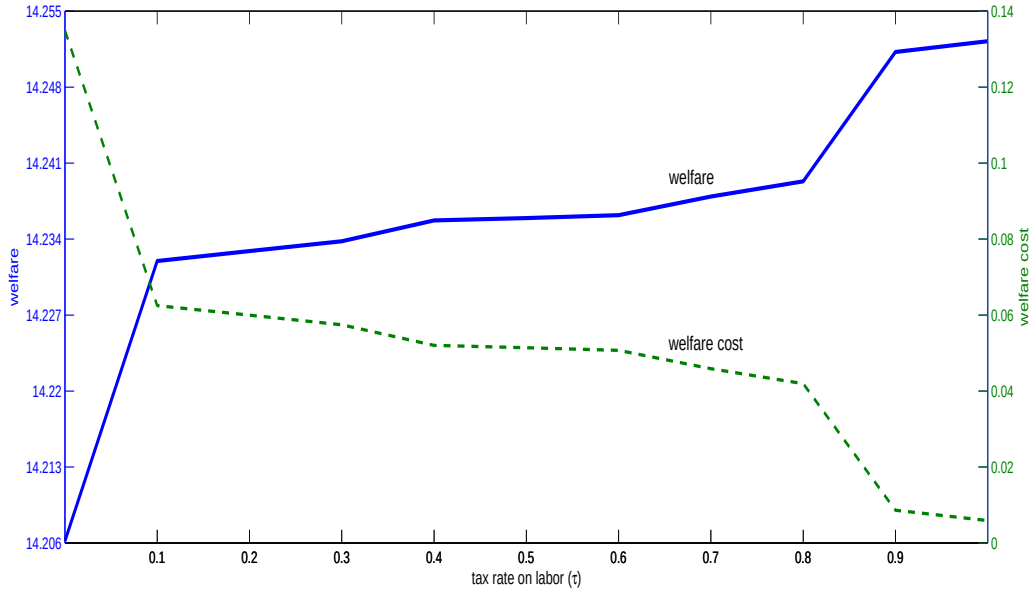
value of debit-card transactions to that of cash ($\mathbf{D} = 1.65$). The model parameterized with $(\Omega, \eta, \kappa) = (2.0 \times 10^{-3}, 8.0 \times 10^{-4}, 1.15 \times 10^{-4})$ implies $\mathbf{S} = 0.512$ and $\mathbf{D} = 1.50$. As reported in Figure 7, this variation does not change optimality of the unity tax rate on labor.

Figure 7: Fixed cash-handling cost for sellers



As another robustness check, we consider the case in which the cash-withdrawing cost for a buyer is fixed, whereas the cash-handling cost for a seller is proportional. That is, withdrawing cash incurs a fixed cost $\bar{\eta}$ for a buyer, while cash-handling cost for a seller increases with the amount of cash transaction at a rate of $\tilde{\kappa}$. In computing a stationary equilibrium for this case, we again choose $(\bar{\eta}, \tilde{\kappa})$ to fit the model to the data. The model parameterized with $(\Omega, \bar{\eta}, \tilde{\kappa}) = (3.5 \times 10^{-3}, 1.01 \times 10^{-3}, 1.0 \times 10^{-3})$ implies $\mathbf{S} = 0.568$ and $\mathbf{D} = 1.56$. Figure 8 shows that optimality of the unity tax rate on labor is also immune to this variation.

Figure 8: Fixed cash-withdrawing cost and proportional cash-handling cost



5. Concluding Remarks

In this paper, we have explored an optimal allocation of the social cost for electronic payment system by incorporating a standard Ramsey taxation approach into an off-the-shelf matching model of money. Our results suggest that economy of scale in the use of electronic payment technology is enhanced with the tax rate on labor. This then decreases not only per transaction cost of electronic payment and cash withdrawing cost, but also welfare loss due to the distortionary consumption taxation. As a result, the unit tax rate on labor (or zero tax rate on consumption) yields an optimal allocation of the social cost for electronic payment system.

Finally, we have focused on an allocation scheme of the social cost for electronic payment system without considering its implications from the viewpoint of industrial organization. For instance, we do not deal with an issue on how to impose fees on merchants and consumers

by a profit-maximizing card issuer. We leave to future research the generalization of our model in which the private and the social cost can be explored systematically.

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