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Abstract

The welfare cost of inflation is explored via a search-theoretic model in which along with non-interest-bearing cash, interest-bearing liquid and illiquid assets are available. With inflation, agents are willing to replace higher-return illiquid assets with lower-return liquid assets for consumption purchases. The opportunity cost incurred by this adjustment turns out to have quantitatively significant implications on the cost of inflation. A parameterized version of the model suggests that the cost of 10% inflation with liquid and illiquid interest-bearing assets is almost 3 times larger than that in a cash-only model. This implies that most existing measures of inflation cost with narrow money are substantially underestimated.

Keywords: cash, narrow money, broad money, portfolio shift, inflation cost
JEL classification: E31, E40, E50

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1. Introduction

Since the 1980s, interest-bearing checkable deposits such as the NOW account have been steadily expanded and in July 2011, the Regulation Q that banned interest payment on demand deposits was eventually repealed. In this regard, Cysne (2003), Jones et al. (2004), and Cysne and Turchick (2010), among others, show that the presence of interest-bearing liquid assets lessens the cost of inflation in a shopping-time or a money-in-utility-function model. These works essentially take notice that the effect of inflation on the demand for interest-bearing liquid assets is not identical to that for cash.[1]

We revisit the issue via search-theoretic monetary models wherein the benefit of monetary exchange is spelled out explicitly. We first consider the model by Lagos and Wright (2005) where at the end of each period, an agent chooses a portfolio consisting of non-interest-bearing cash and interest-bearing checkable deposit. An agent can use cash as well as a debit card based on her checkable deposit to pay for consumption purchases. The cost associated with a debit-card transaction is proportional to the transaction amount and is borne by a seller. In equilibrium, the wealth distribution is degenerate and agents do not demand more monetary wealth than what they need to trade.

In the model, the implication of an interest-bearing liquid asset on the inflation cost is basically identical to that in the aforementioned literature: i.e., the presence of interest-bearing liquid assets reduces the cost of inflation by alleviating the negative effect of inflation on M1 (cash+checkable deposit) demand and hence on consumption.

It is worth noting, however, that the model with narrow money (e.g., M1) mainly focuses on the effect of inflation on money demand stemming from the accrued interest on liquid assets. This M1-demand-oriented approach does not take into account explicitly the cost

[1]For instance, in Jones et al. (2004) and Cysne and Turchick (2010), the user cost of interest-bearing liquid asset is invariant to inflation, whereas the user cost of cash is proportional to inflation.

associated with portfolio shifts, in particular, between liquid and illiquid assets. Put in another way, as inflation goes up, agents are willing to replace a higher-return illiquid asset with lower-return liquid assets for consumption purchases and this change in portfolios would incur the loss of return. To the best of our knowledge, no existing work has analyzed the cost of inflation by shedding light on this point.

In order to capture the effect of the portfolio shift on the cost of inflation, broad money (e.g., M3) rather than narrow money should be taken into account. Hence, we adopt the search-theoretic model of Zhu and Wallace (2007) in which other than cash and checkable deposit, there is an illiquid one-period coupon bond that accrues an exogenous coupon payment at maturity. In equilibrium, the wealth distribution is nondegenerate and a sufficiently rich agent demands an illiquid asset because she holds idle wealth, "idle" in the sense that it is not necessary for an immediate consumption purchase. In this sense, monetary wealth here can be interpreted as broad money.

In the model, the welfare cost of 10% inflation turns out to be almost threefold as large as that in a cash-only model. In particular, the cost associated with the portfolio shift from an illiquid asset to a liquid asset accounts for around two-thirds of the welfare difference between 0% and 10% inflation. This result suggests that the opportunity cost incurred by the portfolio shift due to inflation has significant implication on the cost of inflation and therefore, existing measures of it with cash only or M1 (cash + interest-bearing liquid asset) are substantially underestimated.

Finally, we check the robustness of the result to different settings. We first consider the model in which the return rate of illiquid asset is endogenized. More specifically, we endogenize the return rate of illiquid asset by having it depend on the government's budget constraint and the demand for illiquid asset across agents with different wealth. We also consider the model in which a proportional cost of debit-card transaction is replaced with a fixed cost. These variations of model, however, do not change our main result qualitatively:

i.e., the cost of 10% inflation with broad money still turns out to be considerably large compared to that in a cash-only economy.

2. Background Environment

In what follows, we will study two search-theoretic models of exchange, one is a model wherein monetary wealth can be interpreted as narrow money (e.g., M1) and the other is a model wherein monetary wealth can be interpreted as broad money (e.g., M3). We first describe the background environment that is common to both models.

There is a durable object called money that can be deposited into a checkable deposit from which a debit card can be used as a means of payment. The information on checking accounts is kept by the government which has technology for record keeping on intra-temporal financial histories associated with the accounts, but not on agents' trading histories. The government can access linear technology $G = \theta\mathfrak{D}$ where $\theta > 0$, G is the quantity of (perishable and divisible) general good produced and $\mathfrak{D}$ is the balance of the deposit at maturity. At the end of each period, the government redeems the balance in the checking account with θ units of general good per unit of matured deposit. Therefore, the government budget is always in balance. There is no return for the early-withdrawn deposit for consumption purchases. An agent obtains utility $\mathcal{U}(g)$ from consuming g units of the general good where $\mathcal{U}'' < 0 < \mathcal{U}'$, $\mathcal{U}(0) = 0$, and $\mathcal{U}'(0)$ is sufficiently large. Each agent maximizes discounted expected utility with the discount factor $\beta \in (0, 1)$.

In the decentralized market (hereinafter "DM"), each agent is randomly matched with another agent. In the meeting, trading histories are private and agents cannot commit to their future actions, which make a medium of exchange essential. (See, for example, Kocherlakota 1998, Wallace 2001, Corbae et al. 2003, Aliprantis et al. 2007, and Lagos and Wright 2008.) An agent obtains utility $u(q)$ from consuming q units of a perishable and divisible DM-good

where $u(\cdot)$ satisfies all the properties of $\mathcal{U}(\cdot)$ mentioned above. Production of q units of a DM-good incurs disutility q.

A buyer in a pairwise trade makes a take-it-or-leave-it offer (q, p) where q denotes the quantity of DM-good produced by a seller and p denotes the amount of money transferred to a seller. If a buyer uses a debit card to pay p, it is withdrawn from the buyer's account and transferred to the seller in cash where the disutility φp is borne by the seller. This disutility can be regarded as the transaction fee paid by retailers to debit-card providers in the real world. Meanwhile, there is no such disutility if a buyer uses cash, which conforms to the survey result.[2]

3. Inflation Cost with Narrow Money

We first study the welfare cost of inflation with a search-theoretic model wherein monetary wealth can be interpreted as narrow money (e.g., M1). Specifically, we incorporate the background environment of Section 2 into the framework of Lagos and Wright (2005).

Time is discrete and continues forever. A unit mass of infinitely-lived agents trade DM-good in a frictional DM and general good in a frictionless centralized market (hereinafter "CM"), which open sequentially in each date. Money is divisible and total stock evolves deterministically at a (gross) growth rate μ; i.e., $M_t = \mu M_{t-1}$ where M_t denotes the stock of money at the beginning of period t. It is well known that there is no equilibrium for $\mu < \beta$ and hence we assume $\mu \geq \beta$ with the understanding that $\mu = \beta$ is the limit as $\mu \to \beta$.

In DM, each agent becomes either a buyer or a seller with probability $\sigma \leq 1/2$, respectively, and enjoys utility given by $u(q) - q$. At the beginning of CM, each agent receives lump-sum transfer of new money and the government redeems the balance in the checking account to each agent with the promised return of θ units of general good per unit of ma-

[2]The survey done by the Food Marketing Institute shows that transaction cost of accepting a debit card is much more expensive than cash (see Humphrey 2004).

tured deposit. All agents can also produce, consume general good and choose a portfolio $\boldsymbol{\omega} = (C, D) \in \mathbb{R}^2_+$ that is carried into the next period where C and D denote cash and a checkable deposit, respectively. Production of q units of general good in CM incurs disutility q as in DM.

Turning to the equilibrium, let ϕ_t denote the unit price of money in period t in terms of general good. We will drop the time subscript t hereinafter and index the next (previous) period variable by $+1$ (-1), if there is no risk of confusion. We here consider a stationary equilibrium in which the real balance of monetary wealth is constant in CM: namely, $\phi M = \phi_{+1} M_{+1}$, which implies $\phi/\phi_{+1} = \mu$.

In CM, $W(\hat{\boldsymbol{\omega}})$, the value function for an agent who enters CM with $\hat{\boldsymbol{\omega}} = (c, d) = (\phi C, \phi D)$, satisfies

$$W(\hat{\boldsymbol{\omega}}) = c + d\,[1 + (\theta/\phi)] + \tau + \max\,[\mathcal{U}(g) - g] + \max\,[\beta V(\hat{\boldsymbol{\omega}}_{+1}) - \mu(c_{+1} + d_{+1})]$$

where τ is the real lump-sum transfer of new money $[\tau = (\mu - 1)\phi M]$ and $V(\hat{\boldsymbol{\omega}}_{+1})$ is the value function for an agent who enters DM with $\hat{\boldsymbol{\omega}}_{+1} = (c_{+1}, d_{+1})$. As in Lagos and Wright (2005), W is linear and the choice of $\hat{\boldsymbol{\omega}}_{+1} = (c_{+1}, d_{+1})$ does not depend on $\hat{\boldsymbol{\omega}} = (c, d)$, which conveniently allows us to restrict attention to the case where the portfolio distribution is degenerate at the beginning of each period. In DM, $V(\hat{\boldsymbol{\omega}})$ satisfies

$$\begin{aligned} V(\hat{\boldsymbol{\omega}}) \;\; = \;\; & \sigma\,\{u(q) + W\,[(c - \hat{p})\mathbb{I}^c, d - (\hat{p} - c)\mathbb{I}]\} + \\ & \sigma\,\{W(c + \hat{p}, d) - [q + (\varphi/\phi)(\hat{p} - c)\mathbb{I}]\} + (1 - 2\sigma)W(\hat{\boldsymbol{\omega}}) \end{aligned}$$

where $\hat{p} = p\phi$, $\mathbb{I} = 1$ if and only if $\hat{p} \in (c, c + d]$ and $\mathbb{I}^c = 1 - \mathbb{I}$. Notice that a linearity of W implies $q = \hat{p} - [(\varphi/\phi)(\hat{p} - c)\mathbb{I}]$ where the second term captures the disutility borne by the seller from accepting a debit card. By substituting $V(\cdot)$ into $W(\cdot)$, we have the following

portfolio-choice problem:

$$\max_{(c_{+1},d_{+1})} \left\{ \begin{array}{l} \sigma\left[u(q_{+1}) + W\left((c_{+1}-\hat{p}_{+1})\mathbb{I}^c_{+1}, d_{+1} - (\hat{p}_{+1}-c_{+1})\mathbb{I}_{+1}\right) - W(\hat{\boldsymbol{\omega}}_{+1})\right] \\ -\left[ic_{+1} + \left(i - \frac{\theta}{\phi_{+1}}\right) d_{+1}\right] \end{array} \right\} \tag{1}$$

where $i \equiv (\mu - \beta)/\beta$ is the opportunity cost of cash holdings which can be interpreted as a nominal return rate of the illiquid bond where the real return rate of an illiquid bond (r) satisfies $\beta(1+r) = 1$ in equilibrium and hence the standard Fisher equation, $i = (\mu-\beta)/\beta = [\mu - (1+r)^{-1}](1+r) = \mu(1+r) - 1$. (See also Lester et al. 2012.)

We now consider a cash-only economy. Since d is always equal to zero, $\mathbb{I} = 0$ and hence (1) can be simplified as

$$\max_{c_{+1}} \left\{\sigma\left[u(c_{+1}) - c_{+1}\right] - ic_{+1}\right\}$$

where we use $q^m_{+1} = c_{+1}$ because in an equilibrium, agents do not bring more money than what they need to trade in DM due to $\mu \geq \beta$. Then the agent chooses c_{+1} that satisfies

$$i = \sigma\left[u'(c_{+1}) - 1\right]. \tag{2}$$

Since $i = [(\mu/\beta) - 1]$, it is straightforward to show that

$$u'(c_{+1}) = u'(q^m_{+1}) = \frac{\mu - \beta}{\beta\sigma} + 1$$

which immediately implies that $q^m < q^* = \arg\max\{u(q) - q\}$ for $\mu > \beta$, $q^m = q^*$ for $\mu = \beta$ (the Friedman rule), and $(\partial q^m/\partial\mu) < 0$. This is the key relationship used in the previous literature to measure the cost of inflation.

We next turn to an economy with cash and interest-bearing checkable deposit. For the existence of equilibrium, we first assume that $\beta[(1/\mu) + r] \leq 1$.[3] Under the assumption,

[3] If $\phi < \beta(\phi_{+1} + \theta)$, there is no equilibrium and therefore in any equilibrium, $\phi \geq \beta(\phi_{+1} + \theta)$. This can

the following lemma suggests that an agent is willing to hold an interest-bearing checkable deposit if the return from the checkable deposit (θ) is not too small.

Lemma 1 *If* $\theta > \bar{\theta} \equiv [\varphi/(1-\sigma)\beta][\mu - \beta(1-\sigma)]$, $d_{+1} > 0$ *and* $q^b > q^m$.

Proof. See Appendix 1. ■

Now, the welfare cost of $i' > 0$ compared to $i = 0$ (the Friedman rule) can be measured by asking how much consumption should be compensated to bring the welfare at $i' > 0$ up to what it is at $i = 0$: i.e., the welfare cost of $i' > 0$ denoted by $\xi_{i'}$ solves

$$\sigma[u(q_0) - q_0] + \mathcal{U}(g^*) = \sigma\left\{u[q_{i'}(1+\xi_{i'})] - q_{i'}\right\} + \mathcal{U}[g^*(1+\xi_{i'})] \tag{3}$$

where $g^* = \arg\max[\mathcal{U}(g) - g]$. Let $\xi_{i'}^m$ be the welfare cost of $i' > 0$ in a cash-only economy and $\xi_{i'}^b$ be that in the presence of an interest-bearing liquid asset. The following proposition shows that the welfare cost of inflation with an interest-bearing liquid asset is smaller than that in a cash-only economy.

Proposition 1 *For* $i' > 0$, $\xi_{i'}^m > \xi_{i'}^b$.

Proof. See Appendix 1. ■

The intuition for the result is straightforward. When $i = 0$ (the Friedman rule), there is no opportunity cost of cash holdings and an economy with cash only is identical to an economy with cash and checkable deposit because the nominal net return rate of the checkable deposit should be also equal to 0 (see Appendix 1). When $i > 0$, however, the opportunity cost of cash holdings is $i = \mu(1+r) - 1$, whereas the cost of checkable deposits is $i - (\theta/\phi_{+1}) = \mu(1+r) - 1 - \mu(\theta/\phi) = \mu[1 + r - (\theta/\phi)] - 1$. Therefore, liquid-asset holdings in an economy with cash and checkable deposit are larger than those in a cash-only economy.

be expressed as $\beta[(1/\mu) + (\theta/\phi)] \leq 1$, which always holds if $\beta[(1/\mu) + r] \leq 1$ because the lower bound of ϕ is $\beta\theta/(1-\beta)$.

In sum, the presence of an interest-bearing liquid asset lessens the opportunity cost of liquid-asset holdings and hence the welfare cost of inflation. This is in line with the previous studies such as Bali (2000), Jones et al. (2004), and Cysne and Turchick (2010) where the overestimation problem of inflation cost under the assumption of a non-interest-bearing M1 is shown via a shopping-time or a money-in-utility-function model.

However, the model with narrow money like above cannot explicitly capture the opportunity cost stemming from the portfolio adjustment due to inflation. Put somewhat differently, inflation will cause portfolio shift across cash, checkable deposits and illiquid bonds, but the result above essentially captures only the effect on the demand for cash and checkable deposit. A potential cost resulting from the portfolio shift, in particular from higher-return illiquid assets to lower-return liquid assets, is left out.

4. Inflation Cost with Broad Money

In order to uncover the implication of the portfolio shift on the cost of inflation, we now incorporate the background environment of Section 2 into Zhu and Wallace (2007) which is a version of the random matching model of Shi (1995) and Trejos and Wright (1995) augmented with distribution of wealth. As we will see, wealth distribution is nondegenerate in equilibrium and some agents hold idle monetary wealth in the form of illiquid assets where "idle" means that it is not necessary for current-period consumption purchases. In the sense, we can interpret the monetary wealth in this model as broad money (e.g., M3).

There is a unit mass of each of $K > 2$ types of infinitely lived agents with K types of a DM-good and a general good. A type $k \in \{1, 2, ..., K\}$ agent produces only good k and consumes only good $k+1$ (modulo K), and enjoys per-period utility given by $u(q_{k+1}) - q_k + \mathcal{U}(g)$. Money is indivisible and symmetrically distributed across K specialized types. Let $\bar{m}$ and Z denote the exogenous average wealth per type and the exogenous upper bound on individual

wealth holdings, respectively. Then the set of possible individual wealth holdings can be denoted by $\mathbf{M} = \{0, 1, ..., Z\}$. At the beginning of a period, an agent with $m \in \mathbf{M}$ chooses a portfolio $\boldsymbol{\omega} = (C, D, B)$ subject to $C + D + B \leq m$ where B denotes the holdings of an illiquid one-period coupon bond with a coupon payment $\tilde{\theta}$ in terms of general good. Since a coupon bond is completely illiquid, without loss of generality, we can assume $\tilde{\theta} > \theta$ due to, for instance, liquidity premium (see, for instance, Lagos 2011).

With chosen portfolios, agents enter DM where each agent is randomly matched with another agent and trades occur in single-coincidence meetings. After pairwise trades, the balance of deposit and illiquid bonds are redeemed to each agent with the promised returns of θ and $\tilde{\theta}$, respectively. Before moving into the next period, money creation occurs in a lump-sum way and an inflation tax is imposed. Specifically, an agent with $m \in \mathbf{M}\backslash\{Z\}$ obtains a unit of money with probability μ and then each unit of money is confiscated with probability τ, which is essentially equivalent to the standard lump-sum money creation with divisible money. (See, for instance, Lucas and Woodford 1994, and Deviatov and Wallace 2001.)

Turning to the equilibrium, we consider a symmetric (across specialization types) stationary equilibrium that consists of (v, π, λ). Here $v(m)$ and $\pi(m)$ denote the value of holding $m \in \mathbf{M}$ and the fraction of each specialization type with $m \in \mathbf{M}$ at the beginning of a period, respectively, and $\lambda(\boldsymbol{\omega})$ is the fraction of each specialization type with a portfolio $\boldsymbol{\omega}$ after the portfolio choices and before the pairwise meetings. The formal definition of a steady state is described in Appendix 2.

For a given steady state (v, π, λ), the expected lifetime utility of a representative agent prior to the assignment of wealth according to π can be expressed as

$$\mathbb{W} = \left(\frac{\sigma}{1-\beta}\right) \pi \mathbf{U} \pi' + \left(\frac{1-\sigma}{1-\beta}\right) \pi \boldsymbol{\mathcal{U}}_{\mathbf{d}}.$$

Here $\sigma \equiv (1/K)$, and the element in row $j \in \mathbf{M}$ and column $k \in \mathbf{M}$ of the matrix $\mathbf{U}$ is $u[q(j,k)] - \tilde{q}(j,k) + \mathcal{U}\left\{\theta[D_j - (p(j,k) - C_j)\mathbf{1}_{\{p(j,k)>C_j\}}] + \tilde{\theta}B_j\right\}$ with $\mathbf{1}_{\{\cdot\}} = 1$ if and only if $p(j,k) > C_j$ and $\tilde{q}(j,k) = q(j,k) + \varphi[p(j,k) - C_j]\mathbf{1}_{\{\cdot\}}$ where the second term, $\varphi[p(\cdot) - C_j]\mathbf{1}_{\{\cdot\}}$, denotes the disutility borne by the seller from accepting a debit card. The $j-th$ component of the $(Z+1)$ vector $\boldsymbol{\mathcal{U}}_{\mathbf{d}}$ is $\mathcal{U}(\theta D_j + \tilde{\theta}B_j)$ where the last term, $\tilde{\theta}B_j$, is not taken into account in the previous studies. As in Section 3, the welfare cost of $x\%$ inflation can be measured by asking how much consumption should be given to buyers in every single-coincidence meeting to bring $\mathbb{W}$ up to what it is at 0% inflation. Specifically, we first find an additive consumption compensation Δ that solves

$$\mathbb{W}_0 = \left(\frac{\sigma}{1-\beta}\right)[\pi(\mathbf{U}_\Delta)\pi']_x + \left(\frac{1-\sigma}{1-\beta}\right)[\pi\boldsymbol{\mathcal{U}}_{\mathbf{d}}]_x$$

where the element in row $j \in \mathbf{M}$ and column $k \in \mathbf{M}$ of $\mathbf{U}_\Delta$ is $u[q(j,k) + \Delta] - \tilde{q}(j,k) + \mathcal{U}\left\{\theta[D_j - (p(j,k) - C_j)\mathbb{I}_{\{p(j,k)>C_j\}}] + \tilde{\theta}B_j\right\}$ and $[\kappa]_x$ denotes κ in the steady state of $x\%$ inflation.[4] And the welfare cost of $x\%$ inflation is expressed as a ratio of Δ to the average consumption in the steady state of $x\%$ inflation.

Hereinafter our discussion mainly draws on observations of numerical examples because the endogeneity of non-degenerate wealth distribution rules out closed-form solutions.

4.1. Numerical Environment

In order to solve the model numerically, we parameterize the background environment as follows. We set $K = 3$ which is the smallest number of types eliminating the possibility of double-coincidence of wants in a pairwise meeting. We set $(\bar{m}, Z) = (20, 3\bar{m})$ so that the indivisibility of money and the upper bound on money holdings are not too severe. In this type of model, almost all monetary offers are either 0 or 1 if the indivisibility of money is

[4] We here compute a consumption compensation as an addition rather than as a multiple of consumption because consumption is zero in some single-coincidence meetings.

too severe. However, as we will see, it is not the case in our examples. In addition, $Z = 3\bar{m}$ is large enough in the sense that almost no one is at the upper bound in a steady state and hence the result would be hardly affected even if a larger Z were imposed.

We set $\beta = 0.96^{1/F}$ where 0.96 is a standard annual discount factor and F is the number of model periods per year. We here study a quarterly model period ($F = 4$) and therefore, an annualized growth rate of money implied by a given μ is $(4\mu/\bar{m})$.

We next set the real return rate of checkable deposit (θ) per model period as 0.01% which is close to the real return rate of MZM deposits reported in Šustek (2010). The real return rate of illiquid bond per model period is set to $\tilde{\theta} = 0.18\%$ which is the average real return rate of 3-month AA Financial Commercial Papers from the Federal Reserve Economic Database (FRED).[5]

We assume that $u(q) = q^{1-\eta}/(1-\eta)$ and $\mathcal{U}(g) = \ln(1+g)$ where η together with φ (transaction cost of a debit card) is chosen to fit the model to the data concerning the ratios of (M1/M3) and (cash/M1). In our model, the sum of cash holdings $\left[\mathbb{E}(C) = \sum_{j\in\mathbf{M}} \pi(j)C_j\right]$ and checkable deposit $\left[\mathbb{E}(D) = \sum_{j\in\mathbf{M}} \pi(j)D_j\right]$ can be interpreted as M1 and $\bar{m}$ as M3. Then $(\eta, \varphi) = (0.3865, 2 \times 10^{-4})$ with an inflation rate of 4.4% (average inflation rate of the U.S. economy over the period 1970-2011) implies that the ratios of M1 to M3 [i.e., $(\mathbb{E}(C) + \mathbb{E}(D))/\bar{m}$] and cash to M1 [i.e.,$(\mathbb{E}(C)/(\mathbb{E}(C) + \mathbb{E}(D))$] are 22.182% and 34.972%, respectively, which are very close to those ratios for the U.S. economy over the period 1970-2011 [(M1/M3)$\times 10^2 = 21.889$, (cash/M1)$\times 10^2 = 35.123$].[6]

Finally, we check the overall plausibility of the above parameterizations by considering an economy with $\tilde{\theta} = \theta = 0$. Noting that the portfolios consisting entirely of cash are equilibrium portfolios in this economy, the model is now identical to the existing cash-only

[5]The data on return rate of 3-month AA Financial Commercial Papers are available from January 1997 at http://research.stlouisfed.org/fred2.

[6]The Board of Governors of the Federal Reserve System ceased the publication of M3 in March 2006. Hence, we use the World Bank data at http://databank.worldbank.org.

one. For the case, the welfare cost of 10% inflation turns out to be 1.36%. This is in the range of existing measures of the cost ($1 \sim 1.5\%$) in the context of search-theoretic models with the buyer-take-all bargaining solution (see Nosal and Rocheteau 2011, pp. 154-160).[7] This implies that our numerical environment is not out of the ordinary and is indeed in line with the previous studies.

In addition, the payment patterns predicted by the model are also somewhat consistent with those observed from the real world. Table 1 reports the payment pattern by wealth levels in the steady state with 4.4% inflation. The poor use cash dominantly to pay for their relatively small consumption purchases, whereas the rich typically use debit cards to pay for their relatively large consumption purchases. Arango and Welte (2012) report that the average sizes of cash and debit-card transactions are respectively \$16.9 and \$51.2 in Canada.

Table 1: Payment patterns by wealth levels

	average offer		
		cash fraction	debit-card fraction
$m \leq 5$	0.981	0.999	0.001
$6 \leq m \leq 15$	1.640	0.680	0.320
$16 \leq m \leq 25$	2.500	0.530	0.470
$m \geq 26$	3.522	0.019	0.981

4.2. Portfolio Shift and Inflation Cost

Table 2 reports summary statistics of steady states for an economy with broad money and a cash-only economy ($\tilde{\theta} = \theta = 0$). The welfare level in an economy with broad money is higher than that in a cash-only economy mainly because in the former economy, idle monetary wealth can be held in the form of interest-bearing assets. That is, the value of money in a broad-money economy is higher than in a cash-only economy because the presence of

[7] Craig and Rocheteau (2008) show that these estimates are essentially consistent with the estimates based on the Bailey's (1956) methodology such as Lucas (2000).

interest-bearing assets grants an option feature to money. For instance, a seller who obtains money in a pairwise trade has an option to hold it in the form of cash or to invest it into interest-bearing assets. Therefore, as we can see in Table 2, a seller in an economy with broad money is willing to produce more DM-good in exchange for a unit of money.

Table 2: Composition of portfolio, DM-trade and welfare

	broad-money economy			cash-only economy		
Inflation rate	0%	3%	10%	0%	3%	10%
$\{[\mathbb{E}(C)+\mathbb{E}(D)]/\bar{m}\}\times 10^2$	13.869	22.180	34.400	100.00	100.00	100.00
$\{\mathbb{E}(C)/[\mathbb{E}(C)+\mathbb{E}(D)]\}\times 10^2$	28.335	34.999	69.186	100.00	100.00	100.00
average offer in DM	1.352	2.446	5.423	1.353	2.467	5.494
DM-good per unit of money	0.677	0.353	0.126	0.674	0.350	0.123
welfare	22.988	22.546	21.940	19.914	19.756	19.546
welfare cost	–	1.378	3.874	–	0.492	1.357

However, the welfare cost of inflation with broad money turns out to be almost threefold as large as that of a cash-only economy.[8] This result is in stark contrast to the previous studies with narrow money where the presence of interest-bearing liquid assets lessens the cost of inflation. The main source of this discrepancy stems from portfolio shifts with inflation, particularly from a higher-return illiquid asset to lower-return liquid assets. Table 3 reports the average bond holdings and the associated welfare loss due to inflation, $\mathbf{L} = [\mathbb{E}_0(B) - \mathbb{E}_x(B)]\tilde{\theta}(1-\beta)^{-1}$ where $\mathbb{E}_x(B)$ denotes the average bond holdings across agents with different wealth in the steady state of $x\%$ inflation. With inflation, the average offer in DM increases and hence agents are willing to hold more liquid assets (M1): i.e., the ratio of M1 to M3 [i.e., $(\mathbb{E}(C)+\mathbb{E}(D))/\bar{m}$] increases as an inflation rate goes up (see Table

[8]This is somewhat consistent with Lee (2012) wherein inflation induces the deadweight losses associated with an interest-bearing liquid asset due to its intermediary cost and foregone return and therefore, inflation cost would most likely be underrated in cash-only models. However, his model cannot capture the effect of portfolio shift from illiquid assets to liquid assets.

2). This immediately implies the decline in the general-good consumption (i.e., return from illiquid-asset holdings). Compared to 0% inflation, the welfare losses induced by the foregone return from illiquid asset reach around 0.299 in 3% inflation and 0.739 in 10% inflation. These magnitudes respectively account for around 68% of the welfare difference between 0% and 3% inflation [$0.299/(22.988 - 22.546)$] and around 71% of the welfare difference between 0% and 10% inflation [$0.739/(22.988 - 21.940)$].

Table 3: Welfare loss due to foregone return from illiquid bond

	inflation rate		
	0%	3%	10%
$\mathbb{E}(B)$	17.226	15.565	13.121
$\mathbf{L} = [\mathbb{E}_0(B) - \mathbb{E}_x(B)]\tilde{\theta}(1-\beta)^{-1}$	–	0.299	0.739
welfare cost when $\mathbf{L}$ is disregarded	–	0.444	1.135
(welfare cost in a cash-only economy)		(0.492)	(1.357)

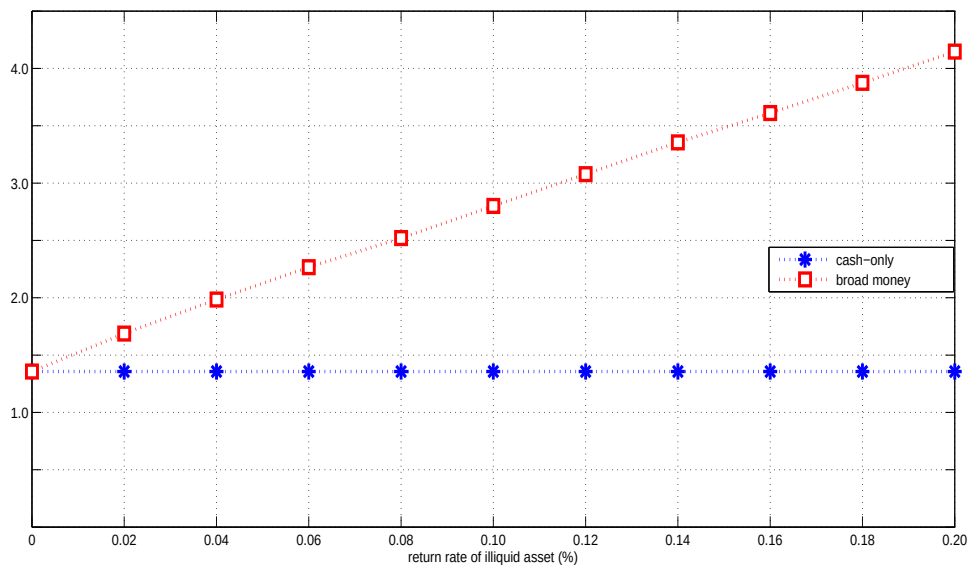
In order to grasp the above magnitudes more concretely, we calculate the welfare cost of inflation by disregarding the welfare loss resulting from the foregone return. If we simply ignore the foregone return, the welfare difference between 0% and x% inflation in a broad-money economy dwindles from 0.442 to 0.143 when $x = 3$ and from 1.048 to 0.309 when $x = 10$. Furthermore, as claimed by the previous studies and Proposition 1 in Section 3, the welfare cost of inflation in an economy with interest-bearing assets is smaller than that in a cash-only economy (see Table 3).

In sum, the opportunity cost incurred by the portfolio shift due to inflation has quantitatively significant implication on the cost of inflation. This result suggests that existing measures of inflation cost with cash only or M1 (cash + interest-bearing liquid asset) are substantially underestimated.

4.3. Robustness

As a robustness check, we first consider the wider range of $\tilde{\theta}$ such as $2\theta \leq \tilde{\theta} \leq 20\theta$. Figure 1 presents the cost of 10% inflation as a function of $\tilde{\theta}$ where, needless to say, $\tilde{\theta} = \theta = 0$ is equivalent to a cash-only economy. The result suggests that in an economy where $\tilde{\theta} > 0$ and some agents hold idle money, M1-demand-oriented approach to the cost of inflation would be misleading and the underestimation problem worsens as $\tilde{\theta}$ increases.

Figure 1: Welfare cost of 10% inflation as a function of $\tilde{\theta}$



We next endogenize the return rate of illiquid bond ($\tilde{\theta}$) by having it depend on the government's budget constraint and the endogenous demand for illiquid bond across agents with different wealth. More specifically, we assume that the proceeds of bond sales $[\mathbb{E}(B)]$ are used to produce general goods according to the decreasing returns-to-scale technology of $\alpha\sqrt{\mathbb{E}(B)}$. This, together with the government's balanced budget condition, implies that $\tilde{\theta} = [\alpha\sqrt{\mathbb{E}(B)}/\mathbb{E}(B)] = \alpha/\sqrt{\mathbb{E}(B)}$. In computing a steady state for this case, we set α to fit the model to the U.S. data concerning the ratios of (M1/M3)$\times 10^2$=21.889 and

(cash/M1)$\times 10^2$=35.123. The model parameterized with $(\alpha, \mu) = (8.33 \times 10^{-3}, 0.22)$ implies that the ratios of M1 to M3 and cash to M1 are 22.181% and 36.011%, respectively, where $\mu = 0.22$ corresponds to an inflation rate of 4.4% (average inflation rate of U.S. for the period 1970-2011). As reported in Table 4, the amount of transaction in DM $[\mathbb{E}(p)]$ increases with inflation and hence, monetary wealth invested into the coupon bond declines. Then $\tilde{\theta} = \alpha/\sqrt{\mathbb{E}(B)}$ implies that for a given α, $\tilde{\theta}$ increases with inflation, which renders the welfare cost of inflation smaller. However, the welfare cost of inflation in an economy with broad money is still more than twice as large as that in a cash-only economy.

Table 4: Welfare cost with $\tilde{\theta} = \alpha/\sqrt{\mathbb{E}(B)}$

	broad-money economy			cash-only economy		
Inflation rate	0%	3%	10%	0%	3%	10%
average offer in DM	1.352	2.446	5.412	1.353	2.467	5.494
average bond holding	17.226	15.565	13.121	–	–	–
illiquid bond return rate (%)	0.201	0.211	0.230	–	–	–
welfare	23.337	23.018	22.589	19.914	19.756	19.546
welfare cost	–	0.991	2.756	–	0.492	1.357

As another robustness check, we change the cost structure of debit-card transactions from a proportional one to a fixed one. That is, the transaction cost of a debit card is now irrelevant to the amount of transaction. In computing a steady state for this case, we again choose the fixed cost of a debit-card transaction ($\bar{\varphi}$) to fit the model to the U.S. data regarding the ratios of (M1/M3)$\times 10^2$=21.889 and (cash/M1)$\times 10^2$=35.123. The model parameterized with $(\bar{\varphi}, \mu) = (6.25 \times 10^{-4}, 0.22)$ implies that the ratios of M1 to M3 and cash to M1 are 22.182% and 34.136%, respectively. As reported in Table 5, our result again turns out to be qualitatively immune to this variation: i.e., the welfare cost of inflation with a fixed cost is similar to that with a proportional cost.

Notice that however, the adjustment pattern of portfolios with inflation in an economy with the fixed cost is quite different from that in an economy with the proportional cost. Since the transaction cost of debit card does not rely on the transaction size under the fixed-cost structure, agents other than the poor are willing to make transactions by debit card. However under the proportional-cost structure, agents are willing to economize on the debit-card transaction as much as possible because the cost increases with the amount of debit-card transaction. Therefore, as transaction size increases with inflation, checkable deposits rise more rapidly in an economy with the fixed cost, whereas cash holdings rise more quickly in an economy with the proportional cost.

Table 5: Alternative cost structure of debit-card transaction

	fixed cost			proportional cost		
Inflation rate	0%	3%	10%	0%	3%	10%
$\{[\mathbb{E}(C)+\mathbb{E}(D)]/\bar{m}\}\times 10^2$	13.869	22.181	34.401	13.869	22.180	34.400
$\{\mathbb{E}(C)/[\mathbb{E}(C)+\mathbb{E}(D)]\}\times 10^2$	48.484	31.917	7.009	28.335	34.999	69.186
average offer in DM	1.352	2.450	5.437	1.352	2.446	5.423
DM-good per unit of money	0.677	0.353	0.125	0.677	0.353	0.126
welfare	22.987	22.550	21.971	22.988	22.546	21.940
welfare cost	–	1.360	3.749	–	1.378	3.874

5. Concluding Remarks

We took an off-the-shelf matching model of monetary exchange and chose the parameter values that conform to the previous studies. This parameterized version of the model implies that the welfare cost of inflation would be substantially underestimated if we simply take into account the demand side of liquid assets. In particular, the return from illiquid asset turns out to have quantitatively significant implications on the cost of inflation.

It is worthwhile to note that we do not consider either a state-contingent optimal money-creation mechanism as in Wallace (2012) or an optimal trading mechanism as in Rocheteau (2012). In order to facilitate the comparisons with the previous literature, we assume a lump-sum way of money creation and a buyer-take-all bargaining solution.

Finally, we can also consider a model in which agents are allowed to trade lotteries on indivisible money. With lottery offers by buyers and narrow money (cash+checkable deposit), Lee (2012) illustrates that the cost of inflation not being captured in a cash-only model matters if an ex-ante net return rate of liquid asset is far off from zero. This implies that our main result will remain intact qualitatively even if lotteries are introduced.

6. Appendix 1: Proofs

Proof of Lemma 1: Notice that the assumption $\beta[(1/\mu)+r] \leq 1$ implies $\mathbb{I}=1$ for $d_{+1}>0$ and hence (1) can be written as

$$\max_{(c_{+1},d_{+1})} \left\{ \sigma \left[u(q^b) - c_{+1} - \left(1+\frac{\theta}{\phi_{+1}}\right) d_{+1} \right] - \left[ic_{+1} + \left(i - \frac{\theta}{\phi_{+1}}\right) d_{+1} \right] \right\} \tag{4}$$

where $q^b = c_{+1} + [1-(\varphi/\phi_{+1})]d_{+1}$. The first order condition for c_{+1} is identical to (2) and let q^m be the solution to $u'(q^m) = [(\mu-\beta)/\beta\sigma]+1$. The solution $d_{+1}>0$ should satisfy

$$\sigma \left[u'(q^b) \left(1 - \frac{\varphi}{\phi_{+1}}\right) - \left(1+\frac{\theta}{\phi_{+1}}\right) \right] = i - \left(\frac{\theta}{\phi_{+1}}\right). \tag{5}$$

Furthermore, since $\sigma\{u'(q^m)[1-(\varphi/\phi_{+1})] - [1+(\theta/\phi_{+1})]\} \leq [i-(\theta/\phi_{+1})]$ means $d_{+1}=0$, $d_{+1}>0$ is implied by the following inequality:

$$\sigma \left[u'(q^m) \left(1 - \frac{\varphi}{\phi_{+1}}\right) - \left(1+\frac{\theta}{\phi_{+1}}\right) \right] > i - \left(\frac{\theta}{\phi_{+1}}\right). \tag{6}$$

By using (2), (6) can be simplified as

$$u'(q^m)\sigma\varphi < \theta(1-\sigma). \tag{7}$$

Then $\bar{\theta}$ such that $u'(q^m)\sigma\varphi = \bar{\theta}(1-\sigma)$ is uniquely defined as $\bar{\theta} = [\varphi/(1-\sigma)\beta][\mu - \beta(1-\sigma)]$ where we use $u'(q^m) = [(\mu-\beta)/\beta\sigma] + 1$. Hence (7) holds for $\theta > \bar{\theta}$. Now from (5) and (6), we have

$$u'(q^b)\left(1 - \frac{\varphi}{\phi_{+1}}\right) - \left(1 + \frac{\theta}{\phi_{+1}}\right) < u'(q^m)\left(1 - \frac{\varphi}{\phi_{+1}}\right) - \left(1 + \frac{\theta}{\phi_{+1}}\right)$$

which means that $u'(q^b) < u'(q^m)$ for $d_{+1} > 0$ and hence $q^b > q^m$.

Proof of Proposition 1: Since $\phi \geq \beta(\phi_{+1} + \theta)$ in an equilibrium, $(1/\beta) = (1+r) \geq [(\phi_{+1}+\theta)/\phi] = 1 + r_l$ where r_l denotes the net real return rate of the checkable deposit. Then $(1+r)\mu = 1$ for $i = 0$ implies $(1+r_l)\mu \leq 1$. This means that when $i = 0$, the net nominal return rate of the checkable deposit, $\theta/\phi_{+1} = \mu\theta/\phi$, should be also 0 with the understanding that it cannot be negative. Hence, $\theta = 0$ when $i = 0$. If $i = \theta = 0$, $q^m = q^b = q^*$ from (2) and (5). Then from (3), we have $\sigma[u(q^*) - q^*] + \mathcal{U}(g^*) = \sigma\{u[q^m_{i'}(1+\xi^m_{i'})] - q^m_{i'}\} + \mathcal{U}[g^*(1+\xi^m_{i'})]$ and $\sigma[u(q^*) - q^*] + \mathcal{U}(g^*) = \sigma\{u[q^b_{i'}(1+\xi^b_{i'})] - q^b_{i'}\} + \mathcal{U}[g^*(1+\xi^b_{i'})]$ and hence, $\sigma\{u[q^m_{i'}(1+\xi^m_{i'})] - q^m_{i'}\} + \mathcal{U}[g^*(1+\xi^m_{i'})] = \sigma\{u[q^b_{i'}(1+\xi^b_{i'})] - q^b_{i'}\} + \mathcal{U}[g^*(1+\xi^b_{i'})]$. Now, since $\sigma\{[u(q(1+\xi)) - q] + \mathcal{U}[g^*(1+\xi)]\}$ increases with $q \in (0, q^*)$ and $q^b > q^m$ for $i' > 0$ from Lemma 1, the claim is followed.

7. Appendix 2: Steady State for Broad-Money Model

We here present the definition of a stationary symmetric equilibrium for the nondegenerate model discussed in Section 4. Although the definition is that for a non-lottery version of Lee

et al. (2005), we include it here for the convenience of readers. We begin with a portfolio-choice stage. Letting $\mathbf{W} : \mathbf{\Omega} \to \mathbb{R}$ with $\mathbf{\Omega} = \{\boldsymbol{\omega} = (C, D, B) \in \mathbb{Z}_+^3 : C + D + B \leq Z\}$ denote the expected utility after the portfolio choice and before the pairwise meeting, the portfolio choice problem for an agent with m is

$$\mathbf{J}(m, \mathbf{W}) = \max_{\boldsymbol{\omega} \in \Gamma(m)} \mathbf{W}(\boldsymbol{\omega}) \tag{8}$$

where $\Gamma(m) = \{\boldsymbol{\omega} = (C, D, B) \in \mathbb{Z}_+^3 : C + D + B \leq m\}$ is the set of feasible portfolios for an agent with m. Let the set of maximizers in (8) be $\mathbb{S}_1(m, \mathbf{W})$. If $\mathbb{S}_1(m, \mathbf{W})$ contains multiple elements, we allow for all possible randomizations over them. This set of randomizations can be defined as $\Delta_1(m, \mathbf{W}) = \{\delta_m : \delta_m(\boldsymbol{\omega}) = 0 \text{ if } \boldsymbol{\omega} \notin \mathbb{S}_1(m, \mathbf{W})\}$. Then $\Lambda(\mathbf{W}, \pi)$, the set of portfolio distributions on $\mathbf{\Omega}$, can be defined as

$$\Lambda(\mathbf{W}, \pi) = \{\lambda : \lambda(\boldsymbol{\omega}) = \sum_m \pi(m)\delta_m(\boldsymbol{\omega}) \text{ for } \delta_m(\boldsymbol{\omega}) \in \Delta_1(m, \mathbf{W})\}. \tag{9}$$

We next turn to pairwise trades. Consider a generic single-coincidence meeting between a buyer with $\boldsymbol{\omega} = (C, D, B)$ and a seller with $\tilde{\boldsymbol{\omega}} = (\tilde{C}, \tilde{D}, \tilde{B})$. Let $m_{\boldsymbol{\omega}} = (C + D + B)$ and $m_{\tilde{\boldsymbol{\omega}}} = (\tilde{C} + \tilde{D} + \tilde{B})$ denote the total wealth implied by the portfolio $\boldsymbol{\omega}$ and $\tilde{\boldsymbol{\omega}}$, respectively. Further, let $m^l_{\boldsymbol{\omega}} = (C + D)$ and $m^l_{\tilde{\boldsymbol{\omega}}} = (\tilde{C} + \tilde{D})$ denote the total *liquid* wealth implied by the portfolio $\boldsymbol{\omega}$ and $\tilde{\boldsymbol{\omega}}$, respectively. For the meeting, the set of feasible offers from a buyer to a seller can be defined as $\Gamma(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) = \{p : p \in \{0, 1, ..., \min\{m^l_{\boldsymbol{\omega}}, Z - m_{\tilde{\boldsymbol{\omega}}}\}\}\}$. With a tie-breaking rule by which a seller accepts all offers that leave her no worse off, a buyer's problem is

$$\max_{p \in \Gamma(\boldsymbol{\omega}, \boldsymbol{\omega}')} \left(\begin{array}{l} u\left\{\tilde{v}[m_{\tilde{\boldsymbol{\omega}}} + p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}})] - \tilde{v}(m_{\tilde{\boldsymbol{\omega}}}) - \varphi[p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) - C]\mathbf{1}_{\{p>C\}}\right\} + \tilde{v}\left[m_{\boldsymbol{\omega}} - p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}})\right] + \\ \mathcal{U}\left\{[D - (p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) - C)\mathbf{1}_{\{p>C\}}]\theta + B\tilde{\theta}\right\} \end{array} \right)$$

where $\tilde{v} : \mathbf{M} \to \mathbb{R}$ denotes the expected utility after the pairwise meeting and before the

money creation. Let the set of maximizers of the buyer's problem be $\mathbb{S}_2(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})$ and the maximized value of that be $f(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})$. Noting that the payoff with portfolio $\boldsymbol{\omega} = (C, D, B)$ as a seller is $\tilde{v}(m_{\boldsymbol{\omega}}) + \mathcal{U}(D\theta + B\tilde{\theta})$, $\mathbf{W}(\boldsymbol{\omega})$ satisfies

$$\mathbf{W}(\boldsymbol{\omega}) = \sigma \sum_{\tilde{\boldsymbol{\omega}}} \lambda(\tilde{\boldsymbol{\omega}}) f(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v}) + (1-\sigma)\left[\tilde{v}(m_{\boldsymbol{\omega}}) + \mathcal{U}(D\theta + B\tilde{\theta})\right]. \tag{10}$$

Now, we can describe the evolution of wealth distribution induced by pairwise trades. As in the portfolio-choice stage, we allow for all possible randomizations over the elements in $\mathbb{S}_2(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})$. It is convenient to define this set of randomizations over the post-trade wealth of a buyer such that $\Delta_2(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v}) = \{\delta(\cdot; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v}) : \delta(m; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v}) = 0 \text{ if } m \notin \{m_{\boldsymbol{\omega}} - p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})\}$ for $p(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v}) \in \mathbb{S}_2(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})\}$. Then, the set of post-trade wealth distributions on $\mathbf{M}$ can be defined as

$$\Phi(\tilde{v}, \lambda) = \left\{ \tilde{\pi} : \tilde{\pi}(m) = \sigma \sum_{(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}})} \lambda(\boldsymbol{\omega}) \lambda(\tilde{\boldsymbol{\omega}}) \Theta(m; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) + (1-2\sigma) \sum_{\boldsymbol{\omega}} \lambda(\boldsymbol{\omega}) \mathbf{1}_{\{m_{\boldsymbol{\omega}} = m\}} \right\}$$

where $\Theta(m; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) = [\delta(m; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}) + \delta(m_{\boldsymbol{\omega}} + m_{\tilde{\boldsymbol{\omega}}} - m; \boldsymbol{\omega}, \tilde{\boldsymbol{\omega}})]$ for $\delta \in \Delta_2(\boldsymbol{\omega}, \tilde{\boldsymbol{\omega}}, \tilde{v})$.

The evolution of wealth distribution induced by the money creation and confiscation is as follows. Let $\mathbb{M}$ denote the transition matrix of wealth after the money creation. Then, for $(m, m') \in \mathbf{M} \times \mathbf{M}$ with $m \leq m'$, non-zero elements of the upper-triangular matrix $\mathbb{M}$ can be defined as

$$\mathbb{M}_{(m,m')} = \begin{cases} 1-\mu & \text{if } m = m' \\ \mu & \text{if } m = m'-1. \end{cases}$$

Needless to say, $\mathbb{M}_{(Z,Z)} = 1$. Similarly, let $\mathbb{C}$ denote the transition matrix due to money confiscation. Then, for $(m, m') \in \mathbf{M} \times \mathbf{M}$ with $m \geq m'$, non-zero elements of the lower-triangular matrix $\mathbb{C}$ can be defined as

$$\mathbb{C}_{(m,m')} = ({}_m\mathbb{B}_{m'})\,(1-\tau)^{m'} \tau^{m-m'}$$

where ${}_m\mathbb{B}_{m'}$ is a binomial coefficient. Since a proportional confiscation is nothing but normalization, τ should satisfy $\tilde{\pi}\mathbb{M}\mathbb{C}\mathbf{M} = \bar{m}$ for $\tilde{\pi} \in \Phi(\tilde{v}, \lambda)$. Then $\tilde{v}$, the value function defined on the money holdings after the pairwise meetings and before the money creation, satisfies

$$\tilde{v} = \beta\mathbb{M}\mathbb{C}v$$

where β appears because agents carry the balance of money after confiscation to the next period. Finally, the set of distribution on $\mathbf{M}$ after the money creation and confiscation can be written as

$$\Pi(v, \lambda) = \{\pi : \pi = \tilde{\pi}\mathbb{M}\mathbb{C} \text{ for } \tilde{\pi} \in \Phi(\tilde{v}, \lambda)\} \tag{11}$$

where the dependence of Π on v is through the dependence of $\tilde{v}$ on v.

Now, we can complete the conditions for a steady state as follows: (i) $v(m) = \mathbf{J}(m, \mathbf{W})$ where $\mathbf{J}(m, \mathbf{W})$ is given by (8) and $\mathbf{W} : \mathbf{\Omega} \to \mathbb{R}$ is given by (10); (ii) $\pi \in \Pi(v, \lambda)$ where $\Pi(v, \lambda)$ is given by (11); (iii) $\lambda \in \Lambda(\mathbf{W}, \pi)$ where $\Lambda(\mathbf{W}, \pi)$ is given by (9).

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