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Organizational Design with Non-Contractible Quality

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Abstract

When contracting with an agent who is a worker of non-contractible quality, a principal considers mechanisms with an informed third party, a manager. To induce the manager with limited liability to report worker quality truthfully, the principal devises the first-order alignment, an incentive alignment based on the first-order condition with an interval structure. We show that the mechanism of contracting simultaneously with the manager and the agent dominates the optimal "selling the project" mechanism at a low information cost. The interplay between information cost and limited liability results in three optimal organizational structures: simultaneous contracting (manager inside the firm), ex ante contracting (out-sourcing), or partial contracting (no manager). Lastly, we apply this model to explain what may cause the difference in the firm structure across three types of labor markets.

Keywords and Phrases: Non-contractible quality, first-order alignment, boundary of the firm, information cost, limited liability

JEL Classification Numbers: D21, D82, D86

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1 Introduction

Every real-world organization has an organizational structure, and one main role of the structure is to facilitate information acquisition and its transmission within it. This is especially important with regard to those aspects of work quality that are difficult to measure. For instance, a principal can contract on the number of reports or pages written by an agent, but it is hard to verify the quality, content, correctness, *etc.* Computer code may be another example; it is easy to contract on the number of lines written, but harder to verify their quality, robustness, adaptability, readability, and reuse by other programmers. Needless to say, determining work quality – or *type* in general – is crucial to an organization's viability. To overcome such information asymmetry between firms and workers, employers typically consult a third party with appropriate expertise to verify it. Job assignment, promotion, and other internal labor allocations are often constrained by a third party's evaluations rather than measurable outcomes.[1]

The existing theories model worker quality in essentially two ways: unobservable and private value; or observable investment but not verifiable. In a screening or signaling approach, worker quality is a worker's private information since it is observable only to him. In addition, the quality is not a worker's value but the principal's value in the sense that it only affects the principal, but the quality is *paired* with production cost – a worker's private value – so eliciting the cost through the single-crossing reveals the quality as well. In an incomplete contract approach, worker quality is a worker's human capital investment outcome, and this observable but nonverifiable action is settled through a bargaining process. Neither approach typically requires such a third party observing the quality for truthful revelation, however.

This paper suggests a model in which a worker's non-contractible quality is his separate trait and thereby characterizes the optimal mechanism with a third party observing the quality.[2] In this case, a worker with high quality may have a high or low cost of producing each unit (*e.g.* number of pages or lines); that is, the agent's type is *multi-dimensional*. This simple separation, yet, invites new challenges. As in the standard screening or signaling approach, the quality is the agent's private information but the principal's value. Now, with no such pairing between quality and production cost of quantity, it is not possible to elicit the separate dimension of a worker's characteristics, without a third party.

To fix the idea, consider an example:

> A firm's owner assigns different hours to workers for a job. The job has two dimensions, working hours (quantity) and value per hour (quality), since its unit

[1] Anecdotal evidence well documented by Baker, Gibbons and Murphy (1994) not only supports the case that performance is not easily measurable, as intended in their paper, but also the case that worker quality cannot be assessed.

[2] For evidence that such a trait exists as a type, *i.e.*, quality in the present paper, see Burks et al (2015) and Pallais and Sands (2016).

output depends on each worker's quality.

The quality is the agent's private information whereas the quantity is observable, since the former is difficult to measure, unlike the observable measures for the latter.[3] The same type of non-contractible quality has been the main concern in a sizable literature on procurement auctions since Manelli and Vincent (1995). In the classical multi-tasking model of Holmström and Milgrom (1991), such quality can also be found not as an agent's type but as an outcome of an agent's action that is difficult to measure. The present model extends the framework to include a third party observing the quality where it is an additional dimension of an agent's characteristics.

To resolve the problem, the principal may consider a manager to acquire the information.[4] With the manager, the principal can choose one of three different organizational structures: the manager inside the firm, selling a project to the manager, and no manager. In particular, we focus on the emergency of the first structure in which the principal contracts simultaneously with both the manager and the agent. Formally, they are called, *simultaneous contracting, ex ante contracting*, and *partial contracting*, respectively. For all three mechanisms, in light of modern firm structures, the principal of the present paper uses monetary transfers to make contracts with the agent and the manager. However, with the monetary transfers empowering him, the problem of the optimal organization can become less interesting; it is always optimal for him to have the full mechanism with the manager. Then, what can explain different firm structures that exist today?

We suppose that the manager incurs costs when seeking to acquire the information, unlike the agent, after participating in the organization, as the quality is not his intrinsic type, and that he is protected by limited liability. The former implies that having the manager inside the firm is no longer free since the principal must pay the manager more to make him acquire the quality, so he faces the *trade-off* between information acquisition and an extra rent to the manager. The latter changes the price of selling-the-project. Then, the structure with the manager can dominate the selling depending on the magnitude of limited liability. Overall, we provide the optimal organizational structure with the interplay between *information (acquisition) cost and limited liability.* In particular, to the best of the author's knowledge, no previous paper studies the role of limited liability in information acquisition.

Such role of a third party observing an agent's type was originated by Tirole (1986). The quality as a separate task in this paper departs from the seminal paper in which an

[3] If the relationship between two parties is one-time, as in a standard adverse selection model such as the buyer-seller setting where a seller's product quality, *e.g.*, a used car's quality, also directly affects a buyer, the quality is not contractible with a different rationale; typical terms of trade protect a seller so that a buyer cannot make payment contingent on it, that is, paying *after* using a product. To resolve such non-contractibility, reputation building, as in Kreps (1990), can be used if an uninformed party can observe an outcome after each period, but in this model, a quality outcome is difficult to measure, at least in the short term.

[4] The principal could also have expertise to verify worker quality, but he may not have time to do so. See an expository writing, Arrow (1964), for the relevance of the problem, called "the span of control."

agent's action and type are *accrued* to a one-dimensional task. In other words, the role of a third party in the present paper is to report non-contractible quality, which is related to non-contractible quality in Manelli and Vincent (1995) and Holmström and Milgrom (1991). In particular, we consider worker quality as a pure adverse selection problem like the former, but to solve the problem, we rely on two different aspects of an agent's outcome like the latter in which one aspect is contractible but the other aspect is not.[5] Solving simultaneous contracting requires the manager's truth-telling about what he observes, and in this model, the manager's payoff is only monetary compensation from the principal, as in Tirole (1986). Then, our model with the two aspects of an agent's outcome concerns the following key question: How can the principal possibly elicit the quality from the manager when his payoff is only a monetary transfer?

The benchmark of this framework is the second-best case – the maximum possible payoff – in which the principal, like the manager, can acquire information on worker quality, while production cost remains the agent's private information. Without the information cost, achieving the second best is almost trivial; a constant payment induces the manager to report the quality truthfully, leaving no surplus to him, but even an infinitesimally small positive cost discourages the manager from acquiring information on it. The same problem arises for other mechanisms making use of the fact that the quality is known both to the agent and the manager, for example, punishing them for two different quality reports.[6] Another convenient solution is the selling-the-project mechanism, ex ante contracting. However, if the manager is protected by (ex post) limited liability, the ex ante contracting mechanism can be suboptimal.[7]

We start by observing that the well-known simple contracts fail to incentivize the manager's information acquisition because in this model, the principal must tackle *combinatorial deviations*: The manager can misreport the agent's quality after not acquiring the information.[8] That is, for the optimal contract to achieve the second best, the principal must

[5]The key difference between this model and relational contracts is a type of information structure. In this paper, the quality is a pure adverse selection problem. Further, the quantity is observable and verifiable and with the manager, the quality is contractible, which enables us to use a mechanism design approach. As a result, in the present paper, monetary transfers play an important role in shaping a firm structure, whereas the total payment must be fixed via the standard "money burning" in relational contracts.

[6]See Section 4 for more details. Another potential problem with the scheme is collusion through communication between them.

[7]Selling and a selling-like contract cannot explain a firm's structure based on the informational aspect because it fails to show the emergence of an integrated large firm in reality, instead of multiple disintegrated firms. The ex ante contract can be interpreted in two ways; a contract within an organization is executed in exactly the same way as selling the project, and the project is "literally" sold. In the former, the principal retains ownership; in the latter, ownership of the project is transferred to the manager. Nonetheless, retaining the ownership combined with the selling does not require any information flow between them, so the two are treated virtually the same. The limited liability in the former can be translated into outsourcing with limited liability or selling with liquidity constraint in the context of the latter.

[8]See Myerson (1982) for such deviations and the difference between the truthful or honest incentive

provide truthful incentive compatibility together with an incentive for information acquisition to resolve the problem. On the other hand, to circumvent the direct effect of limited liability of selling the project, any base compensation for limited liability has to be *decoupled* from incentive compatibility. Thus, providing truthful incentive compatibility in this model is a delicate problem due to the incentive for information acquisition and limited liability. It is not just for its theoretical contribution *per se* but, more interestingly, for its implications for the optimal organizational structure.

The first main result is to propose a mechanism in which the principal aligns the manager's optimality condition for truth-telling with the principal's optimality for the second best, through the first-order condition. We term the first order condition for decoupling the *first-order alignment*. The incentive alignment is in fact shown to be achieved with a natural contractual form. The principal sets a target quantity, and if the agent's quantity turns out to be greater than that, he rewards the manager such that the compensation depends on the manager's report on quality. Thus, it has an *interval structure* for observable volume of output, generalizing the interval delegation by Holmström (1977, 1984).[9] The necessary condition of reporting worker quality truthfully becomes *sufficient* by connecting it to the reverse hazard rate dominance between the two dimensions of a type, quality and cost, without an exogenously given single-crossing.[10] The first main result shows that if information acquisition is not very costly, the simultaneous contracting mechanism attains the second-best payoff, dominating the selling.[11]

The linear characterization of the principal's payoff from simultaneous contracting enables us to derive a simple formula for the maximum cost that incentivizes the manager's information acquisition. Then, if an information cost is greater than the maximum admissible cost, simultaneous contracting is not feasible. However, the bound can be *loosened* with an extra rent to the manager, which leads the principal to face the trade-off between information acquisition and the rent. Since the magnitude of limited liability affects the principal's payoff from simultaneous contracting via the base compensation, this in turn results in the trade-off between information acquisition and limited liability.

The trade-off provides the second main result of this paper, a rich outcome on the *bound-*

compatibility and obedience incentive compatibility.

[9]To implement simultaneous contracting, the principal can suggest a set of contracts, leaving the manager to choose one, as a standard screening problem. Consequently, the delegation of this model can be rather effortlessly achieved.

[10]The cost distribution conditional on a higher quality dominates the one on a lower quality with respect to the reverse hazard rate, which is widely used in auction theory (see, Maskin and Riley (2000) and Kirkegaard (2012)) for ranking auctions, but no paper applied it to eliciting a type.

[11]At least two more reasons for the dominance of a simultaneous contracting mechanism over selling can be suggested: management cost and partial commitment. First, if the principal is an expert in management, only the manager incurs a "management cost" when he becomes the mechanism designer (buys the project). Second, within an organization, people lower in the hierarchy (like the manager relative to the principal) typically have more specific information but less commitment power.

ary of the firm. In essence, the interplay between the extent to which limited liability constrains the principal and the size of the information cost determines whether simultaneous contracting or ex-ante contracting is optimal. If the magnitude of "effective" limited liability is not that high, then to make the principal indifferent between simultaneous contracting and ex ante contracting, the former's decoupling can increase an additional unit of limited liability, without requiring any sacrifice of information cost, while to make him indifferent between ex ante contracting and partial contracting, the exchange rate between them becomes 1. On the other hand, if the negative effects from limited liability are severe, then ex ante contracting is not optimal at all, and for the indifference between simultaneous contracting and partial contracting, in order to increase an additional unit of limited liability, less than one unit of information cost suffices. The full characterization shows the intricate nature of the optimal organizational structure. The analysis can be further extended to answer an intriguing question raised by Holmström and Roberts (1999) about the difference in firm structure between U.S. firms (exercising simultaneous contracting more) and Japanese firms (exercising ex-ante contracting more), in the presence of the hold-up problem.

We discuss the related literature in the following section. Section 3 introduces a model. Section 4 provides the main results: three optimal mechanisms and the full characterization of the optimal organizational structure. We extend the results to include multiple target quantities in Section 5 and to show the uniqueness of simultaneous contracting in Section 6. Section 7 concludes, and the proofs are collected in an appendix.

2 Related literature

Study on a third party inside an organization has been a subject of interest over the last several decades. The third party's role is to monitor an agent (or agents) in an organization owned by a principal, and the role can be further classified into two cases: monitoring the agent's action or type. Alchian and Demsetz (1972) propose a monitor observing agents' actions in a team, suggesting monitoring as an origin of a firm's structure. Tirole (1986) introduces a supervisor observing an agent's type in the three-tier hierarchical structure, with its focus on collusion between them; a three-tier hierarchical structure is assumed as there is no collusion with selling.[12] Unlike both papers, the principal's goal in the present paper is to solve a *screening problem*: How the principal can assign different volumes of output to a worker of varying quality with the manager's report, not how to incentivize a worker to exert certain efforts.[13] Furthermore, a principal in the present paper hires a manager observing an agent's quality, as in the latter, but with a different model and purpose. In

[12]There is a vast literature on collusion with a third party following Tirole (1986); we wish to relate them to the future research of this model as commented in the concluding remarks.

[13]That is, unlike the standard moral hazard models, there is no observable noisy signal from an action in this screening model, which differs from models with such a signal structure and extended models with mediated contracts, *e.g.* in Rahman (2012) and Strausz (2012) among others.

Tirole (1986), the agent's type and action are added to produce a one-dimensional output, and monitoring incurs no cost. In the present paper, the agent's action and type represent two distinct dimensions of a single output, quantity and quality, in addition to the two-dimensional private information, and equally importantly, a manager incurs an information cost for monitoring, while protected by limited liability.

The present paper's main purpose is to characterize a mechanism that makes a structure with a manager inside a firm optimal, and thereby provide the rationale for why a third party is inside an organization in the first place. Despite the resonating influence of such a firm structure from its inception, the question has been rather neglected in the literature. We answer the question with the first order alignment and two key *factors*: information cost and limited liability. The interplay between them yields three optimal organizational structures, while a single factor fails to account for different structures; without limited liability, the optimal contract is always selling, and without information cost, the optimal contract must be to include a third party inside the firm, by having him acquire the information. Since such a "mix" of costly information acquisition and limited liability has not been investigated in the literature, it is a separate theoretical contribution. Still, the present paper treats information cost and limited liability differently from previous papers in each of the environments.

The different treatment of the two well-known environments is due to non-contractibility of quality. The treatment of non-contractible quality of this paper is most closely related to the one in Manelli and Vincent (1995) and that in the information structure of Holmström and Milgrom (1991), as discussed earlier. Recently, Lopomo, Persico and Villa (2023) have provided a general framework to encompass both Myerson (1981) and Manelli and Vincent (1995) such that for the no lemon problem, its optimal mechanism becomes the former and for the severe lemon problem, it becomes the latter; hence, its optimal mechanism handles any case in between. In particular, in the presence of the adverse selection problem in procurement, they propose a new optimal mechanism with a floor price. We advance them, in a screening model, such that there is now a third party observing the quality.

In the multitasking model by Holmström and Milgrom (1991), if one activity cannot be measured at all, incentivizing the other activity that is measured can affect the effort made for the unobservable action through their interactions from a cost function (for example, two activities can be complements or substitutes). In this model, one dimension, worker quality, is the agent's type, not an action. Thus, an interaction between two dimensions to induce the manager's truthful report is provided through correlation between the observable volume of output and worker quality, not through a cost function.[14] An interesting, recent development of Bénabou and Tirole (2016) includes screening into the multitasking model by incorporating a talent parameter or an intrinsic motivation as a type of agent. Since the type

[14] Utilizing the conditional expectations with correlation between quality and cost is essential, as in Crémer and McLean (1988), but their mechanism, despite the generality, is not applicable to the manager because it does not consider limited liability; without limited liability, no reason to resort to them is found, with the convenient selling option in this model. In addition, the principal of the present paper has to tackle limited liability while incentivizing the manager's information acquisition.

is associated with a contractible action in their paper, a screening approach can be adopted for truthful revelation of the type, but in this model, without a third party observing worker quality, the principal cannot screen it at all.

The costly information acquisition in a production model of Baron and Myerson (1982) was first introduced by Crémer, Khalil and Rochet (1998a,b). This paper also considers a Baron-Myerson setting as the baseline model, but unlike their papers, in the present paper, it is a third-party, the manager, who acquires the agent's information in a three-tier hierarchical model as in Tirole (1986). Although this paper shares some common features with them such as the production model and the optimal mechanism to induce information acquisition, we focus on the *third-party's information acquisition*.

In the presence of limited liability, Innes (1990) shows that the optimal contracts have a binary form with a threshold to motivate debt-style contracts, which is further generalized by Poblete and Spulber (2012), in particular, without the monotone likelihood ratio property.[15] The optimal contracts with a threshold or an interval in that literature is to incentivize an agent's efforts, whereas an interval structure from target quantity levels in the present paper is adopted to induce a third party's truth-telling for the agent's type. Thus, the nature of the problem that the present paper studies is different, so given that the manager is not involved in any production, we can obtain the optimal contracts as long as the manager's individual rationality is binding. Yet, the principal must incentivize the manager to acquire the information with a positive cost, together with inducing his truth-telling. Hence, choosing an optimal interval structure becomes a delicate problem when the cost is high; any structure with the binding individual rationality suffices otherwise.

This paper is also related to two more strands of literature. The quality is not observable by the principal, so a bargaining procedure in incomplete contract literature from the property rights approach by Grossman and Hart (1986) and Hart and Moore (1990) built on transaction costs (Williamson (1975, 1985)) cannot be extended to this not-observable quality case. Last, monetary transfers make this model attain the delegation as an implementation of the mechanism with no friction, similar to Krishna and Morgan (2008) with full commitment but unlike the optimal delegation literature based on a bias as in Crawford and Sobel (1982).[16]

3 Model

A principal contracts with a manager and an agent (a worker). The agent's output consists of two dimensions, quantity and quality. He produces $q \geq 0$ units with ω quality per unit such

[15]The present paper's first-order alignment is conceptually different from the first-order approach for this literature as well as that on subjective evaluations: The alignment is to connect the manager's incentive with the principal's to induce truth-telling via their first-order optimality to tackle limited liability and, further, it provides the sufficiency for global optimality through the reverse hazard rate dominance.

[16]The literature is too large to review in its entirety in this paper.

that quantity q is verifiable, whereas quality ω is not contractible. In particular, the latter is the agent's private information but only affects the principal. The feature that a worker's quality is relevant only for the principal's payoff is common in adverse selection problems.[17] The present model departs from them in that the quality is a separate characteristic of the agent; the agent's type consists of two dimensions, quality ω and (marginal) cost θ such that they are drawn from a joint distribution.

Without any fixed pairing of quality ω and cost θ, the quality seizes a new and intriguing role: The quality is the *agent's private information but the principal's value.* With this distinctive but natural role of quality, the principal cannot elicit worker quality from the agent, as shown later. A third party is introduced to tackle the non-contractibility of the agent's quality. The manager has the expertise to acquire information on the agent's quality at cost $c > 0$.

If the principal assigns the agent of quality ω the production q by making a monetary transfer t to him, the agent obtains

$$t - \theta q. \tag{1}$$

The quality does *not appear* above since unlike cost θ, the agent's privately-informed quality ω affects not the agent's payoff but the principal's. Another important feature of this model that makes it differ from the existing approaches is that we allow monetary transfers. With the transfers, the optimal organizational structure is shaped by a new rationale – the interplay between information cost and limited liability of the manager.

The manager's payoff S is only based on the monetary compensation from the principal, as in Tirole (1986) as well as in other papers following it; *i.e.*, S is both a contract for the manager and his payoff. We denote by $\overline{U}$ the manager's reservation payoff, and by B his *ex post* limited liability (or liquidity constraint), assuming that they are any two real numbers with the reservation payoff greater than the limited liability, $\overline{U} > B$.[18] The agent's reservation payoff, on the other hand, is normalized to 0. The quality is the agent's innate characteristic, whereas it is monitored by the manager, so the agent and the manager must differ in their behavior of acquiring the information. The agent observes quality ω without any cost before participating in a mechanism, but the manager incurs a cost to acquire information on it *after* participating. In particular, the latter allows us to consider an ex ante individual rationality for the manager.[19]

[17]In the textbook signaling and screening models with human capital, following Spence (1973), a worker's type consists of a quality and cost pair; *e.g.*, a good type has a high quality and a low cost.

[18]A negative value for B can have a more natural interpretation in our context; for example, the manager is liable for a loss up to, say, $B = -\$100$ or constrained by liquidity up to B.

[19]As the owner of the organizational structure, the principal has a legitimate right to choose when to hire the manager and, by doing so, grant him the opportunity to acquire information on the agent's quality: Timing for the manager is chosen by the principal, not by "Nature." Then, it is clear that for the principal, hiring the manager in the ex ante stage weakly dominates hiring the manager in the interim.

If the principal makes such assignment and transfer, he obtains

$$v(\omega, q) - t, \tag{2}$$

where v is twice-differentiable, strictly increasing in each of ω and q, strictly concave in q, and ω and q are complementary.[20] Then, the total surplus of the principal and the worker is $v(\omega, q) - \theta q$, and for each ω, to guarantee an interior solution, we assume $v_q(\omega, 0) > \underline{\theta}$ and $v_q(\omega, 0) \leq \overline{\theta}$ so that the former ensures that the principal has an incentive to have the worker produce if the cost is the lowest, and the latter ensures that he has no such incentive if it is the highest; and that $\lim_{q\to\infty} v_q(\omega, q) < \underline{\theta}$ so that the optimal output for any type of worker is finite.

We let a two-dimensional type (θ, ω) be drawn from a non-empty subset $\Theta \times \Omega$ of $\mathbb{R}^2$, where $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ and $\Omega \equiv [\underline{\omega}, \overline{\omega}]$. As it is convenient to use a conditional distribution rather than a joint one for the analysis, we denote the conditional cumulative distribution function of cost θ given quality ω by $F(\theta|\omega)$, while the cumulative distribution function of ω by $G(\omega)$. Their density functions are denoted by $f(\theta|\omega) > 0$ for all $\theta \in \Theta$ and $g(\omega) > 0$ for all $\omega \in \Omega$, respectively. We assume that $F(\theta|\omega)$ satisfies the standard monotone hazard rate condition for $F(\theta|\omega)$ for each fixed ω; that is, the ratio $\frac{F(\theta|\omega)}{f(\theta|\omega)}$ is nondecreasing in θ. Finally, quality ω and cost θ are related in such a way that for any pair $\omega' > \omega$, $F(\theta|\omega')$ dominates $F(\theta|\omega)$ in terms of the reverse hazard rate, that is, $\frac{f(\theta|\omega')}{F(\theta|\omega')} > \frac{f(\theta|\omega)}{F(\theta|\omega)}$ for all $\theta \in (\underline{\theta}, \overline{\theta})$, which implies the first-order stochastic dominance.[21]

We find the principal's maximum feasible payoff, called the second-best payoff, before embarking on the main analysis, which serves as our benchmark.[22] Suppose hypothetically, unlike the model, that there is no manager but the principal *can* observe quality ω if he incurs the same opportunity cost and information (acquisition) cost, $\overline{U} + c$, like the manager. Then, a direct mechanism Γ^s consists of functions, q_s and t_s, where $q_s(\cdot, \omega) : \Theta \to \mathbb{R}_+$ and $t_s(\cdot, \omega) : \Theta \to \mathbb{R}$. For each observed ω, the incentive compatibility condition given different values of θ becomes a "reverse" problem of the well-known nonlinear pricing (see, *e.g.*, Mussa and Rosen (1978)). The mechanism yields the principal the following payoff using the standard procedure:

$$V(\Gamma^s) = \int_\Omega \int_\Theta \left[v(\omega, q_s(\theta, \omega)) - q_s(\theta, \omega)\phi(\theta, \omega)\right] f(\theta|\omega) d\theta dG(\omega), \tag{3}$$

[20]Its partial derivatives are $v_\omega > 0, v_q > 0$; the second partial derivative is $v_{qq} < 0$; and the cross partial is $v_{\omega q} \geq 0$ for all $(\omega, q) \in \Omega \times \mathbb{R}_+$. Throughout the paper, we use a subscript to denote a partial derivative of a function.

[21]See Krishna (2002) for its property.

[22]The details for the second best with an example and its relationship with partial contracting can be found in Appendix B.1. Even if quality ω is observable, cost θ still remains the agent's private information, so the benchmark is called the second-best optimal payoff that the principal obtains, not the first best. Note also that the agent observes both dimensions, so throughout our analysis, clearly, no rent extraction-type mechanism utilizing conditional probabilities is applicable to the agent.

where $\phi(\theta, \omega)$ is the virtual cost defined as $\phi(\theta, \omega) \equiv \theta + \frac{F(\theta|\omega)}{f(\theta|\omega)}$. Then, the principal chooses q_s to maximize the term with the virtual cost inside the integral in (3), under the pointwise maximization, not the total surplus $v(\omega, q) - \theta q$, the sum of (1) and (2). Then, the second-best optimal allocation is given as

$$\begin{array}{l} \text{(i) if } v_q(\omega, 0) - \phi(\theta, \omega) \leq 0, q_s(\theta, \omega) = 0; \\ \text{(ii) otherwise, } v_q(\omega, q_s(\theta, \omega)) - \phi(\theta, \omega) = 0. \end{array} \tag{4}$$

We denote the maximum value of (3) by V_s subject to incentive compatibility and individually rationality conditions of the agent, which yields the principal's second-best payoff from the benchmark, $V_s - \overline{U} - c$.

By the two conditions on $F(\theta|\omega)$ above, ϕ is a strictly increasing function of θ but a strictly decreasing function of ω. This together with the properties on v yields that, for interior solutions in (4), the agent's production function $q_s(\theta, \omega)$ is a strictly decreasing function of marginal cost θ but a strictly increasing function of quality ω; these relationships are intuitive.[23] We denote by $V(\theta, \omega)$ the maximized term inside the integral in (3) by substituting the optimal $q_s(\theta, \omega)$ in (4), such that

$$V(\theta, \omega) \equiv v(\omega, q_s(\theta, \omega)) - q_s(\theta, \omega)\phi(\theta, \omega). \tag{5}$$

With $V(\theta, \omega)$, V_s can be neatly written as $V_s = \int_\Omega \int_\Theta V(\theta, \omega) f(\theta|\omega) d\theta dG(\omega)$. The optimal contract in (4) is an immediate extension of the aforementioned well-known problem for our benchmark with observable quality ω. Importantly, for the main analysis with the manager in the next section, we use this optimality condition so that the manager's truth-telling condition can be aligned with it.

The other polar case is partial contracting where for each pair $\omega \neq \omega' \in \Omega$, $q(\theta, \omega) = q(\theta, \omega')$ for almost all $\theta \in \Theta$. We denote the optimal partial contracting payoff by V_p. The maximization of the second best case with observable ω requires that for a higher value of ω, the agent must produce more, whereas it is constant for the partial contracting. The proof for $V_s > V_p$ simply confirms this intuition. Hence, for $\overline{U}$ satisfying $\overline{U} < V_s - V_p$, if $c > 0$ is sufficiently small, we have $V_s - \overline{U} - c > V_p$; the second best yields a higher payoff to the principal than the partial contracting. That is, under the condition, the principal hires the manager in an attempt to achieve the second best if he cannot observe the quality; he would not have an incentive to do so at any cost otherwise. Thus, $V_s - \overline{U} - V_p > 0$ is necessary for information acquisition, which is maintained in what follows.

[23]For each ω, one obtains analogous results with the nonlinear pricing, including the "no distortion at the top and the downward distortion below the top" such that the agent with the lowest marginal cost $\underline{\theta}$ chooses the first-best quantity, whereas the agent with a marginal cost greater than $\underline{\theta}$ chooses a quantity lower than the first best.

4 Optimal contracts

This section contains the two main results of this paper. First, we find the optimal mechanism given each of the following three organizational structures. In the first structure, the principal designs a mechanism only with the agent – with no third party, and in the second structure, the principal sells the project to the manager. In particular, if there is no manager, the principal cannot elicit quality, even with the full monetary transfers empowering the principal. With no information transmission between the principal and the manager for either structure, we consider the third structure where the principal hires the manager to elicit information on worker quality. Second, by comparing the three organizational structures, we fully characterize the optimal organizational structure with the interplay between information cost and limited liability. The three structures are called partial contracting, ex ante contracting, and simultaneous contracting, respectively.

4.1 Three different mechanisms

First, we study partial contracting in which the principal designs a mechanism only with the agent. Since the agent's type (θ, ω) is multi-dimensional, a direct mechanism Γ^p consists of two-dimensional functions, q and t, where $q : \Theta \times \Omega \to \mathbb{R}_+$ and $t : \Theta \times \Omega \to \mathbb{R}$. In light of the revelation principle, the agent is asked to report both θ and ω, and the principal assigns the agent the production of $q(\theta, \omega)$ and commits to paying him $t(\theta, \omega)$. A mechanism is said to be incentive compatible if for each $(\theta, \omega), (\theta', \omega') \in \Theta \times \Omega$,

$$t(\theta, \omega) - q(\theta, \omega)\theta \geq t(\theta', \omega') - q(\theta', \omega')\theta. \tag{6}$$

A direct mechanism can be implemented such that the principal suggests a set of pairs (q, t), leaving the agent to choose one, as a standard screening problem.

We show that, without the manager, assigning different quantity allocations depending on different quality reports is not possible.

Proposition 1 *For any mechanism between the principal and the agent, the incentive compatibility (6) requires that for each pair $\omega \neq \omega' \in \Omega$, $q(\theta, \omega) = q(\theta, \omega')$ for almost all $\theta \in \Theta$.*

The idea behind the result is straightforward. Since the quality does not affect the agent – principal's value, to satisfy the incentive compatibility (6), the agent must obtain the same payoff for any two different quality reports. This in turn implies that different quality levels produce "essentially" an identical volume of output (quantity) for the same cost type. On the other hand, it is not trivial either as the result holds with any level of correlation between quality and cost (see also Schmitz (2002)).[24]

[24] In a seller and a buyer setting, Schmitz (2002) establishes a powerful result, related to this finding, in which the first best may not be achieved due to two types of post-contractual informational asymmetries arising from the combination of hidden action and hidden information, but both of this model's two dimensions are hidden information.

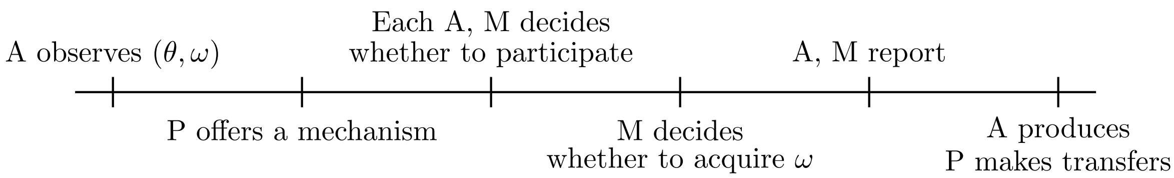

Figure 1: Timeline

Second, we can find the optimal ex-ante contract with a simple procedure. After buying the project from the principal, the manager, as an informed mechanism designer, obtains V_s like the benchmark in Section 3, but he is constrained by limited liability unlike the principal. An ex ante contracting mechanism is said to be individually rational if $V_s - c - \alpha \geq \overline{U}$, where α denotes a sale price. It satisfies limited liability given α if $V(\theta, \omega) - c - \alpha \geq B$ for all $(\theta, \omega) \in \Theta \times \Omega$; or $\min_{(\theta,\omega)} V(\theta, \omega) - c - \alpha \geq B$, where $V(\theta, \omega)$ is the manager's (ex-post) optimal payoff, previously derived in (5). If limited liability is "effective" in the sense that $V_s - \overline{U} > \min_{(\theta,\omega)} V(\theta, \omega) - B$, then the ex ante contracting cannot achieve the second best payoff.[25] Taking into account the role of the effective limited liability, the optimal sale price from the ex ante contracting is

$$\alpha = \min \left\{ V_s - \overline{U}, \min_{(\theta,\omega)} V(\theta, \omega) - B \right\} - c. \tag{7}$$

Third, for simultaneous contracting, to extract worker quality, the principal designs a mechanism that simultaneously contracts with both the manager and the agent. The game's timeline after Nature chooses a type (θ, ω) is shown in Figure 1. The essential feature for contracting with the manager is to provide incentive compatibility while satisfying both an incentive for information acquisition and limited liability. There are simple contracts that satisfy incentive compatibility together with either condition, partially. On one hand, a constant payment and punishment for two different reports satisfy incentive compatibility and limited liability. However, unlike the standard case in which the principal uses either form of contracts only for truthful incentive compatibility, the principal of this model must also incentivize information acquisition. That is, he must handle *combinatorial deviations* – the manager's misreport of quality following no information acquisition. Yet, for those simple contracts, the information cost makes an optimal or equilibrium payoff *without* acquiring the information greater than that with acquiring it.[26] On the other hand, for ex-ante contracting, after buying the project, the manager *himself* finds it optimal to acquire the information if the condition $V_s - \overline{U} - c > V_p$ in the previous section is satisfied. Hence, it can provide the

[25] It is not determinant whether limited liability is effective or not, since, in spite of $\min_{(\theta,\omega)} q(\theta, \omega) = 0$, we may have $\min_{(\theta,\omega)} V(\theta, \omega) > 0$; for instance, in Example 1, it is $\min_{(\theta,\omega)} V(\theta, \omega) = T > 0$, and B can take a negative value.

[26] We relegate related discussions to Appendix B.2. Otherwise, as separate issues, we can have a multiplicity problem and collusion.

incentive for information acquisition, but its direct negative effects are already discussed.[27]

The key idea to design a simultaneous contract in order to circumvent the direct negative effect of limited liability, unlike the ex-ante contract, while providing incentive compatibility is to decouple the former from the latter. This can be done if the manager's IC is provided via the first-order optimality. In this way, a constant limited liability simply disappears in the process of taking the derivative. Although it sounds simple, recall that there is no manager's payoff function exogenously given satisfying nice properties, *e.g.*, differentiability and concavity; that is, the principal *designing it endogenously* is the integral part of the mechanism. To incentivize information acquisition via the correlation between the two dimensions of a type, the principal elicits worker quality ω only from the manager. We call this new way of making the alignment the first-order alignment.[28]

Formally, a direct mechanism Γ consists of two-dimensional functions q, t, and S, where $q : \Theta \times \Omega \to \mathbb{R}_+$, $t : \Theta \times \Omega \to \mathbb{R}$ and $S : \Omega \times \Theta \to \mathbb{R}$. If the agent reports θ and the manager reports ω, then the principal assigns the agent the production of $q(\theta, \omega)$ and commits to paying $t(\theta, \omega)$ to the agent and $S(\omega, \theta)$ to the manager. A contract for the manager S is said to be *incentive compatible* (IC) if for each $\omega, \omega' \in \Omega$,

$$\mathbb{E}_\theta[S(\omega, \theta)|\omega] \geq \mathbb{E}_\theta[S(\omega', \theta)|\omega]. \tag{8}$$

Like the agent's payoff in (1), a quality ω that the manager observes does not affect his payoff $S(\omega', \theta)$; that is, it affects neither the manager's "direct" payoff nor the agent's. However, the true quality ω does affect the manager's expected payoff through the *conditional distribution* of θ given ω. A contract for the manager is *individually rational* (IR) if

$$\int_\Omega \mathbb{E}_\theta[S(\omega, \theta)|\omega] dG(\omega) - c \geq \overline{U}, \tag{9}$$

and it satisfies *limited liability* (LL) if for each $\omega \in \Omega$ and every $\theta \in \Theta$,

$$S(\omega, \theta) - c \geq B. \tag{10}$$

If the manager does not acquire the information, then he chooses a report $\widehat{\omega}$ maximizing his *ex-ante* payoff without incurring the cost c. A contract for the manager incentivizes

[27] One can argue that selling in fact provides the incentive for information acquisition as well; thus, it is a particular class of simultaneous contracting. The ex-ante contracting, however, is constrained by limited liability in a direct manner. On the other hand, the virtue of the simultaneous contracting's first-order alignment is to remove such direct effect of limited liability on the principal's payoff.

[28] Any contract for the manager that utilizes such correlation must make it contingent on the agent's quantity $q_s(\theta, \omega)$, so it is related to Crémer and McLean (1988), but this incentive alignment differs from them in that the alignment applies correlation to the first-order condition for the decoupling. The literature that extended Crémer and McLean (1988) based on their Theorem 1 adopts "lotteries" only for the rent extraction, while incentive compatibility is provided by a Vickrey-Clarke-Groves (VCG) mechanism. However, the VCG for the manager of this model (for any third party as in Tirole (1986)) is a constant payment that fails to provide IA.

information acquisition (IA) if for each $\widehat{\omega} \in \Omega$,

$$\int_{\Omega} \mathbb{E}_{\theta}[S(\omega, \theta)|\omega] dG(\omega) - c \geq \int_{\Omega} \mathbb{E}_{\theta}[S(\widehat{\omega}, \theta)|\omega] dG(\omega). \tag{11}$$

IA is not satisfied unless IC is satisfied with strict inequality over a measurable set of ω, as, for example, a constant payment; this is an additional constraint, unlike the standard screening problem, to handle combinatorial deviations.

The principal obtains his expected payoff from a mechanism Γ such that

$$V(\Gamma) = \int_{\Omega} \int_{\Theta} [v(\omega, q(\theta, \omega)) - t(\theta, \omega) - S(\omega, \theta)] f(\theta|\omega) d\theta dG(\omega). \tag{12}$$

Unlike the payoff from the second best in (3), now, the principal pays the manager $S(\omega, \theta)$ to make him acquire information on the quality. Once the conditions for the manager's contract S, (8) - (11), are satisfied, any direct mechanism (q, t, S) implementing the second-best payoff must be (q_s, t_s, S), where (q_s, t_s) is the optimal choice from the second-best solution in (4), which makes the *truthful report of* θ the agent's weakly dominant strategy (*i.e.*, the agent's IC & IR hold). Then, a simultaneous contracting mechanism Γ is optimal if it maximizes the above payoff $V(\Gamma)$ subject to IC, IR, LL and IA, with the optimal contract for the agent (q_s, t_s).

The manager's payoff is only based on the monetary compensation from the principal, so one instead needs to utilize the principal's payoff function and his first-order condition to solve the delicate problem. First, we reformulate the principal's interior optimality condition in (4) with a cost type $\theta = p(\omega', q)$ (its formal definition is given in Lemma 1 taking care of even the case $q = 0$) satisfying

$$q_s(p(\omega', q), \omega') = q. \tag{13}$$

For $q > 0$, $p(\omega', q)$ is the cost that produces a quantity q given quality ω', and it is simply an inverse function of $q_s(\theta, \omega')$ with respect to q and θ, holding ω' fixed. Note that $q_s(\theta, \omega') > q \Leftrightarrow \theta < p(\omega', q)$, where their inverse relationship is depicted in Figure 2. With this, we align the manager's optimality condition for truth-telling with the principal's optimality condition in (4) for the second best such that

$$\underbrace{\frac{\partial \mathbb{E}_{\theta}\big[S(\omega', \theta) \mid \omega\big]}{\partial \omega'}\bigg|_{\omega'=\omega}}_{\text{M's truth-telling optimality}} = \underbrace{v_q(\omega, q) - \phi(p(\omega, q), \omega)}_{\text{P's second-best optimality}} = 0. \tag{14}$$

With the first-order alignment, the principal can provide the strict IC (8) for IA (11) – together with sufficiency for the optimality – given the reverse hazard rate dominance. Furthermore, the optimal contract is unique up to a positive linear affine transformation. This degree of "freedom" enables the principal to choose a contract such that the manager obtains only up to his outside option IR (9), while satisfying LL (10), as explained in detail subsequently.

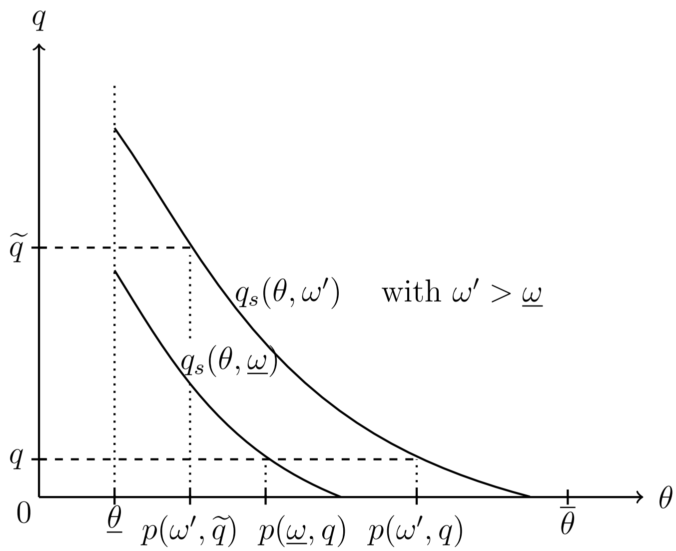

Figure 2: q_s and p mappings

In this section, we suggest a natural class of contracts to show that they can be aligned with the principal's first-order optimality. However, in Section 6, we prove that the other implication holds as well. That is, if any contractual form resorts to the first-order condition, it must belong to this class of contracts we suggest: contracts with an interval structure.

Let us consider an interval structure with a "target" quantity q_0. The principal pays the manager $s_0(\omega')$ for his report ω' if the agent's quantity $q_s(\theta, \omega')$ turns out to be greater than q_0. That is, for a target quantity q_0,

$$S(\omega', \theta) = \begin{cases} \underline{S} + s_0(\omega') & \text{if } q_s(\theta, \omega') > q_0, \\ \underline{S} & \text{otherwise,} \end{cases} \tag{15}$$

where $\underline{S}$ is a constant for the base compensation handling LL, which disappears given the first order condition via the decoupling.[29] Moreover, to make the contracts generating $\underline{S} + s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 | \omega)$ satisfy IC, by the reverse hazard rate dominance, a condition like the single-crossing property between the manager's report ω' and the true ω is *endogenously created* in his conditional expected payoff below, as shown in the proof of Theorem 1 in Appendix A.

If the manager observes ω, by reporting ω', this yields him the expected payoff such as $\mathbb{E}_\theta[S(\omega', \theta)|\omega] = \underline{S} + s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 | \omega)$. Then, by incorporating the most costly type $p(\omega', q)$ in (13), the expected payoff above can be rewritten as

$$\mathbb{E}_\theta[S(\omega', \theta)|\omega] = \underline{S} + u(\omega', \omega, q_0), \tag{16}$$

where denote $u(\omega', \omega, q_0) \equiv s_0(\omega') F(p(\omega', q_0)|\omega)$.

[29] $S(\omega', \theta)$ depends on q_0 as well as $\underline{S}$ and other parameters, but to simplify notations, the two critical arguments – the manager's report and the agent's report – are explicitly written in it.

The following lemma establishes the differentiability of $s_0(\omega')$ as well as important properties of $u(\omega', \omega, q_0)$.

Lemma 1 *If $S(\theta, \omega)$ satisfies IC, it satisfies the following properties.*

(i) $u(\omega', \omega, q_0) > 0$ for all $\omega, \omega' \in \Omega$, where $p(\omega', q_0) \equiv \min\{\theta \in [\underline{\theta}, \overline{\theta}] : q_s(\theta, \omega') = q_0\}$.

(ii) $s_0(\omega')$ is differentiable and strictly decreasing on Ω.

The first result of the Lemma implies that $s_0(\omega') > 0$ *and* $F(p(\omega', q_0)|\omega) > 0$ for all $\omega, \omega' \in \Omega$. If the former is violated, IC has a conflict with the reverse hazard rate dominance. The latter requires a target quantity to yield a positive probability for any report. If it is violated, the manager can misreport a quality level that can give him a positive expected payoff. More interestingly, the second result shows that $s_0(\omega')$ has a negative relationship with the manager's report on worker quality: a "penalty" on the manager's over-reporting incentive of quality for higher output. Together, the intuition behind the result is that if the agent's output meets a target, the manager is rewarded with a positive payoff but the magnitude itself is strictly *decreasing* in his report on the quality.

Now, to find out IC contracts among contracts with an interval structure, consider the partial derivative of $u(\omega', \omega, q_i)$ from (16) with respect to the manager's report ω' (see the proof of the following theorem for the details). If the principal chooses $s_0(\omega)$ such that for each $\omega \in \Omega$ and $q \geq 0$,

$$p(\omega, q_0) - \frac{s_0(\omega) p_\omega(\omega, q_0)}{s_0'(\omega)} = v_q(\omega, q_0), \tag{17}$$

then $u(\omega', \omega, q_i)$ can be rewritten as

$$u_{\omega'}(\omega', \omega, q_0) = -s_0'(\omega') f(p(\omega', q_0)|\omega) \left[v_q(\omega', q_0) - \phi(p(\omega', q_0), \omega) \right].$$

The two optimality conditions are aligned through the first order condition as in (14).

Theorem 1 establishes that a contract for the manager is in fact *unique* up to a positive linear affine transformation. Furthermore, the principal always attains the second-best payoff, $V_s - \overline{U} - c$ for a low information cost $c > 0$; the mechanism dominates the optimal ex ante contracting.

Theorem 1 *The manager's compensation satisfying (8) - (11) is unique up to a positive affine transformation such that its marginal compensation $s_0(\omega)$ is given by*

$$s_0(\omega) = \overline{s}_0 e(\omega, q_0), \tag{18}$$

where the e function is uniquely determined as

$$e(\omega, q_0) = \exp\left\{ \int_\omega^{\overline{\omega}} \frac{p_\omega(x, q_0)}{v_q(x, q_0) - p(x, q_0)} dx \right\},$$

where $\overline{s}_0 > 0$. By choosing the base compensation $\underline{S} = B + c$, the optimal simultaneous contracting mechanism makes the manager's individual rationality (9) binding, while satisfying *the limited liability (10), which dominates the optimal ex ante contracting mechanism. If the information cost c is sufficiently small, this contract incentivizes information acquisition (11).*

The uniqueness characterization with respect to $e(\omega, q_0)$ of the theorem yields the manager's ex-ante expected payoff from acquiring quality ω such that

$$\int_\Omega \mathbb{E}_\theta[S(\omega,\theta)|\omega]dG(\omega) = \underline{S} + \overline{s}_0 U(q_0), \tag{19}$$

where we denote $U(q_0) \equiv \int_\Omega e(\omega, q_0)F(p(\omega, q_0)|\omega)dG(\omega)$. On the other hand, if the manager maximizes his ex-ante payoff *without* acquiring the information, his payoff from reporting $\widehat{\omega}$ is

$$\int_\Omega \mathbb{E}_\theta[S(\widehat{\omega},\theta)|\omega]dG(\omega) = \underline{S} + \overline{s}_0 \widehat{U}(\widehat{\omega}, q_0), \tag{20}$$

where denote $\widehat{U}(\widehat{\omega}, q_0) \equiv e(\widehat{\omega}, q_0)\int_\Omega F(p(\widehat{\omega}, q_0)|\omega)dG(\omega)$. Hence, $\widehat{U}(\widehat{\omega}, q_0)$ is a unit-weight ($\overline{s}_0 = 1$) *ex-ante* payoff from a misreport $\widehat{\omega}$ without acquiring ω, whereas $U(q_0)$ is a unit-weight ex-ante payoff from truthful reports after acquiring it.

4.2 The optimal organization: boundary of the firm

Now, with the three optimal contracts from the previous subsection, we compare them to characterize the optimal organization fully. The size of limited liability and the magnitude of information cost affect the comparison and determine which organization is optimal. To see that, we first begin with the optimal ex-ante contracting (EC). The principal's payoff from (7) can be written explicitly as a function of them such that

$$\mathcal{V}^{EC}(B, c) = \min\{V_s - \overline{U}, V_m - B\} - c. \tag{21}$$

The simple optimal payoff of the principal yet provides an interesting perspective on the relationship between them, if limited liability is effective. For an additional unit of information cost, to make the principal indifferent, one simply needs to decrease one additional unit of limited liability. That is, the rate of change for the trade-off between the two factors is 1 in the case of ex-ante contracting.

Characterizing that relationship for simultaneous contracting is our first main task. In the previous subsection, if the information acquisition is not costly, the optimal simultaneous contract extracts all possible rent from the manager. However, if the cost $c > 0$ is not negligible, it may be optimal for the principal to leave some surplus to the manager and thereby incentivize him to still acquire the costly information. To formalize the trade-off between information acquisition and the manager's extra rent, we denote it by $R \geq 0$ with the binding IR from (9), $\int_\Omega \mathbb{E}_\theta[S(\omega,\theta)|\omega]dG(\omega) - c = \overline{U} + R$. Then, it follows from IA in (11) that the *threshold cost* given an extra rent R that admits a simultaneous contracting is

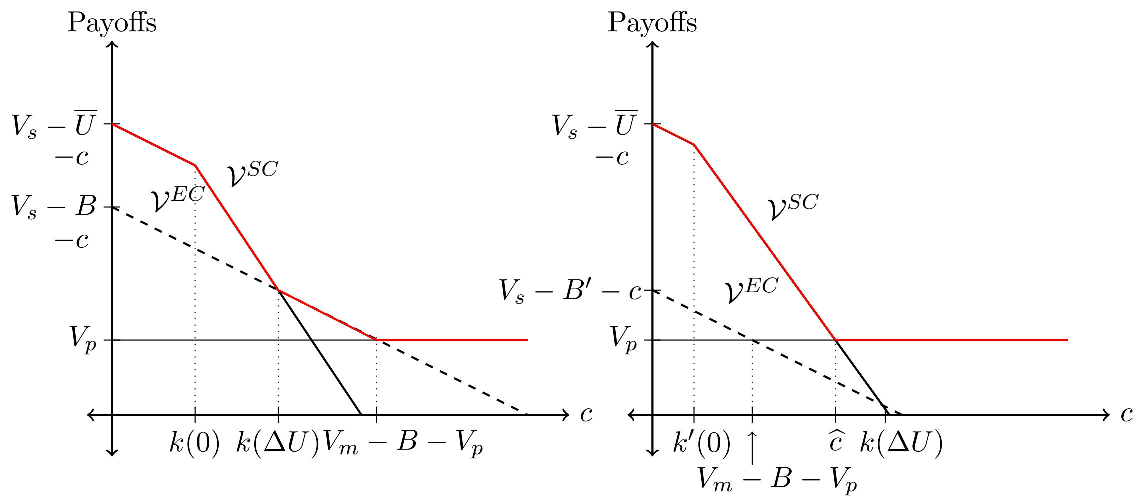

Figure 3: Mild limited liability B (left) and severe limited liability $B' > B$ (right)

$k(R) \equiv \int_\Omega \mathbb{E}_\theta[S(\omega,\theta)|\omega]dG(\omega) - \int_\Omega \mathbb{E}_\theta[S(\widehat{\omega},\theta)|\omega]dG(\omega)$. By incorporating (19) and (20) into that, we obtain the following simple formula

$$k(R) = (\overline{U} - B + R)\left[1 - \frac{\widehat{U}(\widehat{\omega}, q_0)}{U(q_0)}\right]. \tag{22}$$

Note that the extra rent R is a monetary *transfer*, unlike $\overline{U}$, between the principal and the manager, which plays a crucial role for the formula. If the information cost c is lower than $k(R)$ – the equality also satisfying IA – the principal can implement simultaneous contracting as in Theorem 1 but with rent R. The optimal rent that the principal chooses is to minimize it: $R = k^{-1}(c)$ if $c < k(R)$ and $R = 0$ if $c \leq k(0)$. This observation results in the principal's payoff from the optimal simultaneous contracting (SC) such that

$$\mathcal{V}^{SC}(B,c) = \begin{cases} V_s - \overline{U} - c & \text{if } c \leq k(0), \\ V_s - B - \frac{2-y}{1-y}c & \text{if } c > k(0), \end{cases} \tag{23}$$

where to simplify the notation, denote $y \equiv \frac{\widehat{U}(\widehat{\omega},q_0)}{U(q_0)}$ given the fraction in (22).

One can find the optimal organization by considering the three payoff functions, (21) and (23) and the principal's payoff V_p from partial contracting (PC), as in Figure 3, where the non-constant solid lines depict the payoff of simultaneous contracting from (23), and the dotted lines depict the payoff of ex-ante contracting from (21). If c is high, then the principal must choose a large rent R to satisfy IA, which makes simultaneous contracting suboptimal, as illustrated in the figure. That is, there is a *maximum rent* that the principal is willing to pay to incentivize the manager to acquire information on worker quality. There are two different maximum sizes for the rent, one compared with the ex ante contracting and the other compared with the partial contracting. In particular, in the former case, it is denoted by

$$\Delta U \equiv \max\{V_s - \overline{U} - (V_{\min} - B), 0\}. \tag{24}$$

This yields the first intersection $k(\Delta U)$ in the left-panel of Figure 3, and the value $\widehat{c}$ in the lemma below yields that in the right-panel of Figure 3.

Lemma 2 *Suppose the effective limited liability $\Delta U > 0$. The threshold admissible cost $k(R)$ is derived as (22), so there exists a threshold rent $\widehat{R} > 0$ to make simultaneous contracting dominate each of ex ante contracting and partial contracting for $c < k(\widehat{R})$ such that*

$$\widehat{R} = \begin{cases} \Delta U & \text{for ex ante contracting,} \\ V_s - \overline{U} - V_p - \widehat{c} & \text{for partial contracting,} \end{cases}$$

where $\widehat{c}$ satisfies $\widehat{c} = k(V_s - \overline{U} - V_p - \widehat{c})$.

By the lemma, we can also quantify how small is sufficient in Theorem 1's "sufficiently small" cost, which is $c \leq k(0)$.

In Figure 3, the payoffs are drawn in terms of c such that in the left-panel where limited liability is relatively mild, if the cost is in the low range, the optimal mechanism is to include the manager inside the firm; if it is in the middle range, the optimal mechanism is to sell a project; and if it is high, the optimal mechanism is to not resort to a third party at all. What is more interesting is that we have different organization characterizations depending on different sizes of B. The characterization with the three structures of the left-panel of Figure 3 can reduce to two in the right-panel of Figure 3 if limited liability can affect the principal severely for selling: It is optimal for the principal to "bypass" ex ante contracting.

As discussed previously, an informed mechanism designer has an incentive to acquire the information as long as $c < V_s - \overline{U} - V_p$. Hence, under the condition, if the principal sells the project, the manager is willing to observe the information. Despite such willingness on the part of the manager, the optimality of the partial contracting in the two cases means that the principal shall not *confer* such an opportunity, because to do so, the sale price is too high; rather take the partial contracting only with the agent. That is, the manager's chance for the information acquisition is obstructed by the principal. Note that the former case with the three structures (the latter) arises if $V_m - B - V_p > (<)\, k(\Delta U)$, so the threshold limited liability for them is given by $\widehat{B} \equiv V_m - V_p - (V_s - V_m)(1-y)$, where $k(\Delta U) = (V_s - V_p)(1-y)$ given $k(R)$ in (22); another to note is that $k(0) = (\overline{U} - B)(1-y)$, so as B changes, $k(0)$ changes in the right-panel of Figure 3, unlike $k(\Delta U)$.

Now, suppose $\Delta U = 0$ or $B \leq \overline{U} - (V_s - V_m)$ so that limited liability is not effective at all. Then, the principal can always achieve the second-best payoff. Yet, simultaneous contracting is not feasible if the information cost is high, which means that, in contrast to the second best's information acquisition, if $k(0) < c < V_s - \overline{U} - V_p$, the principal still achieves the second best not through information acquisition but only by the selling.

The second main result provides the full characterization of the optimal organizational structure, and its intricate nature is displayed in Figure 4.

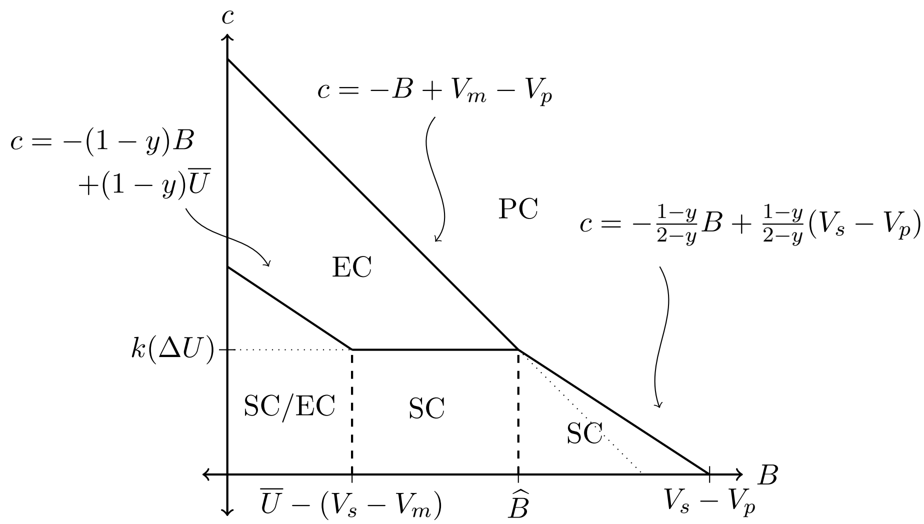

Figure 4: The optimal organizational structure

Theorem 2 *The optimal organization is exclusively given as follows.*

(i) If $c + (1-y)B \leq (1-y)\overline{U}$ and $B \leq \overline{U} - (V_s - V_m)$, the optimal mechanism is SC or EC.

(ii) If $B \geq \overline{U} - (V_s - V_m)$, $c \leq (V_s - V_p)(1-y)$ and $c + \frac{1-y}{2-y}B \leq \frac{1-y}{2-y}(V_s - V_p)$, the optimal mechanism is SC.

(iii) If $c + (1-y)B \geq (1-y)\overline{U}$, $c \geq (V_s - V_p)(1-y)$ and $c + B \leq V_m - V_p$, the optimal mechanism is EC.

(iv) If $c + B \geq V_m - V_p$ and $c + \frac{1-y}{2-y}B \geq \frac{1-y}{2-y}(V_s - V_p)$, the optimal mechanism is PC.

The interaction between the two key factors, alternatively, can be seen if we set the payoffs now in terms of B, as in Figure 5. In the left-panel where information cost is relatively low, if the magnitude of limited liability B is small, the optimal mechanism is simultaneous contracting, but if B is high, the optimal mechanism is partial contracting. On the other hand, if information cost is high, we have a *switch* in that relationship. In the right-panel, if the magnitude of limited liability B is small, the optimal mechanism is now ex-ante contracting, but if B is high, the optimal mechanism is partial contracting.

Now, to answer the question raised by Holmström and Roberts (1999), we consider the comparison only between the simultaneous contracting and the ex-ante contracting. In Japan (especially, in the auto industry), despite the potential hold-up problem, auto makers largely depend on *external* suppliers for designing of parts and manufacturing of other components. They provide a convincing explanation on how it still works in a relationship of repeated transactions and information sharing, but not on what may *originate* such a system. The difference in information cost or difference in limited liability with three types of labor market can be suggested as the origin in this paper.

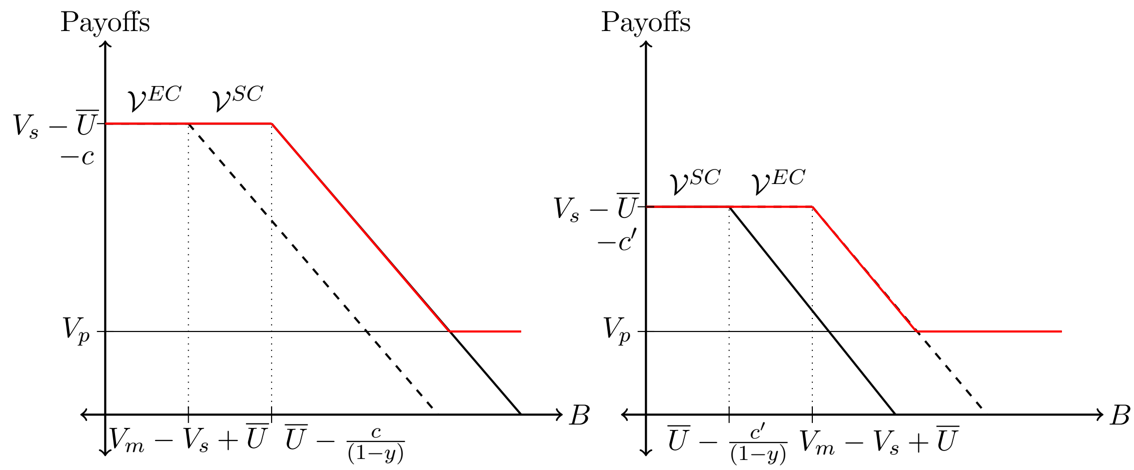

Figure 5: Low information cost c (left) and high information cost $c' > c$ (right)

One immediate answer can be obtained if, for an identical maximum rent ΔU to the manager, one labor market's information cost c' is lower than the other's c; that is, if this difference in the cost accounts for the difference between the U.S. and Japan. On the other hand, if the cost is identical for two regimes, we can consider two different types of labor market in which one market's limited liability is higher such that $\Delta U' > \Delta U$ from (24), implying $k(\Delta U') > k(\Delta U)$.

5 Multiple target levels

We can extend (16) to multiple target levels, not just one. Further, in this section, we pursue the question of choosing target quantity levels endogenously. Yet, we restrict a number of target quantity levels to be finite based on the following two reasonings: First, one ought to consider how plausible it is to have an infinite number (or a continuum) number of target levels in the real world sense, and second, solving the endogenous target quantity levels with an infinite set becomes an *infinite* dimensional problem.

The principal chooses a finite number of partitions for q with $q_0 < \cdots < q_{N-1} < q_s(\underline{\theta}, \underline{\omega})$, where $q_i < q_s(\underline{\theta}, \underline{\omega})$ to make $F(p(\omega', q_0)|\omega) > 0$ in Lemma 1.[30] Then, he considers a collection of compensations $\big(S_i(\omega')\big)_{i=0,\dots,N-1}$ given the manager's report ω' such that the principal pays the manager $S_i(\omega')$ if the agent's output $q_s(\theta, \omega')$ falls into an interval $[q_i, q_{i+1}]$. Formally,

$$S(\omega', \theta) = \begin{cases} \underline{S} & \text{if } q_s(\theta, \omega') = 0, \\ S_i(\omega') & \text{if } q_s(\theta, \omega') \in [q_i, q_{i+1}] \text{ and } q_s(\theta, \omega') \neq 0. \end{cases}$$

Such a collection can be interpreted as a collection of *marginal* compensation payments

[30]To understand it, if the target quantity is set at a high level, for instance, $\widetilde{q}$ in Figure 2, for $\underline{\omega}$, there is simply no cost type that can produce it, which violates a necessary condition for Lemma 1.

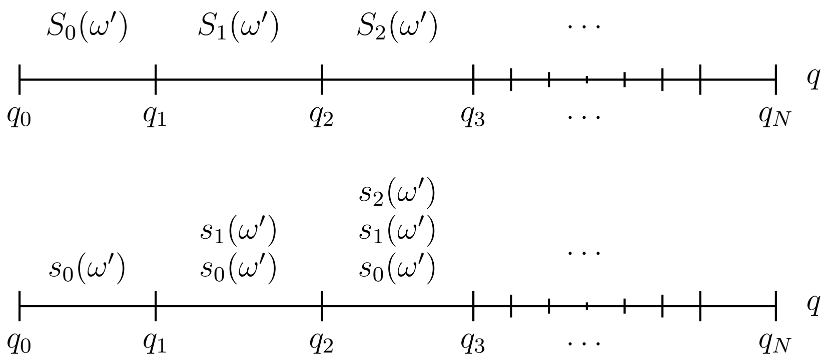

Figure 6: A compensation scheme

$\big(s_i(\omega')\big)_{i=0,\dots,N-1} \in \mathbb{R}^N$ with $s_i(\omega') \equiv \overline{s}_i e(\omega', q_i)$ such that

$$S_i(\omega') \equiv \begin{cases} \underline{S} + s_i(\omega') & \text{if } i = 0, \\ S_{i-1}(\omega') + s_i(\omega') & \text{if } i \geq 1, \end{cases}$$

where the equivalence between them is illustrated in Figure 3. In particular, Lemma 1 shows that the incentive compatible contract with intervals is monotone, $S_0(\omega') < S_1(\omega') < \cdots < S_{N-1}(\omega')$. Thus, the intuition of the Lemma expands: The principal rewards the manager more if the agent's quantity passes an additional target level, but each positive magnitude itself is strictly decreasing in the manager's report on the quality.

Then, we extend the payoff in (16) for multiple targets such as

$$\mathbb{E}_\theta[S(\omega', \theta)|\omega] = \underline{S} + \sum_{i=0}^{N-1} u(\omega', \omega, q_i).$$

Furthermore, with respect to $\overline{s} \equiv (\overline{s}_0, ..., \overline{s}_{N-1})$, (19) and (20) can be expanded such that $\int_\Omega \mathbb{E}_\theta[S(\omega, \theta)|\omega]dG(\omega) = \underline{S} + \sum_{i=0}^{N-1} \overline{s}_i U(q_i)$, and $\int_\Omega \mathbb{E}_\theta[S(\widehat{\omega}, \theta)|\omega]dG(\omega) = \underline{S} + \sum_{i=0}^{N-1} \overline{s}_i \widehat{U}(\widehat{\omega}, q_i)$. Since any $\overline{s}_i$ can be chosen to be zero to make it *ineffective*, the problem of choosing both weights $\overline{s} = (\overline{s}_0, ..., \overline{s}_{N-1})$ and intervals $q = (q_0, ..., q_{N-1})$ can be reduced to the problem of choosing only weights $\overline{s}' = (\overline{s}'_0, ..., \overline{s}'_{\overline{N}-1})$ given the finest possible intervals $q' = (q'_0, ..., q'_{\overline{N}-1})$ with the maximum number of feasible intervals $\overline{N}$. In other words, finding optimal intervals as well as corresponding weights on them can be simplified as only determining optimal weights on all feasible intervals.

To find the maximum feasible information cost, we solve the following continuous minimax problem in $\overline{N}$ dimensions.

$$\begin{aligned}
&\underset{\overline{s}\in\mathbb{R}_{+}^{\overline{N}}}{\text{minimize}}\ \max_{\widehat{\omega}\in\Omega}\ \underline{S}+\sum_{i=0}^{\overline{N}-1}\overline{s}_i\widehat{U}(\widehat{\omega},q_i),\\
&\text{subject to } \underline{S}\geq B+c, \qquad\qquad (25)\\
&\qquad\left[\underline{S}+\sum_{i=0}^{\overline{N}-1}\overline{s}_iU(q_i)\right]-c=\overline{U}+R.
\end{aligned}$$

The principal finds an *optimal interval structure* to minimize the manager's deviation payoff, given the extra rent R, to identify the maximum admissible information cost.

We show the existence of a minimax solution and further linearity of the minmaximized function in R extending the single target case in (22).

Proposition 2 *There exists a solution to the optimal interval structure problem (25), and the minmaximized function is linear in R such that*

$$k(R)=(\overline{U}-B+R)\left[1-\min_{x\in\Delta}\max_{\widehat{\omega}\in\Omega}\sum_{i=0}^{\overline{N}-1}x_i\frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)}\right].$$

With multiple targets, the characterization of Theorem 2 remains intact with the new $k(R)$ above. The value inside bracket above is fixed once it is solved for $x\in\Delta$.[31]

6 Uniqueness on simultaneous contracting

In Section 4, the contract with an interval structure in (15) is suggested as a type of contracts that can be aligned with the principal's optimality. In this section, we aim to establish the equivalence relation: Under IC in (8), a contract for the manager satisfies the principal's first-order optimality if and only if it has an interval structure. Since it was shown earlier that under IC, any contract with an interval structure satisfies the optimality, the other implication remains to be proven. Before proceeding further, we provide these concepts more formally.

The principal's first-order condition under pointwise maximization can be found from the second best in (4), and thereby consider the following: if $\omega'=\omega\in\Omega$,

$$v_q(\omega',q_s(\theta,\omega'))-\phi(\theta,\omega)=0 \text{ satisfying IC.} \qquad (26)$$

If the principal can utilize the condition above for the manager's contract to satisfy IC in (8), the equality means that if the manager reports the quality truthfully, *i.e.*, $\omega'=\omega$,

[31]With the single-peak property of two intervals, we can further state that the optimal number of intervals is simply two and see Appendix B.3.

then the manager's first-order optimality is satisfied. Obviously, not all such contracts from $v_q(\omega', q_s(\theta,\omega')) - \phi(\theta,\omega) = 0$ satisfy IC; that is, a collection of incentive compatible (IC) contracts satisfying the above first-order condition – the derivative being equal to zero – is a *subset* of a set of contracts satisfying only the first-order condition. For example, the lowest cost type $\underline{\theta}$ is to be excluded to make IC hold since otherwise, the true quality ω in $\phi(\theta,\omega) \equiv \theta + \frac{F(\theta|\omega)}{f(\theta|\omega)}$ simply disappears, and even then, the condition is only *necessary* for truth-telling. That is, IC requires the sufficient condition for truth-telling.

On the other hand, more formally, a contract for the manager $S(\omega',\theta)$ is said to have an interval structure if it yields the expected payoff such as

$$\underline{S} + s(\omega', q)\Pr(q_s(\theta,\omega') > q|\omega) \text{ satisfying IC,} \tag{27}$$

where $S(\omega',\theta)$ is given as

$$S(\omega',\theta) = \begin{cases} \underline{S} + s(\omega',q) & \text{if } q_s(\theta,\omega') > q, \\ \underline{S} & \text{otherwise.} \end{cases}$$

Note that in order to generalize (15), in this section, we denote a marginal compensation given any q by $s(\omega', q)$. The interpretation is justified since the simplest and most intuitive contract to generate such a payoff above is the one with an interval structure in (27).[32]

Since (26) $\Leftarrow$ (27) is shown in Section 4, we show that any contract for the manager satisfying the principal's first-order condition comes with such a structure – that is, (26) $\Rightarrow$ (27). We reformulate the optimality in (26) with a cost type $\theta = p(\omega', q)$ from (13). Then the first-order condition in (26) can be rewritten such that if $\omega' = \omega \in \Omega$,

$$v_q(\omega', q) - \phi(p(\omega', q), \omega) = 0 \text{ satisfying IC.} \tag{28}$$

The qualification $\theta > \underline{\theta}$ from (26) for IC is translated to $q = q_s(\theta,\omega)$ for $\theta > \underline{\theta}$.

After incorporating $\phi(\theta,\omega)$, we factor out $p_\omega(\omega', q)$, the partial derivative of $p(\omega', q)$ with respect to ω'. Then, by simple algebra, as shown in the proof of Proposition 3, the above (28) becomes: under IC,

$$\frac{\partial\big[\log(e(\omega',q)F(p(\omega',q)|\omega))\big]}{\partial\omega'} = 0 \text{ if } \omega' = \omega \in \Omega, \tag{29}$$

where $e(\omega, q)$ function is uniquely determined as

$$e(\omega,q) \equiv \exp\left\{\int_{\omega}^{\overline{\omega}} \frac{p_\omega(x,q)}{v_q(x,q) - p(x,q)}dx\right\}. \tag{30}$$

[32] There can be many other forms that can yield (29); for instance, the principal pays the manager $s(\omega', q) + a(\omega', q)$ if $q_s(\theta,\omega') > q$, and pays him $b(\omega', q)$ otherwise, which yields him the same expected payoff in (27) as long as $\Pr(q_s(\theta,\omega') > q \mid \omega)[a(\omega',q) - b(\omega',q)] + b(\omega',q) = 0$. As in any mechanism design problem, we consider the manger's IC in (27) as well as (26), given the agent's truthful report by the contract for the agent (q_s, t_s), the optimal choice from the second best (4).

Since the first-order condition (29) is necessary, the way of integrating (30) provides the sufficiency for IC (see the proof of the following proposition). The solution $e(\omega', q)$ is *scale free*, so we can always put a constant $\overline{s} > 0$ on $e(\omega, q)$ to have $s(\omega', q) = \overline{s}e(\omega', q)$.[33] Then, given that log is increasing, ω' solves $\max s(\omega', q)F(p(\omega', q)|\omega)$ in (27) if and only if it solves the max of logarithm of it. Hence, if (28) holds, $\partial u(\omega', \omega, q)/\partial\omega' = 0$ for all $\omega' = \omega \in \Omega$ satisfying IC, so (26) $\Rightarrow$ (27).

The equivalence relation in the proposition below is established where other details can be found in its proof.

Proposition 3 *Under the manager's incentive compatibility (8), a contract for the manager satisfies the principal's first-order optimality in (26) if and only if it has an interval structure in (27).*

In deriving the contract in (27) from (26), we can find a couple of nice properties for its solution $s(\omega', q) = \overline{s}e(\omega', q)$ where $e(\omega', q)$ is from (30). First, $s(\omega', q) > 0$ and $F(p(\omega', q)|\omega) > 0$ for all $\omega, \omega' \in \Omega$ from $q < q_s(\underline{\theta}, \underline{\omega})$, and $s(\omega', q)$ is positive, differentiable, and strictly decreasing on Ω. If we start with an interval structure (27), to arrive at (26), this means that we need to prove those properties, as shown in Lemma 1.

7 Concluding remarks

This paper studies different organization design problems in which a principal faces an agent whose output consists of two tasks: quantity and non-contractible quality.

We first suggest a novel device to elicit the manager the agent's non-contractible quality truthfully even in the presence of information cost and limited liability: the first-order alignment. Then, by comparing simultaneous contracting with ex ante contracting, we show that the former dominates the latter if the information cost is low. This dominance, combined with the interplay between information cost and limited liability, results in three optimal mechanisms – simultaneous contracting, ex ante contracting, and partial contracting – and their respective indirect organizational structures such as the manager inside the firm, outsourcing, and no quality assessment.

The results of the present paper can be extended in several directions. The principal can implement the optimal contract of simultaneous contracting in the form of delegation to the manager by restricting a feasible set of contracts that the manager chooses for the agent and offering the agent a take-it-or-leave-it decision. Suppose a pre-investment stage where the agent makes an investment. Then, for a positive investment cost, the partial contracting mechanism lowers not only the payoffs to the principal and the agent but also the incentive to make such investment. This decreases the principal's payoff further, compared with the

[33] That is, the fraction of the first order derivative of $s(\omega', q)$ with respect to ω' and $s(\omega', q)$ can be expressed as one functional form, *i.e.*, $\frac{\partial s(\omega', q)/\partial\omega'}{s(\omega', q)} = y(\omega', q)$. This is known as the first-order homogeneous differential equation, which is invariant under changes of scale.

second-best payoff. Finally, the analysis for a single agent can be extended to multiple agents if team production between them has no complementarity, by replicating the analysis for one agent to many agents.[34]

The model can also be developed further for new approaches. If the quality is not a non-contractible type but can be elicited, then the partial contracting's payoff will increase with the truthful report, even without the manager, so the structure including a third party will less likely be optimal. Yet, the exact details in the approach require broad and careful study – *e.g.* quality can be a type that can be elicited or an action with a different type specification – which is beyond our scope, but such quality often exists in the form of a worker's intrinsic type. The agent can also be protected by limited liability, as so often is the case, but the main purpose of the paper is to show whether including the manager inside the firm is optimal, for which the manager's limited liability plays a role, whereas the agent's limited liability will equally affect either case. The focus of this paper is on how the principal can extract relevant information from a third party, the manager, so we restrict the analysis to a coalition-free environment, leaving a full characterization of collusion-proof mechanisms with non-contractible quality for future research.

Appendix A Proofs

Proof of Proposition 1. *Step 1.* We show that the incentive compatibility (6) is equivalently rewritten such that for any pair $(\theta, \omega), (\theta', \omega') \in \Theta \times \Omega$,

$$t(\theta,\omega) - q(\theta,\omega)\theta \geq t(\theta',\omega) - q(\theta',\omega)\theta, \tag{31}$$

$$t(\theta,\omega) - q(\theta,\omega)\theta = t(\theta,\omega') - q(\theta,\omega')\theta. \tag{32}$$

First, show (6) $\Rightarrow$ (31)-(32). If we fix $\omega \in \Omega$ or fix $\theta \in \Theta$, it is immediate that the incentive compatibility (6) implies the incentive compatibility (31)-(32). In particular, for the latter, *i.e.*, for a fixed θ, suppose that ω is the agent's (true) quality. Then, his incentive compatibility is satisfied if the agent obtains a higher payoff from reporting it truthfully compared with the payoff from misreporting it such that

$$t(\theta,\omega) - q(\theta,\omega)\theta \geq t(\theta,\omega') - q(\theta,\omega').$$

On the other hand, suppose that ω' is the agent's quality. Then, his incentive compatibility is satisfied if

$$t(\theta,\omega') - q(\theta,\omega')\theta \geq t(\theta,\omega) - q(\theta,\omega).$$

[34] In the case of a single *indivisible* good – such as a single unit auction – even with interactions between players, a similar analysis can be extended as in Yoo (2017). Technically, in the single-agent case, finding the optimal contract for the manager amounts to deriving an ordinary differential equation, which is still applicable even with multiple agents if there is no complementarity. If the type of their joint production involves complementarity, however, the analysis will not be tractable, since finding the optimal contract requires solving a system of partial differential equations, especially given the nonlinear nature of the problem.

Together, the equality in (32) is derived.[35] In deriving the equality, we use the fact that the true quality does not affect the agent's payoff, so it does not appear in it; in other words, any ω or ω' appearing in the agent's payoff is his report (again, it is not the agent's value but the principal's). Further, (32) implies that for any mechanism inducing the agent to report ω truthfully for a fixed θ, the agent must have an identical payoff.

Now, show (6) $\Leftarrow$ (31)-(32). By combining (31) with (32), it is immediate that the latter implies the former.

Step 2. From the incentive compatibility (31)-(32), a direct mechanism (q, t) is incentive compatible if and only if (i) q is decreasing in θ; (ii) for any $\theta \in \Theta$,

$$t(\theta, \omega) = q(\theta, \omega)\theta + (t(\overline{\theta}, \omega) - q(\overline{\theta}, \omega)\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(x, \omega)dx; \tag{33}$$

and (iii) for each $\omega \neq \omega' \in \Omega$,

$$t(\theta, \omega) - q(\theta, \omega)\theta = t(\theta, \omega') - q(\theta, \omega')\theta.$$

Consider $(\theta, \omega), (\theta, \omega') \in \Theta \times \Omega$ with $\omega' \neq \omega$. Define $D(\theta)$ such that

$$D(\theta) \equiv t(\overline{\theta}, \omega) - q(\overline{\theta}, \omega)\overline{\theta} + \int_{\theta}^{\overline{\theta}} q(x, \omega)dx - \left[t(\overline{\theta}, \omega') - q(\overline{\theta}, \omega')\overline{\theta} + \int_{\theta}^{\overline{\theta}} q(x, \omega')dx \right].$$

Then, from (33), we have

$$t(\theta, \omega) - q(\theta, \omega)\theta = (t(\overline{\theta}, \omega) - q(\overline{\theta}, \omega)\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(x, \omega)dx,$$

$$t(\theta, \omega') - q(\theta, \omega')\theta = (t(\overline{\theta}, \omega') - q(\overline{\theta}, \omega')\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(x, \omega')dx.$$

Then, by (32), $D(\theta) = 0$ for all $\theta \in \Theta$. It follows from the mean value theorem and the fundamental theorem of calculus that $D'(\theta) = 0$ for almost all $\theta \in \Theta$. Hence, for each $\omega \neq \omega' \in \Omega$, $q(\theta, \omega) = q(\theta, \omega')$ and $t(\theta, \omega) = t(\theta, \omega')$ for almost all $\theta \in \Theta$. ■

Proof of Lemma 1. We first show the following claim that will be used in Step 1 repeatedly. Note that $\Pr(q_s(\theta, \omega') > q_0 \mid \omega) > 0$ for all $\omega, \omega' \in \Omega$ is equivalent to $q_0 < q_s(\underline{\theta}, \underline{\omega})$, and we use the claim later for the negation of $q_0 < q_s(\underline{\theta}, \underline{\omega})$, *i.e.* $q_0 \geq q_s(\underline{\theta}, \underline{\omega})$.

Claim. If $q_0 \geq q_s(\underline{\theta}, \underline{\omega})$, for each $\omega, \omega'' \in \Omega$,

$$\Pr(q_s(\theta, \omega') > q_0 \mid \omega) > 0\,(= 0) \Leftrightarrow \Pr(q_s(\theta, \omega') > q_0 \mid \omega'') > 0\,(= 0). \tag{34}$$

[35]One need (31) as well as (32) for (6). That is, the agent's incentive compatibility must also take care of misreporting θ. For instance, consider a contract $q(\theta, \omega) = \omega/\theta$ and $t(\theta, \omega) = \omega$, but if the agent misreports $\theta' < \theta$, then the incentive compatibility is violated: $t(\theta, \omega) - q(\theta, \omega)\theta = 0 < \omega - (\theta'/\theta)\omega$.

Proof. Since q_s is a strictly increasing function of ω, for $q_0 \geq q_s(\underline{\theta}, \underline{\omega})$, there exists a cut-off value $\widetilde{\omega}(q_0) \geq \underline{\omega}$ satisfying $q_s(\underline{\theta}, \widetilde{\omega}(q_0)) = q_0$ such that $q_s(\underline{\theta}, \omega') \leq q_0$ if $\omega' \leq \widetilde{\omega}(q_0)$, whereas $q_s(\underline{\theta}, \omega') > q_0$ if $\omega' > \widetilde{\omega}(q_0)$. Then,

$$\begin{cases} \Pr(q_s(\theta, \omega') > q_0 \mid \omega) = 0 & \text{if } \omega' \leq \widetilde{\omega}(q_0), \\ \Pr(q_s(\theta, \omega') > q_0 \mid \omega) > 0 & \text{if } \omega' > \widetilde{\omega}(q_0), \end{cases} \text{for all } \omega \in \Omega,$$

which yields the claim. ■ That is, whether the probability is positive or not does not depend on the true quality.

Now, the two results of the lemma can be shown with the following three steps.

Step 1. Show that $s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega) \neq 0$ for all $\omega, \omega' \in \Omega$. Suppose, on the contrary, that there exist $\omega, \omega' \in \Omega$ such that $s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega) = 0$, which is equivalent to $q_0 \geq q_s(\underline{\theta}, \underline{\omega})$. First, show that if $s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega) = 0$, then

$$s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega') = 0, \text{that is, conditional on } \omega'.$$

If $s_0(\omega') = 0$, the above trivially holds; now, if $s_0(\omega') \neq 0$, by (34), $\Pr(q_s(\theta, \omega') > q_0 \mid \omega) = 0$ implies $\Pr(q_s(\theta, \omega') > q_0 \mid \omega') = 0$. On the other hand, for target i such that there exists $\omega, \omega'' \in \Omega$ such that $s_0(\omega'') \Pr(q_s(\theta, \omega'') > q_0 \mid \omega) \neq 0$, which implies $s_0(\omega'') \neq 0$ and $\Pr(q_s(\theta, \omega'') > q_0 \mid \omega) > 0$. By (34), $\Pr(q_s(\theta, \omega'') > q_0 \mid \omega) > 0$ implies $\Pr(q_s(\theta, \omega'') > q_0 \mid \omega') > 0$ as well as $\Pr(q_s(\theta, \omega'') > q_0 \mid \omega'') > 0$. Hence,

$$s_0(\omega'') \Pr(q_s(\theta, \omega'') > q_0 \mid \omega') \neq 0.$$

For $s_0(\omega'') > 0$, $s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega') = 0 < s_0(\omega'') \Pr(q_s(\theta, \omega'') > q_0 \mid \omega')$. For $s_0(\omega'') < 0$, $s_0(\omega'') \Pr(q_s(\theta, \omega'') > q_0 \mid \omega'') < 0 = s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega'')$, since if $s_0(\omega') = 0$, $s_0(\omega') \Pr(q_s(\theta, \omega') > q_0 \mid \omega'') = 0$; if not, by (34), $\Pr(q_s(\theta, \omega') > q_0 \mid \omega') = 0$ implies $\Pr(q_s(\theta, \omega') > q_0 \mid \omega'') = 0$. This contradicts IC.

Step 2. From Step 1, $u(\omega', \omega, q_0) = s_0(\omega') F(p(\omega', q_0)|\omega)$ and $s_0(\omega') F(p(\omega', q_0)|\omega) \neq 0$ for all $\omega, \omega' \in \Omega$, which also implies $s_0(\omega) \neq 0$ for all $\omega \in \Omega$. We divide this step into two parts.

Part 1. $s_0(\omega) > 0$ for all $\omega \in \Omega$. *(a)* We first show that $s_0(\omega)$ is a strictly decreasing function of ω. Any incentive compatible $s_0(\omega)$ satisfies that for any $\omega' > \omega$,

$$s_0(\omega) F(p(\omega, q_0)|\omega) \geq s_0(\omega') F(p(\omega', q_0)|\omega).$$

For $q_0 \geq 0$ satisfying Step 1, we have $q_s(p(\omega, q_0), \omega) = q_0$, which guarantees that $p(\omega, q_0)$ is a strictly increasing function of ω. Suppose, on the contrary, that $s_0(\omega') \geq s_0(\omega)$ for $\omega' > \omega$. Then, $s_0(\omega) F(p(\omega, q_0)|\omega) < s_0(\omega') F(p(\omega', q_0)|\omega)$, which contradicts IC.

(b) $s_0(\omega)$ is continuous on $[\underline{\omega}, \overline{\omega}]$. First, note that the separable incentive compatibility IC for $s_0(\omega)$ satisfies that for each $\omega, \omega' \in \Omega$,

$$\begin{cases} s_0(\omega') F(p(\omega', q_0)|\omega') \geq s_0(\omega) F(p(\omega, q_0)|\omega'), \\ s_0(\omega) F(p(\omega, q_0)|\omega) \geq s_0(\omega') F(p(\omega', q_0)|\omega). \end{cases} \tag{35}$$

Now, we examine an interior point and two corner points separately. Show that $s_0(\omega)$ is continuous at $\underline{\omega}$. For each $\omega' > \omega = \underline{\omega}$, IC in (35) is satisfied. Suppose $s_0(\omega)$ is not continuous at $\underline{\omega}$. Then by (a) of Part 1, we have $s_0(\underline{\omega}) > \lim_{\omega \to \underline{\omega}+} s_0(\omega)$. Let $\omega' \to \underline{\omega}$. In the limit, the first IC becomes

$$\lim_{\omega' \to \underline{\omega}+} s_0(\omega')F(p(\underline{\omega}, q_0)|\underline{\omega}) \geq s_0(\underline{\omega})F(p(\underline{\omega}, q_0)|\underline{\omega}) \Leftrightarrow \lim_{\omega' \to \underline{\omega}+} s_0(\omega') \geq s_0(\underline{\omega}),$$

which is a contradiction. Show that $s_0(\omega)$ is continuous at $\overline{\omega}$. For each $\omega' < \omega = \overline{\omega}$, the IC in (35) is satisfied. Suppose $s_0(\omega)$ is not continuous at $\overline{\omega}$. Then by (a) of Part 1, we have $\lim_{\omega \to \overline{\omega}-} s_0(\omega) > s_0(\overline{\omega})$. Let $\omega' \to \underline{\omega}$. In the limit, the second IC becomes

$$s_0(\overline{\omega})F(p(\overline{\omega}, q_0)|\overline{\omega}) \geq \lim_{\omega \to \overline{\omega}-} s_0(\omega)F(p(\overline{\omega}, q_0)|\overline{\omega}) \Leftrightarrow s_0(\overline{\omega}) \geq \lim_{\omega \to \overline{\omega}-} s_0(\omega),$$

and a contradiction. Show that $s_0(\omega)$ is continuous at $\omega \in (\underline{\omega}, \overline{\omega})$. For each $\omega \neq \omega' \in (\underline{\omega}, \overline{\omega})$, the IC in (35) is satisfied. We let $\omega' \to \omega$. Then,

$$\begin{aligned} &\begin{cases} \lim_{\omega' \to \omega} s_0(\omega')F(p(\omega, q_0)|\omega) \geq s_0(\omega)F(p(\omega, q_0)|\omega), \\ s_0(\omega)F(p(\omega, q_0)|\omega) \geq \lim_{\omega' \to \omega} s_0(\omega')F(p(\omega, q_0)|\omega) \end{cases} \\ \Leftrightarrow\ &\lim_{\omega' \to \omega} s_0(\omega') \geq s_0(\omega) \geq \lim_{\omega' \to \omega} s_0(\omega'). \end{aligned}$$

(c) $s_0(\omega)$ is a differentiable function of ω. Consider IC in (35). For each $\omega \neq \omega'$,

$$\begin{aligned} &\begin{cases} s_0(\omega')F(p(\omega', q_0)|\omega') \geq s_0(\omega)F(p(\omega, q_0)|\omega'), \\ s_0(\omega)F(p(\omega, q_0)|\omega) \geq s_0(\omega')F(p(\omega', q_0)|\omega). \end{cases} \\ \Leftrightarrow &\begin{cases} s_0(\omega')F(p(\omega', q_0)|\omega') - s_0(\omega)F(p(\omega', q_0)|\omega') \geq s_0(\omega)F(p(\omega, q_0)|\omega') - s_0(\omega)F(p(\omega', q_0)|\omega'), \\ s_0(\omega)F(p(\omega, q_0)|\omega) - s_0(\omega)F(p(\omega', q_0)|\omega) \geq s_0(\omega')F(p(\omega', q_0)|\omega) - s_0(\omega)F(p(\omega', q_0)|\omega), \end{cases} \end{aligned}$$

which becomes

$$\Leftrightarrow \begin{cases} \frac{s_0(\omega') - s_0(\omega)}{\omega' - \omega}F(p(\omega', q_0)|\omega') \geq -\frac{F(p(\omega, q_0)|\omega') - F(p(\omega', q_0)|\omega')}{\omega - \omega'}s_0(\omega), \\ -\frac{F(p(\omega, q_0)|\omega) - F(p(\omega', q_0)|\omega)}{\omega - \omega'}s_0(\omega) \geq \frac{s_0(\omega') - s_0(\omega)}{\omega' - \omega}F(p(\omega', q_0)|\omega). \end{cases}$$

In the limit, we have

$$\begin{aligned} \left[\lim_{\omega' \to \omega} \frac{s_0(\omega') - s_0(\omega)}{\omega' - \omega}\right] F(p(\omega, q_0)|\omega) &\geq -s_0(\omega)f(p(\omega, q_0)|\omega)p_\omega(\omega, q_0) \\ &\geq \left[\lim_{\omega' \to \omega} \frac{s_0(\omega') - s_0(\omega)}{\omega' - \omega}\right] F(p(\omega, q_0)|\omega). \end{aligned}$$

Part 2. $s_0(\omega) < 0$ for all $\omega \in \Omega$. We show that $s_0(\omega)$ is now strictly increasing on Ω. Suppose, on the contrary, that $s_0(\omega') \leq s_0(\omega)$ for $\omega' > \omega$. Then, given the negative $s_0(\omega) < 0$ for all $\omega \in \Omega$, $s_0(\omega')F(p(\omega', q_0)|\omega') < s_0(\omega')F(p(\omega, q_0)|\omega') \leq s_0(\omega)F(p(\omega, q_0)|\omega')$, which contradicts IC. Then, using the same procedure as Part 1, it can be shown that $s_0(\omega)$ is differentiable.

Step 3. If $s_0(\omega)\Pr(q_s(\theta,\omega) > q_0 \mid \omega) < 0$ for all $\omega \in \Omega$, show that IC is violated. For $\omega' > \omega$ to satisfy the first order condition,

$$\begin{aligned}
0 &= u_{\omega'}(\omega', \omega', q_0) \\
&= s_0'(\omega') f(p(\omega', q_0)|\omega') \left[\frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} + \frac{F(p(\omega', q_0)|\omega')}{f(p(\omega', q_0)|\omega')} \right] \\
&< s_0'(\omega') f(p(\omega', q_0)|\omega') \left[\frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} + \frac{F(p(\omega', q_0)|\omega)}{f(p(\omega', q_0)|\omega)} \right],
\end{aligned}$$

where the last inequality follows from $s_0(\omega') < 0$, $s_0'(\omega') > 0$ from Part 2 of Step 2 and the reverse hazard rate dominance. Then this implies that $\frac{s_0(\omega')p_\omega(\omega',q_0)}{s_0'(\omega')} + \frac{F(p(\omega',q_0)|\omega)}{f(p(\omega',q_0)|\omega)} > 0$, and thus for $\omega' > \omega$,

$$u_{\omega'}(\omega', \omega, q_0) = s_0'(\omega') f(p(\omega', q_0)|\omega) \left[\frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} + \frac{F(p(\omega', q_0)|\omega)}{f(p(\omega', q_0)|\omega)} \right] > 0.$$

Hence, misreporting ω' yields a higher payoff. Together, by Step 3 eliminating $s_0(\omega) < 0$ for all $\omega \in \Omega$, Step 1 and 2 establish (i) and (ii). ■

Proof of Theorem 1. *Part 1* The manager chooses ω' to maximize $u(\omega', \omega, q_0) = s_0(\omega')F(p(\omega', q_0)|\omega)$. By Lemma 1, the partial derivative of $u(\omega', \omega, q_0)$ with respect to ω' yields

$$\begin{aligned}
u_{\omega'}(\omega', \omega, q_0) &= -\, s_0'(\omega') f(p(\omega', q_0)|\omega) \left[-\frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} - \frac{F(p(\omega', q_0)|\omega)}{f(p(\omega', q_0)|\omega)} \right] \qquad (36) \\
&= -\, s_0'(\omega') f(p(\omega', q_0)|\omega) \left[p(\omega', q_0) - \frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} - \phi(p(\omega', q_0), \omega) \right],
\end{aligned}$$

where $\phi(p(\omega', q_0), \omega) = p(\omega', q_0) + F(p(\omega', q_0)|\omega)/f(p(\omega', q_0)|\omega)$. Since for $q_0 \geq 0$ satisfying Lemma 1, $q_s(p(\omega, q_0), \omega) = q_0$, the optimality condition of the second best (4) implies that for each $\omega \in \Omega$, the first-order condition for the manager's truthful report is satisfied such as $u_{\omega'}(\omega, \omega, q_0) = 0$. In other words,

$$v_q(\omega, q_0) - \phi(p(\omega, q_0), \omega) = 0 \Leftrightarrow v_q(\omega, q_s(\theta, \omega)) - \phi(\theta, \omega) = 0.$$

Since for any $\omega' > \omega$, the first order condition is also satisfied such as $u_{\omega'}(\omega', \omega', q_0) = 0$, we have

$$\begin{aligned}
0 &= u_{\omega'}(\omega', \omega', q_0) \\
&= -s_0'(\omega') f(p(\omega', q_0)|\omega') \left[v_q(\omega', q_0) - \phi(p(\omega', q_0), \omega') \right] \\
&> -s_0'(\omega') f(p(\omega', q_0)|\omega') \left[v_q(\omega', q_0) - \phi(p(\omega', q_0), \omega) \right],
\end{aligned}$$

where the inequality follows from $s_0'(\omega') < 0$ from Lemma 1 and the reverse hazard rate dominance, $\phi(p(\omega', q_0), \omega') < \phi(p(\omega', q_0), \omega)$. Then this implies $v_q(\omega', q_0) - \phi(p(\omega', q_0), \omega) < 0$. Hence, we have

$$u_{\omega'}(\omega', \omega, q_0) = -s_0'(\omega') f(p(\omega', q_0)|\omega) \left[v_q(\omega', q_0) - \phi(p(\omega', q_0), \omega) \right] < 0.$$

Similarly, one can show that for any $\omega' < \omega$, $u_{\omega'}(\omega', \omega, q_0) > 0$. Together, the global sufficiency for the manager's truthful report is satisfied. Now, from (17), we find $s_0(\omega)$ explicitly such that

$$\frac{s_0'(\omega)}{s_0(\omega)} = \frac{p_\omega(\omega, q_0)}{p(\omega, q_0) - v_q(\omega, q_0)}, \text{ or } \frac{\partial[\ln s_0(\omega)]}{\partial \omega} = \frac{p_\omega(\omega, q_0)}{p(\omega, q_0) - v_q(\omega, q_0)}.$$

By taking the integral of both sides, we have

$$\int_\omega^{\overline{\omega}} \frac{\partial[\ln s_0(x)]}{\partial x} dx = \int_\omega^{\overline{\omega}} \frac{p_\omega(x, q_0)}{p(x, q_0) - v_q(x, q_0)} dx,$$

which yields (18):

$$\ln s_0(\omega) = \ln \overline{s}_0 - \int_\omega^{\overline{\omega}} \frac{p_\omega(x, q_0)}{p(x, q_0) - v_q(x, q_0)} dx, \text{ so } s_0(\omega) = \overline{s}_0 \exp\left\{\int_\omega^{\overline{\omega}} \frac{p_\omega(x, q_0)}{v_q(x, q_0) - p(x, q_0)} dx\right\}.$$

For the uniqueness, the partial derivative of $u(\omega', \omega, q_0)$ with respect to ω' yields

$$u_{\omega'}(\omega', \omega, q_0) = -s_0'(\omega') f(p(\omega', q_0)|\omega) \left[-\frac{s_0(\omega') p_\omega(\omega', q_0)}{s_0'(\omega')} - \frac{F(p(\omega', q_0)|\omega)}{f(p(\omega', q_0)|\omega)}\right].$$

For any $s_0(\omega)$ satisfying the FOC, by the second best solution in (4),

$$-\frac{s_0(\omega)}{s_0'(\omega)} = \frac{F(p(\omega, q_0)|\omega)}{p_\omega(\omega, q_0) f(p(\omega, q_0)|\omega)} = \frac{v_q(\omega, q_0) - p(\omega, q_0)}{p_\omega(\omega, q_0)},$$

which is identical to the first-order alignment in (18).

Part 2. Dominance. By choosing $\underline{S} = B + c$, we can choose $(\overline{s}_0, ..., \overline{s}_{N-1})$ such that IR binds:

$$B + c + \int_\Omega \sum_{i=0}^{N-1} \overline{s}_0 \exp\left\{\int_\omega^{\overline{\omega}} \frac{p_\omega(x, q_0)}{v_q(x, q_0) - p(x, q_0)} dx\right\} F(p(\omega, q_0)|\omega) dG(\omega) = \overline{U} + c,$$

where $\overline{U} - B > 0$. ■

Proof of Lemma 2. *Part 1.* By the binding IR from (19) and (20),

$$\begin{aligned} k(R) &= \overline{U} + R + c - \left[B + c + \overline{s}_0 \widehat{U}(\widehat{\omega}, q_0)\right] \\ &= (\overline{U} - B + R)\left[1 - \frac{\widehat{U}(\widehat{\omega}, q_0)}{U(q_0)}\right], \end{aligned}$$

where the first equality follows from the binding IR $\int_\Omega \mathbb{E}_\theta[S(\omega, \theta)|\omega] dG(\omega) = \overline{U} + R + c$ and the second from Theorem 2. Then, $k(R)$ is continuous and strictly increasing.

Part 2. (i) The simultaneous contracting versus the ex ante contracting. If $c < k(\Delta U)$, for $0 \le R < \Delta U$, we have $R < \Delta U \Leftrightarrow V_s - \overline{U} - R - c > V_{\min} - B - c$.

(ii) The simultaneous contracting versus the partial contracting. If $c < k(V_s - \overline{U} - c - V_p)$, for $0 \leq R < V_s - \overline{U} - c - V_p$, we have $R < V_s - \overline{U} - c - V_p \Leftrightarrow V_s - \overline{U} - R - c > V_p$. Since c is strictly increasing, whereas $k(V_s - \overline{U} - c - V_p)$ is strictly decreasing in c, there is a unique fixed point $\widehat{c}$ satisfying $\widehat{c} = k(V_s - \overline{U} - V_p - \widehat{c})$ such that for each $c < \widehat{c}$, $c < k(V_s - \overline{U} - c - V_p)$.

Now, $k(0) < c < k(\widehat{R})$, the optimal rent for the principal is the minimum R such that $c = k(R)$. If $c \leq k(0)$, on the other hand, it is $R = 0$. ■

Proof of Theorem 2. We divide the proof into two parts, the ineffective limited liability case and the effective one, that is, whether $V_s - \overline{U} \leq V_{\min} - B$ or $V_s - \overline{U} > V_{\min} - B$. Then the threshold limited liability level for them is given as $B = \overline{U} - (V_s - V_m)$.

Part 1. Suppose $\Delta U = 0$ or $V_s - \overline{U} \leq V_{\min} - B$ so that limited liability is not effective at all. Then, the optimal organization is the simultaneous and the ex ante contracting if $c < k(0)$; it is the ex ante contracting if $k(0) < c < V_s - \overline{U} - V_p$. Hence, the indifference line between them is given as $c = (\overline{U} - B)(1 - y)$, where recall $k(0) = (\overline{U} - B)(1 - y)$.

Part 2. For the proof, to simplify notations, we denote by $\succ_D$ the organizational dominance, together with notations for the three mechanisms such as SC for the simultaneous contracting; EC for the ex ante contracting; and PC for the partial contracting. First, note that $k(\Delta U) < V_s - \overline{U} - V_p$ is satisfied if $\overline{U} - B + \Delta U$ is sufficiently small. Then by Lemma 2,

$$\begin{cases} SC \succ_D EC & \text{if } c < k(\Delta U), \\ EC \succ_D SC & \text{if } c > k(\Delta U), \end{cases}$$

and, by comparing EC with PC, it is straightforward to find a cut-off $V_{\min} - B - V_p$ such that

$$\begin{cases} EC \succ_D PC & \text{if } c < V_{\min} - B - V_p, \\ PC \succ_D EC & \text{if } c > V_{\min} - B - V_p. \end{cases}$$

Now, we divide the proof for Part 2 into two cases, depending on whether $k(\Delta U) > V_{\min} - B - V_p$ or not, that is, whether $B < \widehat{B} \equiv V_m - V_p - (V_s - V_m)(1 - y)$ or not. The former yields the three organizational structure and the latter the two organizational structure.

Case 1. $k(\Delta U) < V_{\min} - B - V_p$. Then for $c < k(\Delta U)$, $SC \succ_D EC$ and $EC \succ_D PC$, so $SC \succ_D EC \succ_D PC$; for $k(\Delta U) < c < V_{\min} - B - V_p$, $EC \succ_D SC$ and $EC \succ_D PC$; and for $c > V_{\min} - B - V_p$, $EC \succ_D SC$ and $PC \succ_D EC$, so $PC \succ_D EC >_D SC$. This yields the first set of the three organizational structure. The indifference line between EC and PC is given as $c = V_{\min} - B - V_p$, and that between SC and EC is $c = k(\Delta U) = (V_s - V_p)(1 - y)$. It remains to show that this case exists for some B. For a sufficiently small negative B,

$$k(\Delta U) = (V_s - V_{\min})(1 - y) < V_{\min} - B - V_p$$

and it follows from $\Delta U > 0$ that $V_{\min} - B - V_p < V_s - \overline{U} - V_p$.

Case 2. $k(\Delta U) > V_{\min} - B - V_p$. Then for $c < V_{\min} - B - V_p$, $SC \succ_D EC$ and $EC \succ_D PC$, so $SC \succ_D EC \succ_D PC$; and for $c > k(\Delta U)$, $EC \succ_D SC$ and $PC \succ_D EC$, so $PC \succ_D > EC \succ_D SC$. Since for $V_{\min} - B - V_p < c < k(\Delta U)$, $SC \succ_D EC$ and $PC \succ_D EC$, it remains to determine whether SC dominates PC or not. First, show that $\widehat{c} \in (V_{\min} - B - V_p, k(\Delta U))$. Recall that by Lemma 2, $\widehat{c} = k(V_s - \overline{U} - V_p - \widehat{c})$. If $c = k(\Delta U)$, then

$$k(V_s - \overline{U} - V_p - k(\Delta U)) = k(\Delta U + V_{\min} - B - V_p - k(\Delta U)) < k(\Delta U) = c,$$

which implies that $\widehat{c} < k(\Delta U)$ since $c - k(V_s - \overline{U} - V_p - c)$ is a strictly increasing function of c. Similarly, if If $c = V_{\min} - B - V_p$, then

$$k(V_s - \overline{U} - V_p - (V_{\min} - B - V_p)) = k(\Delta U) > V_{\min} - B - V_p = c,$$

which implies that $\widehat{c} > V_{\min} - B - V_p$. Then it follows from Lemma 2 that for $c < \widehat{c}$, $SC \succ_D PC$, whereas for $c > \widehat{c}$, $PC \succ_D SC$. Hence, the indifference line between SC and PC is given as $V_s - B - \frac{2-y}{1-y}c = V_p$.

Together, this establishes the four cases (i) - (iv). ■

Proof of Proposition 2. *Part 1.* First, it is clear that $\underline{S} = B + c$. Furthermore, by denoting $x_i \equiv \frac{\overline{s}_i U(q_i)}{\overline{U}-B+R}$ for all i and $x = (x_0, ..., x_{\overline{N}-1})$ with $\sum_{i=0}^{\overline{N}-1} x_i = 1$, the maximization problem in (25) can be succinctly rewritten as

$$\underset{x \in \Delta}{\text{minimize}} \max_{\widehat{\omega} \in \Omega} \sum_{i=0}^{\overline{N}-1} x_i \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)}, \tag{37}$$

given a compact set $\Delta \equiv \{x \in \mathbb{R}_+^{\overline{N}} : \sum_{i=0}^{\overline{N}-1} x_i = 1\}$. For each x, by the Weierstrass theorem, there exists a solution, not necessarily a unique one. Still, by the maximum theorem, the objective function $\max_{\widehat{\omega} \in \Omega} \sum_{i=0}^{\overline{N}-1} x_i \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)}$ is a continuous function of x. Since x is from a compact set, there exists a solution to the minimax problem in (37).

Part 2. For a solution x^* to the reformulation in (37), there is a unique $s^* \equiv (\overline{s}_0^*, ..., \overline{s}_{\overline{N}-1}^*)$ through $x_i \equiv \frac{\overline{s}_i U(q_i)}{\overline{U}-B+R}$, and vice versa. By the binding IR from (19) and (20),

$$\begin{aligned} B + c + \sum_{i=0}^{N-1} \overline{s}_i^* \widehat{U}(\widehat{\omega}, q_i) &= B + c + (\overline{U} - B + R) \left[\sum_{i=0}^{N-1} \frac{\overline{s}_i^*}{\overline{U} - B + R} \widehat{U}(\widehat{\omega}, q_i) \right] \\ &= B + c + (\overline{U} - B + R) \left[\sum_{i=0}^{N-1} x_i^* \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)} \right], \end{aligned}$$

where the last line yields the formula as a linear function of R, since for each i, $x_i^* = \frac{\overline{s}_i^* U(q_i)}{\overline{U}-B+R}$. ■

Proof of Proposition 3. For (26) $\Rightarrow$ (27), we provide omitted steps between (28) and (29). We factor out $p_\omega(\omega', q)$ from (28) to have: for $\omega' = \omega \in \Omega$,

$$p_\omega(\omega', q) \left[\frac{v_q(\omega', q) - p(\omega', q)}{p_\omega(\omega', q)} - \frac{F(p(\omega', q)|\omega)}{f(p(\omega', q)|\omega) p_\omega(\omega', q)} \right] = 0,$$

which can be rewritten as

$$p_\omega(\omega',q)\left[\frac{v_q(\omega',q)-p(\omega',q)}{p_\omega(\omega',q)}-\frac{F(p(\omega',q)|\omega)}{\partial[F(p(\omega',q)|\omega)]/\partial\omega'}\right]=0, \tag{38}$$

where note that to satisfy IC, $q=q_s(\theta,\omega')$ given $\theta>\underline{\theta}$ is necessary as discussed in (26). To satisfy the equality in (38), the term inside the above bracket must satisfy $\frac{p_\omega(\omega',q)}{v_q(\omega',q)-p(\omega',q)}-\frac{\partial[F(p(\omega',q)|\omega)]/\partial\omega'}{F(p(\omega',q)|\omega)}=0$, by switching the numerator with the denominator, which can be further rewritten as $\frac{p_\omega(\omega',q)}{v_q(\omega',q)-p(\omega',q)}-\frac{\partial\log(F(p(\omega',q)|\omega))}{\partial\omega'}=0$. Then, by taking the integral of the both sides, this implies

$$\frac{\partial\left[\int_{\omega'}^{\overline{\omega}}\frac{p_\omega(x,q)}{v_q(x,q)-p(x,q)}dx+\log(F(p(\omega',q)|\omega))\right]}{\partial\omega'}=0 \text{ if } \omega'=\omega\in\Omega, \tag{39}$$

which, by having $\log\left(\exp\left\{\int_{\omega'}^{\overline{\omega}}\frac{p_\omega(x,q)}{v_q(x,q)-p(x,q)}dx\right\}\right)$, leads to (29).

We show that to satisfy IC, the integral must be taken from ω' to $\overline{\omega}$ as in (30). Suppose, on the contrary, that we take the integral with the other direction to have

$$\frac{\partial\left[\int_{\underline{\omega}}^{\omega'}\frac{p_\omega(x,q)}{v_q(x,q)-p(x,q)}dx-\log(F(p(\omega',q)|\omega))\right]}{\partial\omega'}=0 \text{ if } \omega'=\omega\in\Omega. \tag{40}$$

Note that given the opposite direction of the integral from $\underline{\omega}$ to ω', we have minus sign in front of the second term in (40) unlike (39), and, to indicate the difference clearly, denote $\widehat{e}(\omega,q)\equiv\exp\left\{\int_{\underline{\omega}}^{\omega}\frac{p_\omega(x,q)}{v_q(x,q)-p(x,q)}dx\right\}$ and $\widehat{u}(\omega',\omega,q)$ for the manager's expected payoff given $\widehat{e}(\omega,q)$, not $e(\omega,q)$ in (30). The optimality condition in (40) is when the true quality is given as ω, and the same optimality condition applies to the case that the true quality is ω', which is given as

$$\begin{aligned}\widehat{u}_{\omega'}(\omega',\omega',q)=0&=\frac{p_\omega(\omega',q)}{v_q(\omega',q)-p(\omega',q)}-\frac{F(p(\omega',q)|\omega')p_\omega(\omega',q)}{f(p(\omega',q)|\omega')}\\&<\frac{p_\omega(\omega',q)}{v_q(\omega',q)-p(\omega',q)}-\frac{F(p(\omega',q)|\omega)p_\omega(\omega',q)}{f(p(\omega',q)|\omega)},\end{aligned}$$

where the last inequality follows the reverse hazard rate dominance. This implies that for $\omega'>\omega$,

$$\widehat{u}_{\omega'}(\omega',\omega,q)=\frac{\partial\widehat{e}(\omega',q)}{\partial\omega'}-\frac{F(p(\omega',q)|\omega)p_\omega(\omega',q)}{f(p(\omega',q)|\omega)}>0,$$

and misreporting ω' yields a higher payoff, violating IC.

The remaining procedure to show that (30) satisfies IC is identical to the one in the proof of Theorem 1, so it is omitted. ■

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Appendix B For Online Publication Only

B.1 Second best

A mechanism is said to be incentive compatible and individually rational if for each $\omega \in \Omega$, and every $\theta, \theta' \in \Theta$,

$$t_s(\theta,\omega) - q_s(\theta,\omega)\theta \geq t_s(\theta',\omega) - q_s(\theta',\omega)\theta, \tag{41}$$

$$t_s(\theta,\omega) - q_s(\theta,\omega)\theta \geq 0. \tag{42}$$

With a constant output over quality, a partial contracting mechanism Γ^p consists of $q_p(\theta) = q(\theta,\omega)$ and $t_p(\theta) = t(\theta,\omega)$ for all ω. The principal obtains expected payoff from Γ^p such that

$$V(\Gamma^p) = \int_\Omega \int_\Theta \left[v(\omega, q_p(\theta)) - q_p(\theta)\theta - \int_\theta^{\bar{\theta}} q_p(x)dx \right] f(\theta|\omega) d\theta dG(\omega), \tag{43}$$

and a partial contracting mechanism Γ^p is optimal if it maximizes the above payoff subject to incentive compatibility and individual rationality conditions for $(q_p(\theta), t_p(\theta))$

To make the analysis self-contained, with a slight abuse of notations, we denote by $\widehat{F}(\theta,\omega)$ the joint CDF of θ and ω and by $H(\theta)$ the marginal CDF of θ.

Part 1 Find V_p for (43). With (q_p, t_p), Γ^p is incentive compatible if for each $\theta, \theta' \in \Theta$, $t_p(\theta) - q_p(\theta)\theta \geq t_p(\theta') - q_p(\theta')\theta$, and it is individually rational if for each $\theta \in \Theta$, $t_p(\theta) - q_p(\theta)\theta \geq 0$. Under the principal's payoff maximization, by a standard analysis, an incentive compatible & individually rational Γ^p yields $t_p(\theta) = q_p(\theta)\theta + \int_\theta^{\bar{\theta}} q_p(x)dx$, and thus the principal's payoff in (43). Since $\widehat{F}(\theta,\omega) = F(\theta|\omega)G(\omega)$, this is equivalent to:

$$\begin{aligned}
&\int_{\Theta\times\Omega} \left[v(\omega, q_p(\theta)) - q_p(\theta)\theta - \int_\theta^{\bar{\theta}} q_p(x)dx \right] d\widehat{F}(\theta,\omega) \\
&= \int_\Theta \left[\mathbb{E}_\omega[v(\omega, q_p(\theta))] - q_p(\theta)\theta - \int_\theta^{\bar{\theta}} q_p(x)dx \right] dH(\theta) \\
&= \int_\Theta \left[\mathbb{E}_\omega[v(\omega, q_p(\theta))] - q_p(\theta)\Big(\theta + \frac{H(\theta)}{h(\theta)}\Big) \right] dH(\theta).
\end{aligned} \tag{44}$$

The first equality follows since v is the only term that depends on ω inside the integral, where $\mathbb{E}_\omega[v(\omega, q_p(\theta))]$ is the expected value of $v(\omega, q_p(\theta))$ given ω that the principal obtains. The second equality follows from the standard procedure by changing the order of integration. Then, the principal chooses q_p to maximize the term inside the integral, which yields V_p.

Part 2 Find V_s for (3). For the benchmark, its direct mechanism yields the principal's payoff such as

$$\int_\Omega \int_\Theta \left[v(\omega, q_s(\theta,\omega)) - q_s(\theta,\omega)\theta - \int_\theta^{\bar{\theta}} q_s(x,\omega)dx \right] f(\theta|\omega) d\theta dG(\omega),$$

which, by changing the order of integration, is rewritten as (3). The principal chooses q_s to maximize the term inside the integral, and the solution yields V_s.

Part 3 Compare them. Since $\widehat{F}(\theta,\omega) = F(\theta|\omega)G(\omega)$,

$$\begin{aligned}
&\int_{\Theta\times\Omega}\left[v(\omega,q_p(\theta)) - q_p(\theta)\theta - \int_{\theta}^{\bar{\theta}} q_p(x)dx\right] d\widehat{F}(\theta,\omega)\\
=&\int_{\Omega}\int_{\Theta}\left[v(\omega,q_p(\theta)) - q_p(\theta)\theta - \int_{\theta}^{\bar{\theta}} q_p(x)dx\right] f(\theta|\omega)d\theta dG(\omega) \qquad (45)\\
=&\int_{\Omega}\int_{\Theta}\left[v(\omega,q_p(\theta)) - q_p(\theta)\phi(\theta,\omega)\right] f(\theta|\omega)d\theta dG(\omega),
\end{aligned}$$

where we apply changing the order of integration with respect to $f(\theta|\omega)$ this time. It is clear that for all (θ,ω), $v(\omega,q_s(\theta,\omega)) - q_s(\theta,\omega)\phi(\theta,\omega) \geq v(\omega,q_p(\theta)) - q_p(\theta)\phi(\theta,\omega)$. Furthermore, the optimal interior q_s in (4) is strictly increasing in ω, whereas q_p is a constant over all ω, so by comparing the payoff V_s from (3) with V_p, we have the strict inequality such that

$$\begin{aligned}
V_s =& \int_{\Omega}\int_{\Theta}\left[v(\omega,q_s(\theta,\omega)) - q_s(\theta,\omega)\phi(\theta,\omega)\right] f(\theta|\omega)d\theta dG(\omega)\\
>& \int_{\Omega}\int_{\Theta}\left[v(\omega,q_p(\theta)) - q_p(\theta)\phi(\theta,\omega)\right] f(\theta|\omega)d\theta dG(\omega) = V_p.
\end{aligned}$$

where the last equality follows from (45).

To illustrate the second-best solution, consider the example from the introduction with particular functional forms and one worker.

Example 1 A firm's owner assigns different working hours to a worker for a job among the total working hours $T > 0$. The job's outcome depends on the worker's quality, and his cost function is θq for $q \geq 0$ hours. The conditional distribution of cost θ given quality ω is $F(\theta|\omega) = \theta^{\omega}$ for $\theta \in [0,1]$, which yields the virtual cost $\phi(\theta,\omega) = \theta + \frac{\theta}{\omega}$. The principal's payoff from the job is $\omega\ln(q+1)$ if the worker with quality ω invests q hours. The worker can be assigned to the other standard job for the remaining hours $T-q$, which gives the principal payoff $(T-q)$. Together, the principal obtains a total value $v(\omega,q) = \omega\ln(q+1) + (T-q)$. The distribution F and v satisfy the assumptions in Section 2, including the reverse hazard rate dominance. Additionally, the conditions for an interior output require $1 < \underline{\omega} < \overline{\omega} \leq 2$. Then, the second-best solution from (4) is given as $q_s(\theta,\omega) = \frac{\omega^2}{(\theta\omega+\theta+\omega)} - 1$.

B.2 Mechanisms that make both M and A report ω

If both the manager and the agent report ω, a direct mechanism consists of functions $\widehat{q}, \widehat{t}$ and $\widehat{S}$ where $\widehat{q} : \Theta\times\Omega^2 \to \mathbb{R}_+, \widehat{t} : \Theta\times\Omega^2 \to \mathbb{R}$ and $\widehat{S} : \Omega\times\Theta\times\Omega \to \mathbb{R}$. If the agent reports (θ,ω''), and the manager reports ω', then the principal assigns the agent the production of $\widehat{q}(\theta,\omega'',\omega')$ and commits to paying $\widehat{t}(\theta,\omega'',\omega')$ to the agent and $\widehat{S}(\omega',\theta,\omega'')$ to the manager.

Proposition 4 *If the manager's contract in a mechanism does not depend on θ, for any true ω, any report $\widehat{\omega}$ can be an equilibrium in the mechanism. If the manager's contract in a mechanism depends on θ, to induce a unique Bayesian equilibrium, the principal elicits reporting on the quality only from the manager.*

Proof. *Case 1.* Suppose the manager's contract does not depend on θ. If it is incentive compatible, then for each truthful report of $\theta \in \Theta$, the following conditions are satisfied: for each true $\omega \in \Omega$ and every reports $\omega', \omega'' \in \Omega$, $\widehat{S}(\omega, \theta, \omega) \geq \widehat{S}(\omega', \theta, \omega)$ for the manager, and $\widehat{t}(\theta, \omega, \omega) - \widehat{q}(\theta, \omega, \omega)\theta \geq \widehat{t}(\theta, \omega'', \omega) - \widehat{q}(\theta, \omega'', \omega)\theta$ for the agent. However, the same incentive compatibility allows all other $\widehat{\omega} \neq \omega$ to arise as an equilibrium because the true quality affects neither directly: that is, for each reports $\omega', \omega'' \in \Omega$, $\widehat{S}(\widehat{\omega}, \theta, \widehat{\omega}) \geq \widehat{S}(\omega', \theta, \widehat{\omega})$ and $\widehat{t}(\theta, \widehat{\omega}, \widehat{\omega}) - \widehat{q}(\theta, \widehat{\omega}, \widehat{\omega})\theta \geq \widehat{t}(\theta, \omega'', \widehat{\omega}) - \widehat{q}(\theta, \omega'', \widehat{\omega})\theta$.

Case 2. Now, suppose the manager's contract depends on θ. If a direct mechanism $(\widehat{q}, \widehat{t}, \widehat{S})$ is incentive compatible, then for each truthful report of $\theta \in \Theta$, the following conditions are satisfied for any $\omega \neq \omega' \in \Omega$: (i) for the manager,

$$\mathbb{E}_\theta[\widehat{S}(\omega, \theta, \omega)|\omega] \geq \mathbb{E}_\theta[\widehat{S}(\omega', \theta, \omega)|\omega], \tag{46}$$

and (ii) for the agent, it is identical to the one from Case 1. It is sufficient to show that eliciting the quality from both the manager and the agent leads to a continuum of non-truthful equilibria. The manager's truthful report requires an additional incentive compatibility condition: for each $\omega \in \Omega$, there exists $\epsilon > 0$ such that for any $\omega' \in (\omega - \epsilon, \omega + \epsilon)$,

$$\mathbb{E}_\theta[\widehat{S}(\omega'', \theta, \omega')|\omega] \leq \mathbb{E}_\theta[\widehat{S}(\omega', \theta, \omega')|\omega] \text{ for all } \omega'' \in \Omega \text{ and } \omega'' \neq \omega'. \tag{47}$$

If not, there exists $\omega \in \Omega$ such that for each $\epsilon > 0$, there exists $\omega' \in (\omega - \epsilon, \omega + \epsilon)$ such that

$$\mathbb{E}_\theta[\widehat{S}(\omega'', \theta, \omega')|\omega] > \mathbb{E}_\theta[\widehat{S}(\omega', \theta, \omega')|\omega] \text{ for some } \omega'' \in \Omega \text{ and } \omega'' \neq \omega'.$$

This implies:

$$\mathbb{E}_\theta[\widehat{S}(\omega'', \theta, \omega')|\omega'] > \mathbb{E}_\theta[\widehat{S}(\omega', \theta, \omega')|\omega'] \text{ for some } \omega'' \in \Omega \text{ and } \omega'' \neq \omega',$$

which is a contradiction with the condition for the manager's truth-telling of ω' from (46). Hence, (47) implies that for each $\omega \in \Omega$, there is a continuum of non-truthful equilibria, both reporting (ω', ω') for $\omega' \neq \omega$. ■

Consider any punishment for two different reports. As shown above, then, for each true $\omega \in \Omega$, any $\widehat{\omega} \in \Omega$ arises as an equilibrium, because true quality ω is *not embedded* in either the manger's payoff or the agent's. Hence, with a positive information cost, if the manager chooses $\widehat{\omega}$ maximizing $\widehat{S}(\omega, \theta, \omega)$ since any $\widehat{\omega} \in \Omega$ arises as an equilibrium, then

$$\widehat{S}(\widehat{\omega}, \theta, \widehat{\omega}) \geq \widehat{S}(\omega, \theta, \omega) > \widehat{S}(\omega, \theta, \omega) - c,$$

which fails to incentivize the manager's information acquisition.

B.3 Optimal number of intervals

For its characterization, we suppose that each $\widehat{U}(\widehat{\omega}, q_i)$ has a single peak, which is denoted by ω_i, that is, $\omega_i = \arg\max_{\widehat{\omega}\in\Omega} \widehat{U}(\widehat{\omega}, q_i)$. We say that an interval q_k is dominant if for all q_j,

$$\frac{\widehat{U}(\omega_k, q_k)}{U(q_k)} \leq \frac{\widehat{U}(\omega_k, q_j)}{U(q_j)}. \tag{48}$$

It is not difficult for one to envision a few figures to verify indeed that if there is a dominant interval, by the single-peak property, the single dominant interval solves the minimax problem.

A more general and interesting case is, however, that there is no such dominant interval, which invites a couple of challenges for the analysis: an arbitrary number of intervals and an irregularity of the function $\widehat{U}(\widehat{\omega}, q_i)$ with respect to $\widehat{\omega}$. If $\widehat{\omega}$ were fixed, that is, with no maximization problem in (37), the minimization problem itself is a simple linear programming with respect to $x = (x_0, ..., x_{\overline{N}-1})$, like the cost minimization from classical consumer or production theory, thanks to the linear characterization of the solution $s_i(\omega) = \overline{s}_i e(\omega, q_i)$ from Theorem 1. Despite the linearity, the minimax problem involves an additional layer of the optimization: for each vector of weights x, there is a corresponding $\widehat{\omega}$ that the manager chooses to deviate to *maximize* the convex combinations of the ratios, $\left(\frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)}\right)_{i=0,...,\overline{N}-1}$. The maximized function $\max_{\widehat{\omega}\in\Omega} \sum_{i=0}^{\overline{N}-1} x_i \frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)}$ in the minimax problem is continuous in its parameter x – so there exists a solution to the minimax problem – but if $\widehat{U}(\widehat{\omega}, q_i)$ is not concave for some q_i, its solution is not necessarily differentiable.[36] On the other hand, conceivably, the concavity for *all* $i = 0, ..., \overline{N} - 1$ requires strong conditions on the distribution functions, F and G, and p.

To delve into the challenges further, we probe the minimax problem only with two arbitrary intervals, as an intermediate step toward the main goal. The study on the two-interval case not only clarifies what the aforementioned difficulty means in detail but provides a clean segue into the second main result of this paper, the *optimal number of intervals*. The minimax with two intervals q_i and q_j is given as

$$\underset{x_i\in[0,1]}{\text{minimize}} \max_{\widehat{\omega}\in\Omega} x_i \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)} + (1 - x_i)\frac{\widehat{U}(\widehat{\omega}, q_j)}{U(q_j)}, \tag{49}$$

where $\frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)}$ and $\frac{\widehat{U}(\widehat{\omega},q_j)}{U(q_j)}$ have single peaks ω_i and ω_j. We denote by $\omega^*_{ij}(x_i)$ a report $\widehat{\omega}$ satisfying the first-order condition for the manager's max problem to misreport worker quality $\widehat{\omega}$, given x_i:

$$x_i \frac{\widehat{U}_{\widehat{\omega}}(\omega^*_{ij}(x_i), q_i)}{U(q_i)} + (1 - x_i)\frac{\widehat{U}_{\widehat{\omega}}(\omega^*_{ij}(x_i), q_j)}{U(q_j)} = 0. \tag{50}$$

Provided that neither is a dominant interval, there is no problem (by the intermediate value theorem) of making x_i *generate* an intersection between the two ratios as the one satisfying

[36] Technically, by the maximum theorem – the solution – the optimal $\widehat{\omega}$ is only *upper hemicontinuous* in x.

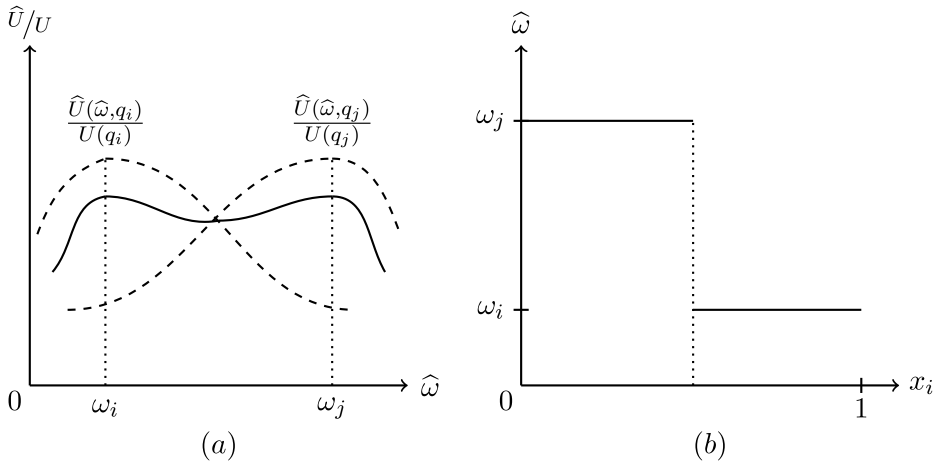

Figure 7: $x_i \frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)} + (1-x_i)\frac{\widehat{U}(\widehat{\omega},q_j)}{U(q_j)}$ and not differentiable $\omega^*_{ij}(x_i)$

the equality above, but it may not satisfy the sufficiency; that is, the weight x_i satisfying the first order optimality above in fact may *not* be a solution to the minimax. As an illustration, in Figure 4, consider a case that the convex combination of the two ratios is rather *strictly convex* between two peaks, ω_i and ω_j. This particular case even exemplifies that the optimal $\widehat{\omega}$ is not a function, but a correspondence having multiple solutions, not differentiable, at least one x_i value. Yet, the minimal condition to solve the minimax problem with the differentiability of $\widehat{\omega}$ with respect to x_i is equivalent to the concavity on those *between the two peaks.*

Lemma 3 *Suppose that $\widehat{U}(\widehat{\omega}, q_i)$ and $\widehat{U}(\widehat{\omega}, q_j)$ have single peaks with $\omega_i < \omega_j$ and that there is no dominant interval. Then, $\omega^*_{ij}(x_i)$ is an implicit function of x_i on $[0, 1]$ if and only if the convex combination of the two ratios is strictly concave on $[\omega_i, \omega_j]$ for $\omega^*_{ij}(x_i)$ satisfying the first-order condition.*

Proof. For the proof, to simplify notations, denote the two ratios by

$$\mathcal{R}_i(\widehat{\omega}) \equiv \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)} \text{ and } \mathcal{R}_j(\widehat{\omega}) \equiv \frac{\widehat{U}(\widehat{\omega}, q_j)}{U(q_j)}.$$

First, show ($\Leftarrow$). Suppose that for each $x_i \in [0, 1]$, $x_i\mathcal{R}_i(\widehat{\omega}) + (1-x_i)\mathcal{R}_j(\widehat{\omega})$ is strictly concave on $[\omega_i, \omega_j]$ for $\omega^*_{ij}(x_i)$ satisfying the first-order condition. Then, such $\widehat{\omega}$ satisfies the second-order sufficiency. Furthermore, since for each $x_i \in [0, 1]$, $x_i\mathcal{R}''_i(\widehat{\omega}) + (1-x_i)\mathcal{R}''_j(\widehat{\omega}) < 0$ for all $\widehat{\omega} \in [\omega_i, \omega_j]$, there exists a unique differentiable implicit function $\omega^*_{ij}(x_i)$ of x_i on $[0, 1]$. Show ($\Rightarrow$). Suppose, on the contrary, that for some x_i, there exists one point $\omega' \in [\omega_i, \omega_j]$ satisfying the first-order condition $x_i\mathcal{R}'_i(\omega')+(1-x_i)\mathcal{R}'_j(\omega') = 0$ such that $x_i\mathcal{R}''_i(\omega')+(1-x_i)\mathcal{R}''_j(\omega') \geq 0$. If $x_i\mathcal{R}''_i(\omega') + (1 - x_i)\mathcal{R}''_j(\omega') > 0$, then ω' is not a solution. It remains to show it for $x_i\mathcal{R}''_i(\omega') + (1 - x_i)\mathcal{R}''_j(\omega') = 0$. The first-order condition for $\widehat{\omega}$ can be rewritten in the following way for x_i such that

$$x_i = \Phi_{ij}(\widehat{\omega}),$$

where denote $\Phi_{ij}(\widehat{\omega}) \equiv -\frac{\mathcal{R}_j'(\widehat{\omega})}{\mathcal{R}_i'(\widehat{\omega})-\mathcal{R}_j'(\widehat{\omega})}$. Hence, an existence of an implicit function is equivalent to an existence of an inverse function, which requires $\Phi_{ij}'(\widehat{\omega}) \neq 0$. Note that

$$\Phi_{ij}'(\widehat{\omega}) = -\frac{\mathcal{R}_i'(\widehat{\omega})\mathcal{R}_j''(\widehat{\omega}) - \mathcal{R}_j'(\widehat{\omega})\mathcal{R}_i''(\widehat{\omega})}{[\mathcal{R}_i'(\widehat{\omega}) - \mathcal{R}_j'(\widehat{\omega})]^2}.$$

By substituting the first-order condition into $x_i\mathcal{R}_i''(\omega') + (1 - x_i)\mathcal{R}_j''(\omega') = 0$, we have $\mathcal{R}_i'(\omega')\mathcal{R}_j''(\omega') - \mathcal{R}_j'(\omega')\mathcal{R}_i''(\omega') = 0$ at ω', which yields a contradiction. ■

The sufficiency for the implicit function is immediate; its necessity requires a formal proof, and more importantly, the consequence of Lemma 3 is that if $\omega_{ij}^*(x_i)$ is an implicit function of x_i on $[0, 1]$, then, a solution to the minimax problem in (49) with only two q_i and q_j must be an *intersection* of the two ratios.[37] The intersection is denoted by ω_{ij} such that

$$\frac{\widehat{U}(\omega_{ij}, q_i)}{U(q_i)} = \frac{\widehat{U}(\omega_{ij}, q_j)}{U(q_j)}.$$

In other words, Lemma 3 enables us to obtain the natural outcome for the manager's optimal misreport: The solution to (49) arises at the intersection; for the optimal x_i, $\omega_{ij}^*(x_i) = \omega_{ij}$. There might be no such intersection between two peaks, but if there is, it must be unique given the single-peak property: for any i, j, $\frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)} - \frac{\widehat{U}(\widehat{\omega},q_j)}{U(q_j)}$ is strictly decreasing on (ω_i, ω_j).

Proposition 5 shows that if $\omega_{kl}^*(x_k)$ is an implicit function where targets q_k and q_l generate the *minimum* intersection, then the optimal number of intervals is just two; the two intervals solve the minimax problem in (37) with $\overline{N}$ potential intervals. Thus, the principal can use only two targets to solve the minimax, which enables the principal to *economize* a number of targets. Furthermore, the result minimizes the scope of the regularity for $\widehat{U}(\widehat{\omega}, q_i)$: the strict concavity of the convex combination with the two ratios (between two peaks).

Proposition 5 *Suppose that each $\widehat{U}(\widehat{\omega}, q_i)$ has a single peak and that there is no dominant interval. Then, if for q_k, q_l solving $\min_{q_i,q_j} \frac{\widehat{U}(\omega_{ij},q_i)}{U(q_i)}$, there exists an implicit function $\omega_{kl}^*(x_k)$ of x_k on $[0, 1]$, then two intervals given q_k and q_l solve the minimax problem.*

Proof. Suppose, on the contrary, there is $x' \in \Delta$ such that

$$\max_{\widehat{\omega}\in\Omega} \sum_{i=0}^{\overline{N}-1} x_i' \frac{\widehat{U}(\widehat{\omega}, q_i)}{U(q_i)} < \frac{\widehat{U}(\omega_{kl}, q_k)}{U(q_k)}.$$

Since given x', the maximization yields $\max_{\widehat{\omega}\in\Omega} \sum_{i=0}^{\overline{N}-1} x_i' \frac{\widehat{U}(\widehat{\omega},q_i)}{U(q_i)} \geq \sum_{i=0}^{\overline{N}-1} x_i' \frac{\widehat{U}(\omega_{kl},q_i)}{U(q_i)}$, we have

$$\frac{\widehat{U}(\omega_{kl}, q_k)}{U(q_k)} > \sum_{i=0}^{\overline{N}-1} x_i' \frac{\widehat{U}(\omega_{kl}, q_i)}{U(q_i)} \geq \min\left\{\frac{\widehat{U}(\omega_{kl}, q_0)}{U(q_0)}, ..., \frac{\widehat{U}(\omega_{kl}, q_{\overline{N}-1})}{U(q_{\overline{N}-1})}\right\}.$$

[37] That is, the sufficiency for the implicit function follows from the implicit function theorem, and then, the intersection from the envelope theorem.

Let the ratio from q_j attain the min level, that is, $\frac{\widehat{U}(\omega_{kl},q_j)}{U(q_j)} = \min\left\{\frac{\widehat{U}(\omega_{kl},q_0)}{U(q_0)}, ..., \frac{\widehat{U}(\omega_{kl},q_{\overline{N}-1})}{U(q_{\overline{N}-1})}\right\}$. Then, from the above, for q_j, we have

$$\frac{\widehat{U}(\omega_{kl}, q_j)}{U(q_j)} < \frac{\widehat{U}(\omega_{kl}, q_k)}{U(q_k)}. \tag{51}$$

Note that the min function of $\widehat{\omega}$ $\min\left\{\frac{\widehat{U}(\widehat{\omega},q_k)}{U(q_k)}, \frac{\widehat{U}(\widehat{\omega},q_l)}{U(q_l)}\right\}$ is related to the intersection of two areas under $\frac{\widehat{U}(\widehat{\omega},q_k)}{U(q_k)}$ and $\frac{\widehat{U}(\widehat{\omega},q_l)}{U(q_l)}$ in Figure 4. With the min function, the proof is divided into two cases.

Case 1. The single peak from q_j, ω_j, is above the min function from q_k and q_l such that

$$\frac{\widehat{U}(\omega_j, q_j)}{U(q_j)} > \min\left\{\frac{\widehat{U}(\omega_j, q_k)}{U(q_k)}, \frac{\widehat{U}(\omega_j, q_l)}{U(q_l)}\right\}.$$

Consider two cases, $\omega_j > \omega_{kl}$ or $\omega_j < \omega_{kl}$ ($\omega_j \neq \omega_{kl}$ for $\widehat{U}(\widehat{\omega}, q_j)$ to be a function). Suppose $\omega_j > \omega_{kl}$. WLOG, let $\omega_k < \omega_l$. Then, by the above and (51),

$$\frac{\widehat{U}(\omega_{kl}, q_k)}{U(q_k)} - \frac{\widehat{U}(\omega_{kl}, q_j)}{U(q_j)} > 0 \text{ and } \frac{\widehat{U}(\omega_j, q_k)}{U(q_k)} - \frac{\widehat{U}(\omega_j, q_j)}{U(q_j)} < 0,$$

which, by the intermediate value theorem, implies that there is an intersection $\omega_{kj} \in (\omega_k, \omega_j)$ between two ratios $\frac{\widehat{U}(\widehat{\omega},q_k)}{U(q_k)}$ and $\frac{\widehat{U}(\widehat{\omega},q_j)}{U(q_j)}$. By the single-peak property, $\frac{\widehat{U}(\omega_{kj},q_k)}{U(q_k)} < \frac{\widehat{U}(\omega_{kl},q_j)}{U(q_j)}$. This contradicts that ω_{kl} generates the lowest intersection. The same argument applies to the case $\omega_j < \omega_{kl}$.

Case 2. The single peak from q_j, ω_j, is below the min function from q_k and q_l such that

$$\frac{\widehat{U}(\omega_j, q_j)}{U(q_j)} \leq \min\left\{\frac{\widehat{U}(\omega_j, q_k)}{U(q_k)}, \frac{\widehat{U}(\omega_j, q_l)}{U(q_l)}\right\}.$$

We show that, in this case, for all q_i, $\frac{\widehat{U}(\omega_j,q_i)}{U(q_i)} \geq \frac{\widehat{U}(\omega_j,q_j)}{U(q_j)}$. Suppose not. That is, there is one q_h such that for ω_j, $\frac{\widehat{U}(\omega_j,q_h)}{U(q_h)} < \frac{\widehat{U}(\omega_j,q_j)}{U(q_j)}$. If there is no intersection not to have a contradiction as in Case 1, it must be that for ω_h, $\frac{\widehat{U}(\omega_h,q_h)}{U(q_h)} \leq \frac{\widehat{U}(\omega_h,q_j)}{U(q_j)}$. Then, we show that for all q_i, $\frac{\widehat{U}(\omega_h,q_i)}{U(q_i)} \geq \frac{\widehat{U}(\omega_h,q_h)}{U(q_h)}$. The same argument keeps applying until we reach the minimum single peak q_m such that

$$\frac{\widehat{U}(\omega_i, q_i)}{U(q_i)} \geq \frac{\widehat{U}(\omega_m, q_m)}{U(q_m)} \text{ for all } i \in \{0, ..., \overline{N} - 1\}.$$

For the minimum single peak, there is no way to avoid an intersection except that for all q_i, $\frac{\widehat{U}(\omega_m,q_i)}{U(q_i)} \geq \frac{\widehat{U}(\omega_m,q_m)}{U(q_m)}$. But, in this case, the interval given q_m is dominant, a contradiction. ■

The idea for the two-interval solution is in fact simple. If there exists an implicit function $\omega^*_{kl}(x_k)$ of x_k on $[0, 1]$, then for the two intervals, the manager chooses a unique intersection between the two ratios as the optimal report if he does not acquire the information. If the intersection is indeed the lowest intersection among all others, it solves the minimax problem; otherwise, the minimax point of a combination of some number – not necessarily two – of multiple ratios is *lower* than the minimax point of the two ratios, and thus, at least one single-peak function from the former must cut through one of the two ratios from the latter, contradicting that the latter yields the lowest minimax for any given two ratios.

To appreciate Proposition 5, we revisit Example 2, in which case in fact the sufficiency for the implicit function is satisfied for a uniform G; that is, *each* $\widehat{U}(\widehat{\omega}, q_i)$ is strictly concave on Ω.[38] This results in the following Corollary.

Corollary 1 *Consider Example 2 and a uniform G. Then, for any finite feasible intervals, the number of intervals to solve the optimal interval structure is at most two.*

[38]The strict concavity can be found from the closed-form $\widehat{U}$, which is derived as, for a uniform G with $[\underline{\omega}, \overline{\omega}]$,

$$\widehat{U}(\widehat{\omega}, q_i) = e^{(\overline{\omega}-\widehat{\omega})} \left(\frac{q_i - 1}{q_i + 1 + \widehat{\omega}}\right)^{(q_i+1)} \left(\frac{\overline{\omega}+1}{\widehat{\omega}+1}\right) \frac{1}{\ln\left(\frac{\widehat{\omega}[\widehat{\omega}-(q_i+1)]}{(\widehat{\omega}+1)(q_i+1)}\right)} \left[\left(\frac{\widehat{\omega}[\widehat{\omega}-(q_i+1)]}{(\widehat{\omega}+1)(q_i+1)}\right)^{\overline{\omega}} - \left(\frac{\widehat{\omega}[\widehat{\omega}-(q_i+1)]}{(\widehat{\omega}+1)(q_i+1)}\right)^{\underline{\omega}}\right].$$