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Optimal Design for an Informed Auctioneer

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Abstract

Each seller's quality, not associated with a contractible action, making a contract incomplete, a principal seeks a better-informed auctioneer to tackle the problem. To induce the auctioneer's truthful quality reports for multiple sellers, the principal constructs the *first-order absolute alignment* that is aligned with a benchmark's absolute criterion. The optimal compensation for the auctioneer highlights a new role for reserve prices as a *revelation device*. In addition, we design a practical auction format to implement the optimal mechanism. Finally, this mechanism is applied to asymmetric seller auctions to relax the long-lasting assumption: common knowledge of asymmetric distributions.

Keywords and Phrases: First-order absolute alignment, Auction, Revelation device

JEL Classification Numbers: D44, D82

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1 Introduction

A considerable number of employees buy things for organizations. In the U.S. alone, in 2014, a total of 443,200 agents were classified as procurement officers or purchasing managers in both public and private sectors.[1] The number manifesting the relevance of the job, its role, according to the Bureau of Labor Statistics, is to evaluate suppliers and their product quality.

Yet, without such a "third-party," a principal implements an allocation in the standard mechanism (e.g., Myerson (1982)). The existing procurement literature, emphasizing the quality of a product in addition to its price, follows the tradition by which quality level can be contractible via a mechanism; that is, in an auction, quality is a *bid*. In practice, a main competitive contracting method of the Federal Acquisition Regulation (FAR) of the U.S. selects suppliers based on both suppliers' prices and their proposals.[2] If a proposal is a bid, then the job does not require a depth of expertise that makes it distinct from the principal's, and thus, in a model, it can be subsumed by him, as a simplification of efficient division of labour in reality.

However, given that firm's product quality levels are hard to change shortly,[3] the view that a proposal is to bid may not be fully satisfactory; often, it is meant to certify a supplier's current performance capabilities, present product features, and other aspects of its existing products. Alternatively, we propose that quality is instead a firm's characteristic to be *verified*, to rationalize the third party's role as information acquisition and report.

Despite the increased use of reverse auctions – e.g. in the U.S., from 2008 to 2012, the tripled auctions from 7,193 to 19,688 expending about 828 million in fiscal year 2012[4]

[1]See Bureau of Labor Statistics, U.S. Department of Labor, Occupational Outlook Handbook, 2016-17 Edition, Buyers and Purchasing Agents. Also find 35,707 contracting officers government wide in fiscal year 2010, e.g., in Warren (2014).

[2]The proposals contain potential suppliers' claims and promises to meet contracting officers' requirements. Specifically, the FAR describes two main competitive contracting methods: sealed bidding and negotiation (FAR Subchapter C). In the former, only price and price-related factors are to be considered in the selection process (FAR Subpart 14.1.), whereas in the latter, factors other than price are to be considered as well. Despite the presumably misleading name "negotiation," competition is at the heart of the procedure (FAR 15.002 Types of negotiated acquisition. (b) Competitive acquisitions).

[3]For example, it is well known that different companies occupy different segments of the market in terms of their quality levels (e.g., the auto industry and the mobile phone industry). Of course, this paper studies the case in which each firm's "exact quality" is known only to the specialist, the informed auctioneer.

[4]See GAO 2013 report, "Reverse auctions, Guidance Is Needed to Maximize Competition and Achieve Cost Savings," December 2013, US Government Accountability Office, Report to Congressional Requesters.

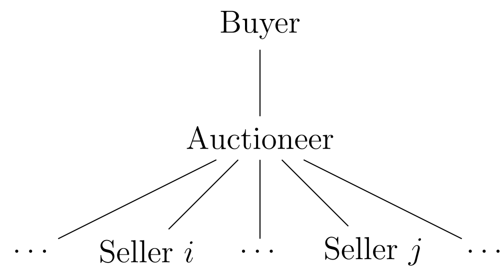

Figure 1: Organizational structure

– evidently, the current practice is far from optimality by any measure; that is, this is a subject to be addressed, as a burgeoning area.[5] It is clear that this paper aims not to justify the current, seemingly failing, system, but to contribute to a theoretical optimal mechanism that could have implications on its re-design for a future system.

The importance of this setting can be appreciated for its wide range of applications, apart from a direct one, procurement. With different levels of quality, capability, or human capital, agents compete: in a contest, players compete to win a prize (e.g., a research grant), in an industry, upstream firms compete to supply to a downstream firm, and in a labor market, job candidates compete to be hired by an organization. All these structures embody an expert, called an auctioneer in this model, who must do the verifying work, as described in figure 1. In the model, a buyer intends to purchase an indivisible good from a potential seller. Each seller has a type with two dimensions, production cost and product quality, that are correlated. Product quality is each seller's endowed capability that affects the buyer's payoff directly, but the quality is not observable by the buyer. The quality, not associated with a contractible action, results in an incomplete contract: without such an expert, there are no different contracts for different levels of quality.

We address two questions with the model. First, this paper finds a mechanism *contracting simultaneously* with an auctioneer and multiple sellers that attains the second best. The second best payoff is the buyer's maximum payoff from the hypothetical benchmark that the buyer can observe the quality. Second, this paper constructs a practical auction format that

[5] Notably, vendors were concerned that the reverse auction system, by only identifying the lowest bidder, may fail to take into account quality dimension of a product, but, in fact, about a quarter of auctions in 2012 were not awarded to the lowest bidder, based on discretion by contracting officers behind their main auction contractor, Fedbid (see p. 28. of GAO report 2013), and, in its conclusion, "We found confusion about who is making final award determinations and the basis for those determinations." (p. 29. of GAO report 2013)

can implement the optimal mechanism. In doing so, we examine both cases – continuous and discrete quality levels – since, in the real world, quality levels are often classified according to a discrete measure; and the two require different approaches to endogenously generate the auctioneer's truth-telling, without a condition like the single-crossing.[6]

The information structure by Yoo (2016), examining a single-agent case with continuous output, shows that a mechanism, based on the *first-order (condition) alignment* with a manager, dominates selling, while it provides an incentive for information acquisition through uniqueness.[7] The aforementioned paper studies the role in a contractual situation, with a single agent, whereas this paper studies it in a mechanism design situation, with multiple sellers. As such, our focus is to achieve the level of generality that is applicable to other mechanism design problems, understanding that the generalization is not mechanical but necessitates a new idea.

An indivisible good, a natural circumstance in an auction, enables us to divide the benchmark optimality into two parts: an *absolute* criterion and a *relative* criterion. The former is about whether a bidder satisfies a standard, whereas the latter is about how a bidder can be compared with the other bidders. The optimal contract of this paper, unlike the Vickrey-Clarke-Groves (VCG) mechanism's alignment, aligns the auctioneer's truth-telling with the benchmark's absolute criterion through the first-order condition; the novelty of this approach is that it entertains the idea of using the *alignment*.[8] In this way, the auctioneer's compensation for reporting one seller's quality can be separated from that for another, with which we can solve the problem of finding an ordinary differential equation, not a system of partial differential equations. This separable alignment with the absolute criterion is not feasible with a continuous case. The alignment coupled with the reverse hazard rate dominance results in the strict Bayesian incentive compatibility for the auctioneer, and a uniqueness

[6]The non-differentiability demands a different approach, especially non-trivial for an arbitrary finite number of quality levels. For the global optimality, the conditional probability distribution of cost given quality in the former requires the reverse hazard rate dominance, whereas the conditional probability distribution in the latter requires the (local) cumulative sum ratio dominance.

[7]Note that a complete information approach to both the manager and the agent observing the quality fails in such an incentive due to multiple equilibria. It applies to two well-known mechanisms: a constant payment and punishing if two reports differ. In addition, selling may not attain the benchmark due to the limited liability. With the same arguments, those conventional mechanisms fail to achieve both the second best and an information incentive in this model with multiple sellers. See Yoo (2016) for the detailed explanations.

[8]In an auction, this means that he obtains the portion only when the seller participates. In a buyer or a procurement auction, the reserve price for seller i is given as the "maximum bid" for the seller.

with each seller's weakly dominant strategy.[9] Overall, with the mechanism, the buyer can extract all surplus from the auctioneer while providing information incentive and the limited liability.

We make additional innovations with the proposed *first-order absolute alignment*. The mechanism can dispense with an interior condition for the auctioneer's strict Bayesian incentive compatibility and encompass a discrete quality case, which can include various real-world applications. Furthermore, in acknowledging the practical aspect of auctions, it is shown that the optimal mechanism with the auctioneer can be implemented in a feasible auction format, called a *quality-adjusted second price auction*, with a profile of reserve prices.[10] The buyer constrains a type of auction format that the auctioneer can choose, and "ties" it to his reserve prices announcement through a one-to-one mapping between a reserve price and a quality weight. Hence, the implementation is related to the delegation of Holmström (1977, 1984), but this model's delegation is not given as an interval delegation. The auction format turns out to be simple, but, initially, it faces two challenges: in the competition between sellers, each seller has a two-dimensional type,[11] and the auctioneer has an incentive to *manipulate* the auction by over-reporting quality weights of sellers. The dominant strategy equilibrium for the competition between the sellers resolves the two interwoven problems.[12]

To show the power of the first-order absolute alignment, we apply this mechanism to asymmetric seller auctions, a mirror image of a procurement auction. The topic of asymmetric auction has generated a voluminous stream of literature: for the equilibrium of asymmetric first price auctions (Lebrun (1999) and Maskin and Riley (2000b) among others), and for ranking asymmetric auctions, especially between the first price auction (FPA) and the

[9]In this paper, the role of the strict Bayesian incentive compatibility is to provide the incentive to acquire costly information. In the literature, the *strict* incentive compatibility has received less attention, with a few exceptions. For example, see Aoyagi (1998), who uses it for an alternative condition under which a Bayesian incentive compatible mechanism exists with budget balance.

[10]Theoretically, McAfee and McMillan (1989) consider discriminating reserve prices, and, empirically, incorporating asymmetric reserve prices is a small but growing area; see, e.g., Flambard and Perrigne (2006).

[11]A multi-dimensional type space, in general, causes an existence-of-equilibrium problem.

[12]The manipulation is not an issue in the simultaneous contracting mechanism with the auctioneer, in which the auctioneer and the sellers report simultaneously; but, to implement the actual auction, the quality weights must be announced prior to the competition between the sellers. The auction rule is designed so that bidding each seller's production cost is weakly dominant, regardless of the auctioneer's quality-weights announcement and other sellers' bids. The second price auction's dominant equilibrium is well known, but no paper has related it to its powerful role in the organizational structure with an informed third party.

second price auction (SPA) (e.g. Maskin and Riley (2000a) and Kirkegaard (2012)).

However, implicit is the assumption that bidders' asymmetric distributions are *common knowledge*, which involves two "layers": a seller knows who has which distribution, and each buyer knows the others'. The literature has been mute on the assumption, given pre-existing challenges even with the restriction.[13] Yet, given that much effort has been put forth, e.g., by econometricians, starting from Laffont, Ossard and Vuong (1995), to identify and estimate such distributions, it is not realistic to assume that this knowledge comes for free, without any acquisition costs. Having each buyer's type distribution in the field translated into a seller's quality in our model, we show that the results in that literature can in fact dispense with the common knowledge assumption by applying the first-order absolute alignment for the expert's truth-telling. The reinterpretation of the optimal auction design by Bulow and Roberts (1989), unlike the asymmetric auctions, requires only the first layer of common knowledge: in the multidimensional market setting, there is no role of consumers' beliefs in one market about the other markets. Still, it is assumed that the monopoly *knows* what the different demand schedules of different markets are. In our model, this assumption can be replaced by the monopoly hiring an expert who can estimate such demand functions and making him report them truthfully.

The questions that this paper asks differ from mechanism design with contractible quality (see Laffont and Tirole (1987), McAfee and McMillan (1987) and Riordan and Sappington (1987)). In that literature, quality can be contractible, so firms bid both price and quality (see e.g., Che (1993), Branco (1997) and Asker and Cantillon (2008)).[14]

This paper is also related to a large literature on the theory of the firm. The organization of Alchian and Demsetz (1972) embodies a monitor observing free-riding in team production, and that of Tirole (1986) a supervisor monitoring an agent's action. We provide an answer to the fundamental question, why those structures should have a monitor in the first place, based on the simultaneous mechanism's dominance over selling with an information acquisition incentive. The sellers' information structure with non-contractible information makes a contract incomplete, which makes its foundation differ from papers based on transaction

[13]Moreover, no FPA equilibrium is known even when we assume just the first layer: a seller knows who has which distribution, but each bidder knows only a distribution of type distributions of the others, not the other bidders' type distributions.

[14]In particular, Asker and Cantillon (2008) was the first to study multidimensional types in procurement auctions, but with multidimensional cost types. In this model, each seller has a two-dimensional type, quality and cost, with quality being not contractible.

costs (Williamson 1975, 1985).[15] Hence, we find its solution by designing the first-order absolute alignment with the better-informed auctioneer, not by allocating property rights (see, e.g., Grossman and Hart (1986) and Hart and Moore (1990)).

McAfee and Reny (1992)'s rent extraction result with continuous types makes the powerful results of Crémer and McLean (1988) encompass important applications with such environments.[16] The main result, however, considers the rent retraction only given that the VCG mechanism provides the incentive compatibility for the agents. The generalization is not applicable to the auctioneer in this paper, since we must provide the strict incentive compatibility to him – only the VCG mechanism for the auctioneer is a constant compensation. The first-order absolute alignment achieves the goal by making it possible for its aligned optimality with the reverse hazard rate dominance to provide the incentive compatibility, without the single-crossing condition in his payoff.[17]

The model is in Section 2, and the benchmark is in Section 3. The optimal direct mechanism with the auctioneer is analyzed in Section 4. The case with discrete quality levels is in Section 5, and the implementation with an auction rule is found in Section 6. The model's extension and applications are discussed in Section 7 and 8, respectively. The concluding remarks are in Section 9, and all proofs are collected in an appendix.

2 Model

A buyer purchases an indivisible good from $N \geq 2$ potential sellers where $I \equiv \{1, ..., N\}$. Seller i produces the good with quality ω_i at cost θ_i, and the buyer obtains a value $v(\omega_i)$ from it. The quality, as an endowed characteristic, is an additional dimension of seller i's type, but the non-contractible ω_i, unlike θ_i, affects the buyer, not seller i. This single aspect of the private information is observable by the auctioneer with his expertise.

[15] See also critics on the lack of foundations of incompleteness (Maskin and Tirole (1999) and Tirole (1999)) and its alternative approach (e.g. Segal (1999)).

[16] Specifically, Theorem 1 of Crémer and McLean (1988), given a VCG mechanism environment, only provides the rent extraction, and it is Theorem 2 that achieves both incentive compatibility and the rent extraction based on the Farkas' Lemma. In addition, clearly, any type of contract based on correlation is not applicable to sellers in this paper because they observe both dimensions.

[17] See supplementary results of McAfee and Reny (1992) attaining the incentive compatibility as well as the rent extraction require additional structures on type distributions. In addition, the first-order absolute alignment satisfies the limited liability. Even for the discrete case, the rent extraction does not provide the limited liability, e.g., see Robert (1991).

$$\ldots,\ \underbrace{\underbrace{\overbrace{(\omega_i,\theta_i)}^{\text{Correlated}}}\ ,\ldots,\ \underbrace{\overbrace{(\omega_j,\theta_j)}^{\text{Correlated}}}}_{\text{Independent}},\ldots$$

Figure 2: Correlation and Independence

Seller i's type (θ_i,ω_i) is drawn from a non-empty subset $[\underline{\theta},\overline{\theta}]\times[\underline{\omega},\overline{\omega}]$ of $\mathbb{R}^2$, according to a differentiable cumulative distribution function $G_i(\theta_i,\omega_i)$ with its density function $g_i(\theta_i,\omega_i)>0$ for all $(\theta_i,\omega_i)\in[\underline{\theta},\overline{\theta}]\times[\underline{\omega},\overline{\omega}]$. Its marginal cumulative functions are denoted by $G_{\theta_i}(\theta_i)$ and $G_{\omega_i}(\omega_i)$, and the conditional cumulative distribution function of θ_i given ω_i is $F_i(\theta_i|\omega_i)$ with its conditional density function $f_i(\theta_i|\omega_i)$. With (θ_i,ω_i) and (θ_j,ω_j) for $i\neq j$ being independently drawn and a vector of production costs and a vector of quality levels, respectively, denoted by $\theta=(\theta_1,...,\theta_N)$ and $\omega=(\omega_1,...,\omega_N)$, (θ,ω) is drawn from a cumulative distribution function G, the product of distributions G_i for all $i\in I$, with its support $\Theta\times\Omega$, where $\Theta\equiv\left[\underline{\theta},\overline{\theta}\right]^N$ and $\Omega\equiv[\underline{\omega},\overline{\omega}]^N$. This difference in their information structures (see figure 2) and the type distributions are common knowledge to them.

We assume that for each $i\in I$, for any pair $\omega_i,\omega_i'\in[\underline{\omega},\overline{\omega}]$ with $\omega_i'>\omega_i$, $F_i(\theta_i|\omega_i')$ dominates $F_i(\theta_i|\omega_i)$ in terms of the reverse hazard rate.[18] That is, for any pair $\omega_i'>\omega_i$, $\frac{f_i(\theta_i|\omega_i')}{F_i(\theta_i|\omega_i')}>\frac{f_i(\theta_i|\omega_i)}{F_i(\theta_i|\omega_i)}$ for all $\theta_i\in(\underline{\theta},\overline{\theta})$. The reverse hazard rate dominance implies that a higher quality's cost distribution $F_i(\theta_i|\omega_i')$ first-order stochastic dominates $F_i(\theta_i|\omega_i)$; a higher quality is more likely to incur higher costs. We also study a *reversed* reverse-hazard-rate case in which a higher quality is more likely to incur lower costs, in the extension, Section 7. Each seller i's "virtual cost" is defined as

$$\phi_i(\theta_i,\omega_i)\equiv\theta_i+\frac{F_i(\theta_i|\omega_i)}{f_i(\theta_i|\omega_i)}. \tag{1}$$

For each $i\in I$, assume the standard, "regular" distribution such that ϕ_i is a strictly increasing function of θ_i. From the reverse hazard rate dominance, the virtual cost is a strictly decreasing function of ω_i.

The buyer's expected payoff is $\sum_{i\in I}[q_i v(\omega_i)-t_i]$, with a strictly increasing v, if he purchases the good from seller i with probability q_i and makes a monetary transfer t_i to seller i. Seller i's payoff is $t_i-\theta_i$ if he supplies the good to the buyer. We let the auctioneer's reservation payoff be $\overline{U}$, assuming that it is greater than his limited liability B, and each seller's

[18]This condition implies the first-order stochastic dominance and is widely used in auction theory. See, e.g., Maskin and Riley (2000a), Krishna (2002) and Kirkegaard (2012).

reservation payoff is normalized to zero. The auctioneer incurs an information acquisition cost $c > 0$, and can observe ω only after being hired by the buyer.[19]

3 Benchmark: second best

Suppose that the buyer, like the auctioneer, can observe each seller's quality, with the same outside option $\overline{U}$. Then, a direct mechanism consists of measurable functions q and t_i for all $i \in I$ such that

$$q(\cdot, \omega) : \Theta \to \Delta \text{ and } t_i(\cdot, \omega) : \Theta \to \mathbb{R}, \tag{2}$$

where $\Delta \equiv \{(q_1, ..., q_N) : q_i \geq 0 \text{ and } \sum_{i \in I} q_i \leq 1\}$. We study this observable ω case, not for its analysis *per se*, a simply modified Myerson (1981), but for its payoff, serving as the second-best to aim, and its optimality, serving as a condition to align, in the next section with no such buyer's observability. If seller i reports θ_i for all $i \in I$, the buyer commits to paying $t_i(\theta, \omega)$ to seller i, and purchases the good from seller i with probability $q_i(\theta, \omega)$. A type profile of all sellers but i, $(\theta_{-i}, \omega_{-i})$, is drawn from a cumulative distribution function G_{-i} with its support $\Theta_{-i} \times \Omega_{-i} \equiv [\underline{\theta}, \overline{\theta}]^{N-1} \times [\underline{\omega}, \overline{\omega}]^{N-1}$. For seller i's type (θ_i, ω_i), the expected probability that the buyer purchases the good from seller i, $Q_i : [\underline{\theta}, \overline{\theta}] \times [\underline{\omega}, \overline{\omega}] \to [0, 1]$, and the expected transfer that the buyer makes to seller i, $T_i : [\underline{\theta}, \overline{\theta}] \times [\underline{\omega}, \overline{\omega}] \to \mathbb{R}$, are respectively defined as $Q_i(\theta_i, \omega_i) = \int_{\Theta_{-i} \times \Omega_{-i}} q_i(\theta_i, \omega_i, \theta_{-i}, \omega_{-i}) dG_{-i}(\theta_{-i}, \omega_{-i})$, and $T_i(\theta_i, \omega_i) = \int_{\Theta_{-i} \times \Omega_{-i}} t_i(\theta_i, \omega_i, \theta_{-i}, \omega_{-i}) dG_{-i}(\theta_{-i}, \omega_{-i})$, from which seller i's interim expected payoff is $T_i(\theta_i, \omega_i) - \theta_i Q_i(\theta_i, \omega_i)$. Unlike the benchmark, in the following main analysis, ω_i is what either seller i or the auctioneer reports to the buyer, not what the buyer can observe.

A direct mechanism is incentive compatible and individually rational if and only if for each $i \in I$, and every $\omega_i \in [\underline{\omega}, \overline{\omega}]$,

$$T_i(\theta_i, \omega_i) - \theta_i Q_i(\theta_i, \omega_i) \geq T_i(\theta_i', \omega_i) - \theta_i Q_i(\theta_i', \omega_i) \text{ for all } \theta_i, \theta_i' \in [\underline{\theta}, \overline{\theta}], \tag{3}$$

$$T_i(\theta_i, \omega_i) - \theta_i Q_i(\theta_i, \omega_i) \geq 0 \text{ for all } \theta_i \in [\underline{\theta}, \overline{\theta}], \tag{4}$$

and from a typical analysis, an incentive compatible and individually rational mechanism yields the buyer's expected payoff such that $\sum_{i \in I} \int_{\Theta \times \Omega} q_i(\theta, \omega) [v(\omega_i) - \phi_i(\theta_i, \omega_i)] \, dG(\theta, \omega)$. The buyer's "virtual payoff" from seller i is defined as

$$\pi_i(\theta_i, \omega_i) \equiv v(\omega_i) - \phi_i(\theta_i, \omega_i). \tag{5}$$

[19]With this natural institutional setting, we consider an ex-ante individual rationality, not an interim one, for the auctioneer.

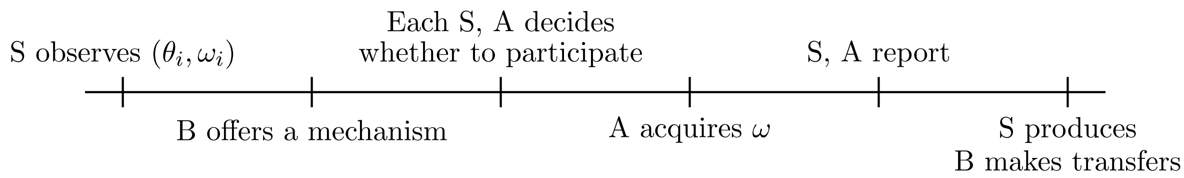

Figure 3: Timeline

For each (θ, ω), the buyer chooses probability $q_i(\theta, \omega)$ for all $i \in I$ to maximize the payoff.

Proposition 1 *The optimal allocation rule is given as*

$$q_i(\theta, \omega) = \begin{cases} 1 & \text{if } \pi_i(\theta_i, \omega_i) > 0, \text{ and } \pi_i(\theta_i, \omega_i) > \pi_j(\theta_j, \omega_j) \; \forall j \in I, j \neq i; \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The optimal mechanism consists of two parts: $\pi_i(\theta_i, \omega_i) > 0$ and $\pi_i(\theta_i, \omega_i) > \pi_j(\theta_j, \omega_j)$ $\forall j \in I, j \neq i$; the winning is based on both the former, an absolute criterion, and the latter, a relative criterion. Suppose an interior absolute criterion, denoted by $p_i(\omega_i)$, satisfying $\pi_i(p_i(\omega_i), \omega_i) = 0$. With the assumptions on $v(\omega_i)$ and $\phi_i(\theta_i, \omega_i)$, including the reverse hazard rate dominance, p_i is strictly increasing. The intuition is straightforward: a bidder with a higher quality is *favorably* treated. We employ the absolute condition to induce the auctioneer's truth-telling in the main analysis.

4 Optimal design for auctioneer

The buyer designs a mechanism that contracts simultaneously with the auctioneer and the sellers. The game's timeline after Nature chooses (θ, ω) is described in figure 3.

Both seller i and the auctioneer observe ω_i, but neither "direct" payoff relies on it. Quality is a non-contractible type for both parties, for different reasons: it is the seller's endowed characteristic and it is not the auctioneer's own type. Such non-contractibility results in conventional mechanisms failing to provide an incentive to acquire costly information.[20] In particular, any mechanism that does not exploit the correlation between cost and quality results in multiple equilibria, and any that exploits the correlation must do so by eliciting quality only from the auctioneer, not from sellers, in order to prevent the multiplicity, as observed by Yoo (2016).

[20]For example, paying a constant wage to the auctioneer or punishing them if their reports differ.

Then, a direct Bayesian mechanism consists of measurable functions, q, t_i for all $i \in I$ and the auctioneer's compensation S, where

$$q : \Theta \times \Omega \to \Delta,\ t_i : \Theta \times \Omega \to \mathbb{R} \text{ and } S : \Omega \times \Theta \to \mathbb{R}_+. \tag{7}$$

To tackle the multiplicity problem, we consider a mechanism with the auctioneer's strict incentive compatibility: for any quality profile $\omega = (\omega_1, ..., \omega_N)$, reporting truthfully is a unique solution. The auctioneer's contract is said to be strictly incentive compatible if

$$\forall \omega \neq \omega' \in \Omega,\ \mathbb{E}_\theta[S(\omega, \theta)|\omega] > \mathbb{E}_\theta[S(\omega', \theta)|\omega]. \tag{8}$$

The quality ω affects neither payoff, but it does affect the auctioneer's expected payoff through the conditional distribution of θ given ω. In addition, the auctioneer's contract is individual rational if

$$\int_\Omega \mathbb{E}_\theta[S(\omega, \theta)|\omega] dG_\omega(\omega) - c \geq \overline{U}, \tag{9}$$

where G_ω is the product of distributions G_{ω_i} for all $i \in I$, and, finally, his contract satisfies the limited liability if

$$S(\omega, \theta) - c \geq B,\ \forall \omega \in \Omega, \forall \theta \in \Theta. \tag{10}$$

The buyer solves the following simultaneous contracting mechanism problem:

$$\underset{q, t_1, \ldots, t_N, S}{\text{Maximize}} \int_{\Theta \times \Omega} \Big\{ \sum_{i \in I} [q_i(\theta, \omega) v(\omega_i) - t_i(\theta, \omega)] - S(\omega, \theta) \Big\} dG(\theta, \omega),$$

$$\text{subject to } (3), (4);\ \text{and } (8), (9), (10).$$

The maximization problem, although it appears to be complex at first, can be simplified. To attain the benchmark payoff from the previous section, for each $i \in I$, seller i's production and transfer are identical to those of the benchmark, the optimal choice of q and t_i from (6). It remains to identify the auctioneer's contract that satisfies (8), (9) and (10). Then, combined with each seller's such *dominant* strategy, the auctioneer's strict incentive compatibility leads to a unique Bayesian implementation.

The main idea to induce the auctioneer's strict incentive compatibility with his individual rationality and limited liability is the first-order condition alignment. We align his optimality with the benchmark's absolute criterion, through the first-order condition, such that for each $i \in I$, there exists some $\tau_i(\omega_i) > 0$ such that for any $\omega'_{-i} \in \Omega_{-i}$,

$$\underbrace{\frac{\partial \mathbb{E}_\theta[S(\omega'_i, \omega'_{-i}, \theta)|\omega]}{\partial \omega'_i}\Bigg|_{\omega'_i = \omega_i}}_{\text{Auctioneer's optimality}} = \tau_i(\omega_i) \times \underbrace{[v(\omega_i) - \phi_i(\theta_i, \omega_i)]}_{\text{Benchmark's absolute criterion for seller } i} = 0. \tag{11}$$

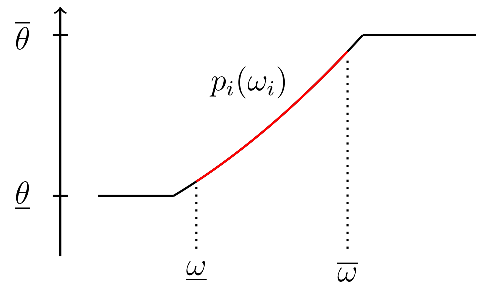

Figure 4: Interior case

We employ the first-order alignment based on absolute criterion for three reasons. First, the alignment makes a contract unique up to a positive linear affine transformation. Since a change in its "scale" is such a transformation, the scheme makes it possible for the buyer to extract all the surplus from the auctioneer, while satisfying the limited liability. Second, the suggested alignment connects the reverse hazard rate dominance with the global optimality, which in turn results in the strict Bayesian incentive compatibility in (8). To be clear, there could be simpler schemes that produce a similar first-order condition for truth-telling but fail at the global optimality.[21] Third, this scheme can also apply to multiple agents, i.e., sellers in this paper, as with the suggested first-order absolute alignment, the auctioneer's incentive compatibility for seller i's quality report is separated from that for seller j's, for $j \neq i$; otherwise, solving a system of partial differential equations is intractable. Additional innovation of the proposed mechanism is its domain-freedom: it applies even to non-interior cases.

From the benchmark's absolute criterion, a threshold cost of seller i, $p_i(\omega_i)$, is defined such that only $\theta_i < p_i(\omega_i)$ wins if $\pi_i(\theta_i, \omega_i) > \pi_j(\theta_j, \omega_j)$ $\forall j \in I, j \neq i$, where

$$p_i(\omega_i) = \begin{cases} \underline{\theta} & \text{if } v(\omega_i) \leq \phi_i(\underline{\theta}, \omega_i), \\ \theta_i \text{ s.t. } v(\omega_i) = \phi_i(\theta_i, \omega_i) & \text{if } v(\omega_i) > \phi_i(\underline{\theta}, \omega_i), v(\omega_i) < \phi_i(\overline{\theta}, \omega_i), \\ \overline{\theta} & \text{if } v(\omega_i) \geq \phi_i(\overline{\theta}, \omega_i). \end{cases} \tag{12}$$

We first consider an interior case and then extend the argument to non-interior cases. By assuming that for each $i \in I$, $v(\underline{\omega}) > \phi_i(\underline{\theta}, \underline{\omega})$ and $v(\overline{\omega}) < \phi_i(\overline{\theta}, \overline{\omega})$, the threshold yields an interior value $p_i(\omega_i) \in (\underline{\theta}, \overline{\theta})$ for all ω_i, as in figure 4.

[21] For example, if for each seller i, the auctioneer is asked to make a payment equal to the squared difference between the cost and the expected cost conditional on the reported quality, i.e. $\left(\theta_i - \int_{\underline{\theta}}^{\overline{\theta}} \theta_i dF_i(\theta_i|\omega_i')\right)^2$, the first-order condition is satisfied, but the second-order condition, the global optimality, is not necessarily satisfied, especially with the sign $\partial f_i(\theta_i|\omega_i')/\partial\omega_i'$ not being determined.

To achieve the alignment (11), while providing the information acquisition incentive, consider the following compensation scheme: given the auctioneer's report ω', the buyer pays the auctioneer $S(\omega', \theta)$ such that

$$S(\omega', \theta) = B + c + \sum_{i=1}^{N} s_i(\omega_i')\mathbf{1}_{\{\theta_i < p_i(\omega_i')\}}, \tag{13}$$

where $\mathbf{1}_{\{\theta_i < p_i(\omega_i')\}}$ is an indicator function. The auctioneer obtains a portion of the compensation from announcing seller i's quality only when the seller's cost is below the zero virtual-payoff threshold. Implementing the optimal mechanism as an auction, which will be discussed in Section 6 in detail, means that the auctioneer is awarded seller i's portion only if the seller participates in the auction.

Then, the auctioneer's expected payoff $\mathbb{E}_\theta[S(\omega', \theta)|\omega]$ from announcing a vector of quality levels ω', conditional on a vector of true quality levels ω, is derived as $\mathbb{E}_\theta[S(\omega', \theta)|\omega] = B + c + \sum_{i \in I} s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i)$, which can be rewritten as

$$\mathbb{E}_\theta[S(\omega', \theta)|\omega] = B + c + \sum_{i \in I} u_i(\omega_i', \omega_i), \tag{14}$$

where $u_i(\omega_i', \omega_i) \equiv s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i)$ denotes a portion of the expected payoff from seller i.

A contract for the auctioneer is incentive compatible if and only if for each $i \in I$,

$$u_i(\omega_i, \omega_i) \geq u_i(\omega_i', \omega_i) \text{ for all } \omega_i', \omega_i \in [\underline{\omega}, \overline{\omega}]. \tag{15}$$

Unlike the formulation of Myerson (1981), for the first-order absolute alignment, we first must establish the differentiability of a contract for the auctioneer. The lemma below shows that for each $i \in I$, any incentive compatible $s_i(\omega_i)$ is strictly decreasing and differentiable in ω_i.

Lemma 1 *For each $i \in I$, any incentive compatible $s_i(\omega_i)$ is strictly decreasing and differentiable in ω_i.*

The negative relationship between the compensation and quality report is derived to "balance" the auctioneer's incentive to over-report seller i's quality in favor of higher production. In addition, a contract for the auctioneer is individually rational with the limited liability if and only if

$$B + \int_\Omega \sum_{i \in I} u_i(\omega_i, \omega_i) dG_\omega(\omega) \geq \overline{U}. \tag{16}$$

The cost c is cancelled out by substituting (14) into (9); $\int_\Omega \mathbb{E}_\theta[S(\omega, \theta)|\omega] dG_\omega(\omega) - c \geq \overline{U}$ to $B + c + \int_\Omega \sum_{i \in I} u_i(\omega_i, \omega_i) dG_\omega(\omega) - c \geq \overline{U}$. Each seller's contract is adopted from the benchmark, so a simultaneous contracting mechanism is incentive compatible and individually rational if and only if the auctioneer's incentive compatibility and individual rationality are satisfied.

The auctioneer chooses ω' to maximize $u_i(\omega_i', \omega_i)$ for all i, and from Lemma 1, the partial derivative of it with respect to ω_i' yields $-s_i'(\omega_i') f_i(p_i(\omega_i')|\omega_i) \left[p_i(\omega_i') - \frac{s_i(\omega_i')p_i'(\omega_i')}{s_i'(\omega_i')} - \phi_i(p_i(\omega_i'), \omega_i)\right]$, where $\phi_i(p_i(\omega_i'), \omega_i) = p_i(\omega_i') + \frac{F_i(p_i(\omega_i')|\omega_i)}{f_i(p_i(\omega_i')|\omega_i)}$. If we choose $s_i(\omega_i)$ such that for each ω_i,

$$p_i(\omega_i) - \frac{s_i(\omega_i)p_i'(\omega_i)}{s_i'(\omega_i)} = v(\omega_i), \tag{17}$$

the derivative can be rewritten as $\frac{\partial u_i(\omega_i', \omega_i)}{\partial \omega_i'} = -s_i'(\omega_i') f_i(p_i(\omega_i')|\omega_i) \left[v(\omega_i') - \phi_i(p_i(\omega_i'), \omega_i)\right]$. As we aim, the two optimality conditions are *aligned*: from the zero virtual-payoff (5), $\frac{\partial u_i(\omega_i, \omega_i)}{\partial \omega_i'} = 0$, which implies $\left.\frac{\partial \mathbb{E}_\theta[S(\omega_i', \omega_{-i}', \theta)|\omega]}{\partial \omega_i'}\right|_{\omega_i' = \omega_i} = 0$ for all ω_{-i}' in (11).

Suppose the interior assumption is not satisfied. This is not innocuous when it meets with the strict incentive compatibility: to achieve it, the truthful report must give the auctioneer a payoff greater than $B + c$. A transformation of $p_i(\omega_i)$ converts the previous corner points into interior ones in order to "retain" the first-order absolute alignment. We relegate its formal proof to the appendix.

Finally, by showing that the auctioneer's contract satisfies the *strict* Bayesian incentive compatibility in (8), $u_i(\omega_i, \omega_i) > u_i(\omega_i', \omega_i)$ for all $\omega_i' \neq \omega_i$, the first main result establishes the existence of a simultaneous contracting mechanism that implements the second best as a unique Bayesian equilibrium. Furthermore, the simultaneous contracting mechanism yields the buyer the same payoff as that from the benchmark.

Theorem 1 *There exists an incentive compatible and individually rational simultaneous contracting mechanism that implements the second-best payoff as a unique Bayesian equilibrium such that (i) a contract for seller i* (q_i, t_i) *satisfies (6), and (ii) a contract for the auctioneer S satisfies the strictly Bayesian incentive compatibility with the individual rationality condition (9) binding. Furthermore, if the information acquisition cost is smaller than a threshold, the mechanism provides the auctioneer an incentive to acquire the true quality levels.*

If the auctioneer does not acquire the quality information, then he will choose a quality report that maximizes his *ex-ante* expected payoff, given the proposed contract. With no acquisition cost, acquiring the information yields a higher payoff to him, from the strict incentive compatibility, so it is optimal for him to acquire the information if the cost is lower than a threshold cost level, the difference between these two payoffs.[22]

5 Optimal design with discrete quality levels

We let sellers have the same support for quality as in the continuous case: for each $i \in I$, $\omega_i \in \{\omega^1, ..., \omega^J\}$ for $J \geq 2$ and a set of quality levels $\omega^1 > \omega^2 > \cdots > \omega^J$ with a partial order. The remaining dimension, cost type, is still continuous.

Utilizing the discrete nature of quality, just like the number of agents, the notations for the marginal cumulative distribution of ω_i and the conditional cumulative distribution of θ_i given ω_i are simplified such that for $\omega_i = \omega^j$, $\lambda_{ij} \equiv G_{\omega_i}(\omega^j)$ and $F_{ij}(\theta_i) \equiv F_i(\theta_i|\omega^j)$. That is, λ_{ij} is the probability that seller i's quality ω^j is realized, and $F_{ij}(\theta_i)$ is the conditional probability that seller i's cost is lower than θ_i given seller i's quality ω^j. Similarly, convenient notations are employed for $p_i(\omega_i)$ and $s_i(\omega_i)$ from (12) and (13) such that for $\omega_i = \omega^j$, we let $p_{ij} \equiv p_i(\omega^j)$ and $s_{ij} \equiv s_i(\omega^j)$.

Recall that in (13), the use of $p_i(\omega_i)$ as a threshold enables us to satisfy the ex-post limited liability since it is the way that the contract was *built*. Relying on the the benchmark's absolute criterion threshold through p_{ij} in (12) as an alignment, we design a discrete counterpart of the first-order derivative in the continuous quality case, called the discrete first-order absolute alignment. In other words, the benchmark's absolute criterion threshold can still be found from the first-order condition, but the auctioneer's quality report in this section is not differentiable, so no equivalent relationship between them can be established, unlike (11) in the previous section.

The starting point is to observe that the partial order of ω^j and the monotonicity of $p_i(\omega_i)$ entail $p_{ij} > p_{ik}$ for any pair $j < k$. With them, a contract for the auctioneer is incentive compatible if and only if for each $i \in I$, $s_{ij}F_{ij}(p_{ij}) \geq s_{ik}F_{ij}(p_{ik})$ for all $k \neq j$, and, in addition, a contract for the auctioneer is individually rational[23] if and only if

$$B + \sum_{i \in I} \sum_{j=1}^{J} \lambda_{ij} s_{ij} F_{ij}(p_{ij}) \geq \overline{U}. \tag{18}$$

Like the continuous case, in the discrete case, an incentive compatible contract for the auctioneer pays less for a higher quality report, meaning that for each $i \in I$, $s_{ij} < s_{ik}$ for all

[22] See Yoo (2016) for a broad discussion for the optimal information acquisition.

[23] Note that as in (16) with the continuous quality, the acquisition cost c is cancelled out such that $B + c + \sum_{i \in I} \sum_{j=1}^{J} \lambda_{ij} s_{ij} F_{ij}(p_{ij}) - c \geq \overline{U}$.

$j < k$. Suppose, on the contrary, $s_{ij} \geq s_{ik}$. Then, $p_{ij} > p_{ik}$ yields $s_{ik}F_{ik}(p_{ik}) < s_{ij}F_{ik}(p_{ij})$, a contradiction with the incentive compatibility $s_{ik}F_{ik}(p_{ik}) \geq s_{ij}F_{ik}(p_{ij})$.

A discrete-case condition corresponding to the reverse hazard rate dominance in the continuous case is called the cumulative sum ratio dominance.[24] The condition is defined such that for any pair $\omega_i' > \omega_i$, $F_i(\theta_i|\omega_i')$ dominates $F_i(\theta_i|\omega_i)$ in terms of the cumulative sum ratio if for all $\theta_i' > \theta_i$, $\frac{F_i(\theta_i'|\omega_i')}{F_i(\theta_i'|\omega_i)} > \frac{F_i(\theta_i|\omega_i')}{F_i(\theta_i|\omega_i)}$. In other words, the ratio $\frac{F_i(\theta_i|\omega_i')}{F_i(\theta_i|\omega_i)}$ is a strictly increasing function of θ_i. The two conditions are equivalent.

Lemma 2 *F_i satisfies the reverse hazard rate dominance if and only if it satisfies the cumulative sum ratio dominance.*

We aim to find a mechanism that implements the second-best payoff as a unique Bayesian equilibrium, and finding such a mechanism reduces to finding a strictly incentive compatible contract for the auctioneer. A two-quality-level case with H and L and $\omega^H > \omega^L$ is introduced first to illustrate the difference between two quality levels and an arbitrary number of quality levels. The strict Bayesian incentive compatibility for the auctioneer is satisfied if and only if for each $i \in I$ and for every $j \in \{H, L\}$, there exist $\epsilon_{iH} > 0$ and $\epsilon_{iL} > 0$ such that $s_{iH}F_{iH}(p_{iH}) - s_{iL}F_{iH}(p_{iL}) = \epsilon_{iH}$ and $s_{iL}F_{iL}(p_{iL}) - s_{iH}F_{iL}(p_{iH}) = \epsilon_{iL}$. Farkas' Lemma shows that the minimum condition is the "local" cumulative sum ratio dominance, it being satisfied in only two points: $\frac{F_{iH}(p_{iH})}{F_{iL}(p_{iH})} > \frac{F_{iH}(p_{iL})}{F_{iL}(p_{iL})}$. For the two-quality case, Farkas' Lemma can be used, but it is used in a way completely different from Crémer and McLean (1988) because one of two dimensions, cost type, is continuous, and the contract must satisfy the limited liability.

More importantly, for an arbitrary number of quality levels that includes the two-quality case, the discrete first-order absolute alignment does not utilize the Lemma at all. The local dominance is a necessary and sufficient condition for the existence of an incentive compatible and individually rational simultaneous contracting mechanism.

Proposition 2 *For $J = 2$, the local cumulative sum ratio dominance is satisfied if and only if there exists an incentive compatible and individually rational simultaneous contracting mechanism that implements the second-best payoff as a unique Bayesian equilibrium.*

If J is large, the problem becomes easily complicated, losing the close, direct connection from $J = 2$ between Farkas' Lemma and the cumulative sum ratio dominance, which makes the Lemma ineffective in utilizing the alignment to handle the limited liability and the acquisition cost. We employ an approach, the discrete first-order absolute alignment, that

[24]The term is well defined in statistics. See, e.g., Hinkley (1971).

does not rely on Farkas' Lemma; first, find, for any two ω^k and ω^{k+1}, a bilateral Bayesian incentive compatibility (BBIC) condition for $\epsilon_{ik} > 0$ such that $s_{ik}F_{ik}(p_{ik}) = s_{ik+1}F_{ik}(p_{ik+1}) + \epsilon_{ik}$ and $s_{ik+1}F_{ik+1}(p_{ik+1}) = s_{ik}F_{ik+1}(p_{ik})$, and later show that a solution from all bilateral Bayesian incentive compatibility conditions is a global solution. The proof requires the local cumulative sum ratio dominance to be satisfied for *every two adjacent* quality levels.

Theorem 2 *Suppose that the local cumulative sum ratio dominance is satisfied. There exists an incentive compatible and individually rational simultaneous contracting mechanism that implements the second-best payoff as a unique Bayesian equilibrium such that (i) a contract for seller i (q_i, t_i) satisfies (6), and (ii) a contract for the auctioneer S satisfies the strictly Bayesian incentive compatibility with the individual rationality condition (18) binding. Furthermore, if the information acquisition cost is smaller than a threshold, the mechanism provides the auctioneer an incentive to acquire the true quality levels.*

6 Implementation

The buyer can implement the second-best outcome through an auction.[25] The auction may be called a quality-adjusted SPA with a profile of reserve prices. The quality adjustment applies for the relative criterion of the optimal mechanism, and asymmetric reserve prices applies for its absolute criterion. Note, however, that we apply only the absolute criterion to induce the auctioneer to report truthfully, so in an auction, *other, not necessarily optimal, relative criteria can be implemented*, without interfering with the auctioneer's truth-telling. See, in Section 8, how standard auction rules can suit the third party's first-order absolute alignment.

We denote by $r \equiv (r_1, ..., r_N)$ a profile of reserve prices and by $b \equiv (b_1, ..., b_N)$ a bid profile. The auction is implemented with the time line below:

1. The auctioneer announces publicly a profile of reserve prices r.

2. The auctioneer implements an auction rule $\mathcal{A}(r)$.

3. The auctioneer's compensation is given as $\widehat{S}(r, b)$.

We elaborate each stage in what follows. Suppose the auctioneer sets a reserve price r_i for bidder i, and bidder i wins with no tie in a SPA. Then the buyer pays bidder i the minimum

[25] The implementation of this section is based on the model of Section 2 and Theorem 1, especially with continuous quality levels, but it can of course be extended to the discrete quality case in the previous section.

of the second lowest bid – in terms of a quality adjustment – and the reserve price for him r_i. That is, the reserve price r_i is the maximum sales price for bidder i that the buyer is willing to pay to him. It is chosen to reflect bidder i's quality as well as his virtual production cost. In this model, different bidders have not only different virtual costs but also different quality levels, which makes the auctioneer choose different reserve prices for them.

Implementing the optimal mechanism as an auction in this setting involves two challenges. First, bidders have two-dimensional types with (θ_i, ω_i) for all $i \in I$. Typically, there is an existence-of-equilibrium problem for incomplete information games with multi-dimensional types. Second, the auction rule $\mathcal{A}(r)$ is designed to consider both aspects of bidders: their bids and their quality levels. For the latter, the rule assigns quality weights to them. The quality weight for bidder i is denoted by ω_i'. It is derived from a one-to-one mapping $\omega_i' = p_i^{-1}(r_i)$, where $p_i^{-1}(r_i)$ is the inverse function of the zero virtual-return threshold $p_i(\cdot)$ in (12) and r_i is from the auctioneer's announcement. However, if the equilibrium outcome depends on the quality weights as well as the reserve prices, the auctioneer may be "tempted" to manipulate the reserve prices to gain a higher payoff.[26] Nevertheless, both can be solved if the auction is implemented as a dominant strategy equilibrium.

The buyer designs the auction rule $\mathcal{A}(r)$ so that it depends on the auctioneer's announcement r. In particular, the rule is based on the quality weights $\omega' \equiv (\omega_1', ..., \omega_N')$ with $\omega_i' = p^{-1}(r_i)$ for all $i \in I$ such that bidder i wins with no tie if $\pi_i(b_i, \omega_i') > \pi_j(b_j, \omega_j')$ for all $j \in I, j \neq i$. When bidder i wins with no tie, bidder i supplies the object with price x_i such that $\pi_i(x_i, \omega_i') = \pi_j(b_j, \omega_j')$, where bidder j is the bidder with the second lowest bid in terms of the quality adjustment, i.e., $j \in \arg\max_{\{k \neq i\}} \pi_k(b_k, \omega_k')$. If there is a tie such that $\pi_i(b_i, \omega_i') = \pi_j(b_j, \omega_j')$, where $j \in \arg\max_{\{k \neq i\}} \pi_k(b_k, \omega_k')$, then each winning bidder supplies the object with equal probability.

Each bidder i's strategy is a mapping from $[\underline{\theta}, \overline{\theta}] \times [\underline{\omega}, \overline{\omega}]$ to $\{\text{No}\} \cup [0, r_i]$, where No refers to the case in which bidder i is not participating. For each announcement r by the auctioneer, this auction rule $\mathcal{A}(r)$ implements a (weakly) dominant strategy equilibrium in which each bidder bids his own cost if that is no greater than his reserve price.

Lemma 3 *For any announcement r, the auction rule $\mathcal{A}(r)$ implements a (weakly) dominant strategy equilibrium such that for all $i \in I$, $b_i = \theta_i$ if $\theta_i \leq r_i$, and $b_i = No$ if $\theta_i > r_i$.*

Then, for the unique truthful revelation of quality weights, that is, $\omega' = \omega$, the principal

[26] This problem does not arise with the direct revelation in the previous section since both the auctioneer and the bidders report their types and observations *simultaneously*, whereas in the implementation, the auction rule should, by nature, be first announced to the bidders.

can design the auctioneer's compensation $\widehat{S}(r,b)$, where b is a profile of bids and r is a profile of reserve prices. Denote $\widehat{s}_i(r_i) \equiv s_i(p_i^{-1}(r_i))$. The auctioneer's payoff is given as $\widehat{S}(r,b) = B + c + \sum_{i=1}^{N} \widehat{s}_i(r_i)\mathbf{1}_{\{b_i<r_i\}}$. From the dominant strategy equilibrium in Lemma 3, the auction rule makes each bidder i participate only when his own production cost θ_i is lower than the reserve price for that bidder. As a result, the auctioneer's payoff is equivalently rewritten as $\widehat{S}(r,b) = B + c + \sum_{i=1}^{N} \widehat{s}_i(r_i)\mathbf{1}_{\{\text{bidder } i \text{ participates}\}}$, where $\widehat{s}_i(r_i)$ is a portion of the compensation from announcing bidder i's reserve price when bidder i participates. Then, the auctioneer's expected payoff from announcing a profile of reserve prices r is

$$B + \sum_{i\in I} \widehat{s}_i(r_i)F_i(r_i|\omega_i). \tag{19}$$

The compensation scheme is constructed in such a way that it is connected to the optimality condition from the benchmark from (14). Hence, the principal implements the benchmark's outcome as a quality-adjusted SPA with a profile of reserve prices.

Theorem 3 *The principal can implement the second-best payoff (6) by delegating a SPA rule $\mathcal{A}(r)$ to the auctioneer, and by paying the compensation $\widehat{S}(r,b)$ to him. Furthermore, if the information acquisition cost is smaller than a threshold, the mechanism provides the auctioneer an incentive to acquire the true quality levels.*

Thus, the quality-adjusted SPA solves the existence problem and manipulation together. The overall implementation including the auctioneer's announcement r cannot be a dominant one, since the auctioneer's strategy depends on the bidders' strategies as well as on their type distributions from (19).

7 Extension

For an interior value $p_i(\omega_i)$ of (12), p_i' yields

$$p_i'(\omega_i) = \frac{v'(\omega_i) - \partial\phi_i(p_i(\omega_i),\omega_i)/\partial\omega_i}{\partial\phi_i(p_i(\omega_i),\omega_i)/\partial\theta_i}, \tag{20}$$

so, given the other assumptions, as discussed earlier, the reverse hazard rate dominance is sufficient for the optimal absolute threshold to be increasing in quality. Despite its appeal to our intuition, canonical signaling models, following Spence (1973), assume a negative, especially deterministic, correlation between quality and cost. They determine both dimensions in favor of a high type: a higher quality and a lower cost, which is obviously violated with the reverse hazard rate dominance in Section 2.

Now, consider the case that for any pair $\omega_i' > \omega_i$, a lower quality's cost distribution $F_i(\theta_i|\omega_i)$ dominates $F_i(\theta_i|\omega_i')$ in terms of the reverse hazard rate; a higher quality is more likely to incur *lower* costs. The sign of the numerator in (20) is no longer determinant, and, moreover, if the magnitude of the opposite reverse hazard rate is sufficiently large, the derivative p_i' has a negative sign, which means, in a procurement auction, that for a higher quality, a lower maximum reserve price is set. This comparative statics result is more problematic, contradicting the intuition, in a continuous output case, studied by Yoo (2016); as the quality increases, the corresponding optimal output *decreases*. Hence, it is reasonable to assume that the increase in quality dominates the magnitude of the reverse hazard rate for this extension so that the sign of the numerator in (20) is still positive.

With the assumption and a reverse hazard rate flipped over, for the auctioneer's truth-telling, we just need to reinvent u_i of (14) differently. For each seller i, define

$$u_i(\omega_i', \omega_i) \equiv s_i(\overline{\omega})F_i(p_i(\overline{\omega})|\overline{\omega}) - s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i). \tag{21}$$

By the envelope theorem, combined with $F_i(\theta_i|\omega_i)$ first-order stochastic dominating $F_i(\theta_i|\omega_i')$, $u_i(\omega_i, \omega_i)$ attains a highest value $u_i(\overline{\omega}, \overline{\omega})$. Hence, for any ω_i, $u_i(\omega_i, \omega_i) \geq 0$ $(= 0$ if $\omega_i = \overline{\omega})$, satisfying the limited liability. The partial derivative of (21) yields $-s_i'(\omega_i')F_i(p_i(\omega_i')|\omega_i) - s_i(\omega_i')f_i(p_i(\omega_i')|\omega_i)$, which can be rewritten as $-s_i'(\omega_i')f_i(p_i(\omega_i')|\omega_i)\left[\phi_i(p_i(\omega_i'), \omega_i) - v(\omega_i)\right]$, by choosing for any $\omega_i \in [\underline{\omega}, \overline{\omega}]$, $\frac{s_i(\omega_i')p_i'(\omega_i)}{s_i'(\omega_i')} - p_i(\omega_i) = v(\omega_i)$, exactly the same as (17). Note that with this reversed $[\phi_i(p_i(\omega_i'), \omega_i) - v(\omega_i)]$, the same logic for the global optimality as Theorem 1 applies to this extension.

8 Asymmetric seller auctions

Consider a seller auction with asymmetric buyers. The seller can adopt either FPA or SPA for an auction setting to sell an indivisible good, neither being an optimal mechanism that can be characterized from its procurement counterpart in Section 4 and implementation in Section 6. As discussed in Section 4 and particularly Section 6, a principal can harness the first-order absolute alignment, based on the absolute criterion, for other, not necessarily optimal, relative criteria.

The literature typically assumes a two-bidder (or two-group) case such that a bidder's identity is identified with his distribution type, e.g., strong bidder and weak bidder. We study a two-bidder case, like a typical setup, but each bidder's distribution type can be one of many distributions. Bidder i has valuation $v_i \in [\underline{v}, \overline{v}], \overline{v} > \underline{v} \geq 0$, and bidder i's type distribution t_i is drawn from a continuum set $[\underline{t}, \bar{t}], \bar{t} > \underline{t}$, which is his private information.[27] Each bidder has a two-dimensional type (v_i, t_i).[28]

We assume that for any pair $t_i' > t_i$, $F_i(v_i|t_i')$ dominates $F_i(v_i|t_i)$ in terms of the hazard rate.[29] That is, $\frac{f_i(v_i|t_i')}{1-F_i(v_i|t_i')} < \frac{f_i(v_i|t_i)}{1-F_i(v_i|t_i)}$ for all $v_i \in (\underline{v}, \bar{v})$. Then, the virtual valuation can be written as for $t_i \in [\underline{t}, \bar{t}]$,

$$\psi_i(v_i, t_i) = v_i - \frac{1 - F_i(v_i|t_i)}{f_i(v_i|t_i)}, \tag{22}$$

where $\psi_i(v_i, t_i)$ is strictly increasing in valuation v_i and strictly decreasing in t_i by the hazard rate dominance. Define an interior reserve price $r_i(t_i)$, in an exactly parallel way as in the buyer auction, such that $\psi_i(r_i(t_i), t_i) = 0$.

Now, we introduce an expert, like the auctioneer in Section 4, who can identify each bidder's type distribution with positive acquisition costs, to replace the common knowledge assumption. With the same timeline in the implementation, Section 6, the seller, the mechanism designer, requires the expert to announce the two bidders' type distributions (t_1, t_2). If a contract for the expert can be designed such that he is truthful, then the second dimension of the common knowledge, each bidder knowing the other's distribution, can be easily resolved upon the expert's public announcement. In SPA, each bidder bids his own valuation v_i, and in FPA, consider an environment in which its unique equilibrium is given as $\beta_1(v_1, t_1)$ and $\beta_2(v_2, t_2)$, if each bidder knows the other's distribution.[30]

Define, applying the results of Maskin and Riley (2000a) and Kirkegaard (2012), a subset of $T_{FP} \subset T \equiv [\underline{t}, \bar{t}] \times [\underline{t}, \bar{t}]$ such that for any $(t_1, t_2) \in T_{FP}$, FPA yields a higher revenue, and for any $(t_1, t_2) \in T_{SP} \subset [\underline{t}, \bar{t}] \times [\underline{t}, \bar{t}]$, SPA yields a higher revenue. Of course, any pair (t_1, t_2) for $t_1 = t_2$ yields the same revenue from the revenue equivalence.

The expert receives $\sigma_i^{SP}(t_i')$ for each $i = 1, 2$ if $(t_1', t_2') \in T \setminus T_{FP}$ and bidder i's bid is greater than $r_i(t_i)$; and he receives $\sigma_i^{FP}(t_i')$ for each $i = 1, 2$ if $(t_1', t_2') \in T_{FP}$ and bidder i's

[27] Maskin and Riley (2000b) allow the case that two type distributions have the same support.

[28] To avoid any potential confusion, in these seller auctions, we adopt different notations for a two-dimensional type, a counterpart of bidder i's type (θ_i, ω_i), cost and quality, from the previous main analysis. It is worth emphasizing, at this juncture, why this mechanism is different from Crémer and McLean (1988). First, we have two correlated types such that two dimensions are of the continuous type; one of them can be extended to a discrete type, as shown in Section 5, but Farkas' Lemma of Crémer and McLean (1988) applies to only discrete type cases. Second, the limited liability is not satisfied in their mechanism.

[29] Kirkegaard (2012) assumes both the hazard rate and reverse hazard rate dominance for additional results.

[30] Finding such environments is beyond our scope; see, e.g., Lebrun (1999) and Maskin and Riley (2000b) among others.

bid is greater than $R_i(t_i)$, where $R_i(t_i) \equiv \beta_i(r_i(t_i), t_i)$. Then, their interim expected payoff can be summarized as follows:

$$\begin{cases} \sum_{i=1,2} \sigma_i^{SP}(t_i')[1 - F_i(r_i(t_i')|t_i)] & \text{if } (t_1', t_2') \in T \setminus T_{FP}, \\ \sum_{i=1,2} \sigma_i^{FP}(t_i')[1 - F_i(r_i(t_i')|t_i)] & \text{if } (t_1', t_2') \in T_{FP}, \end{cases} \tag{23}$$

where from the construction, $\Pr(\beta_i(v_i) > R_i(t_i)) = \Pr(v_i > r_i(t_i'))$, so we must have $\sigma_i(t_i) = \sigma_i^{SP}(t_i) = \sigma_i^{FP}(t_i)$ for all t_i. To apply the same procedure, on how to satisfy the limited liability and the incentive to acquire costly information, of the main analysis in Section 4, for each i, we can redefine u_i of (14) such that

$$u_i(t_i', t_i) \equiv \sigma_i(t_i')[1 - F_i(r_i(t_i')|t_i)]. \tag{24}$$

As in Lemma 1, $\sigma_i(t_i')$ is differentiable, but in contrast to the Lemma, now, $\sigma_i(t_i')$ is strictly increasing to induce the expert's truth-telling. The partial derivative of (24) yields $\sigma_i'(t_i')[1 - F_i(r_i(t_i')|t_i)] - \sigma_i(t_i') f_i(r_i(t_i')|t_i) r_i'(t_i)$, which can be rewritten as $-\sigma_i'(t_i') f_i(r_i(t_i')|t_i)\psi_i(r_i(t_i'), t_i)$ by, for each t_i, choosing $\frac{\sigma_i(t_i) r_i'(t_i)}{\sigma_i'(t_i)} = r_i(t_i)$ for all $t_i \in [\underline{t}, \overline{t}]$. The hazard rate dominance implies $u_i(t_i, t_i) > u_i(t_i', t_i)$ for all $t_i' \neq t_i$ by modifying the proof of Theorem 1, so the expert has no incentive to report any other pair from the same set. That is, if $(t_1, t_2) \in T_{FP}$, $u_1(t_1, t_1) + u_2(t_2, t_2) > u_1(t_1', t_1) + u_2(t_2', t_2)$ for all $(t_1', t_2') \neq (t_1, t_2), (t_1', t_2') \in T_{FP}$.

In addition, the expert has no incentive to report a pair that belongs to the other set: if $(t_1, t_2) \in T_{FP}$, $u_1(t_1, t_1) + u_2(t_2, t_2) > u_1(t_1', t_1) + u_2(t_2', t_2)$ for all $(t_1', t_2') \neq (t_1, t_2), (t_1', t_2') \in T \setminus T_{FP}$. When the number of distribution types is discrete, as in Maskin and Riley (2000a) and Kirkegaard (2012), the results from Section 5 can apply. The same analysis can be applied to the monopoly of Bulow and Roberts (1989).

9 Concluding remarks

This paper studies auctions with a partially informed auctioneer. The sellers' information structure with the non-contractible quality makes it impossible for the buyer to attain the second-best payoff with a typical direct mechanism. The the first-order alignment based on an absolute criterion with the auctioneer is suggested as a solution.

The model's simultaneous mechanism provides a rationale for the presence of procurement officers or purchasing managers commonly observed. This rationale is further enhanced by designing a practical auction format that achieves the goal. The main focus of this paper is how to extract the relevant information from the auctioneer, and implement a feasible auction, while he attains only his outside option.

Collusion between the auctioneer and the sellers can impair the optimal simultaneous mechanism studied in the paper, but its presence and harmful effects largely depend on an institution's characteristics, in particular, its development stage, as well documented in the related literature. Therefore, it is reasonable to consider collusion as a separate subject and leave it for future research.

Appendix: Proofs

Proof of Proposition 1. A mechanism q and t_i with $i \in I$ is incentive compatible if and only if for each $i \in I$ and every $\omega_i \in [\underline{\omega}, \overline{\omega}]$, Q_i is decreasing in θ_i given each ω_i, and

$$T_i(\theta_i, \omega_i) = \theta_i Q_i(\theta_i, \omega_i) + \int_{\theta_i}^{\overline{\theta}} Q_i(x, \omega_i) dx + \left[T_i(\overline{\theta}, \omega_i) - \overline{\theta} Q_i(\overline{\theta}, \omega_i)\right],$$

and is individually rational if and only if for each $i \in I$ and every $\omega_i \in [\underline{\omega}, \overline{\omega}]$, $T_i(\overline{\theta}, \omega_i) - \overline{\theta} Q_i(\overline{\theta}, \omega_i) = 0$. Then, an incentive compatible and individual mechanism yields the buyer's expected payoff as

$$\sum_{i \in I} \int_{\Theta \times \Omega} q_i(\theta, \omega) \left[v(\omega_i) - \phi_i(\theta_i, \omega_i)\right] dG(\theta, \omega).$$

■

Proof of Lemma 1. (a) We first show that $s_i(\omega_i)$ is strictly decreasing. Any incentive compatible $s_i(\omega_i)$ satisfies that for any $\omega_i' > \omega_i$,

$$s_i(\omega_i) F_i(p_i(\omega_i)|\omega_i) \geq s_i(\omega_i') F_i(p_i(\omega_i')|\omega_i).$$

Suppose that $s_i(\omega_i') \geq s_i(\omega_i)$. Then, $s_i(\omega_i) F_i(p_i(\omega_i)|\omega_i) < s_i(\omega_i') F_i(p_i(\omega_i')|\omega_i)$, since $p_i(\omega_i)$ is strictly increasing, so we have a contradiction.

(b) Any incentive compatible $s_i(\omega_i)$ is continuous. Consider an arbitrary ω_i and any incentive compatible $s_i(\omega_i)$ satisfies that for any $\omega_i' \neq \omega_i$,

$$\begin{cases} s_i(\omega_i') F_i(p_i(\omega_i')|\omega_i') \geq s_i(\omega_i) F_i(p_i(\omega_i)|\omega_i'), \\ s_i(\omega_i) F_i(p_i(\omega_i)|\omega_i) \geq s_i(\omega_i') F_i(p_i(\omega_i')|\omega_i), \end{cases} \tag{25}$$

and we let $\omega_i' \to \omega_i$. Note that for any $\omega_i < \overline{\omega}_i$, $s_i(\omega_i) > 0$ since $s_i(\omega_i)$ is strictly decreasing, so we may have $s_i(\omega_i) = 0$ only for $\omega_i = \overline{\omega}_i$.

(i) $s_i(\omega_i)$ is continuous at $\underline{\omega}_i$. For any $\omega_i > \underline{\omega}_i$, the IC in (25) is satisfied. Suppose $s_i(\omega_i)$ is not continuous at $\underline{\omega}_i$, so we have $s_i(\underline{\omega}_i) > \lim_{\omega_i \to \underline{\omega}_i+} s_i(\omega_i)$. Let $\omega_i \to \underline{\omega}_i$. In the limit, the first IC becomes

$$\lim_{\omega_i \to \underline{\omega}_i+} s_i(\omega_i) F_i(p_i(\underline{\omega}_i)|\underline{\omega}_i) \geq s_i(\underline{\omega}_i) F_i(p_i(\underline{\omega}_i)|\underline{\omega}_i),$$

which is a contradiction.

(ii) $s_i(\omega_i)$ is continuous at $\overline{\omega}_i$. For $\omega_i < \overline{\omega}_i$, the IC in (25) is satisfied. Suppose $s_i(\omega_i)$ is not continuous at $\overline{\omega}_i$, so we have $\lim_{\omega_i \to \overline{\omega}_i -} s_i(\omega_i) > s_i(\overline{\omega}_i)$. We let $\omega_i \to \overline{\omega}_i$. If $s_i(\overline{\omega}_i) = 0$, in the limit, the second IC becomes

$$0 = s_i(\overline{\omega}_i)F_i(p_i(\overline{\omega}_i)|\overline{\omega}_i) \geq \lim_{\omega_i \to \overline{\omega}_i} s_i(\omega_i)F_i(p_i(\overline{\omega}_i)|\overline{\omega}_i) > 0,$$

and a contradiction. If $s_i(\overline{\omega}_i) > 0$, in the limit, the second IC becomes

$$s_i(\overline{\omega}_i)F_i(p_i(\overline{\omega}_i)|\overline{\omega}_i) \geq \lim_{\omega_i \to \overline{\omega}_i} s_i(\omega_i)F_i(p_i(\overline{\omega}_i)|\overline{\omega}_i) \Leftrightarrow \ s_i(\overline{\omega}_i) \geq \lim_{\omega_i \to \overline{\omega}_i} s_i(\omega_i),$$

and a contradiction.

(iii) $s_i(\omega_i)$ is continuous at $\omega_i \in (\underline{\omega}_i, \overline{\omega}_i)$. For each $\omega_i \neq \omega_i' \in (\underline{\omega}_i, \overline{\omega}_i)$, the IC in (25) is satisfied. We let $\omega_i' \to \omega_i$. Then,

$$\begin{aligned} &\begin{cases} \lim_{\omega_i' \to \omega_i} s_i(\omega_i')F_i(p_i(\omega_i)|\omega_i) \geq s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i), \\ s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i) \geq \lim_{\omega_i' \to \omega_i} s_i(\omega_i')F_i(p_i(\omega_i)|\omega_i), \end{cases} \\ \Leftrightarrow\ & \lim_{\omega_i' \to \omega_i} s_i(\omega_i') \geq s_i(\omega_i) \geq \lim_{\omega_i' \to \omega_i} s_i(\omega_i'). \end{aligned}$$

(c) $s_i(\omega_i)$ is differentiable. Consider the IC in (25). For any $\omega_i' \neq \omega_i$,

$$\begin{aligned} &\begin{cases} s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i') \geq s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i'), \\ s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i) \geq s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i). \end{cases} \\ \Leftrightarrow &\begin{cases} s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i') - s_i(\omega_i)F_i(p_i(\omega_i')|\omega_i') \geq s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i') - s_i(\omega_i)F_i(p_i(\omega_i')|\omega_i'), \\ s_i(\omega_i)F_i(p_i(\omega_i)|\omega_i) - s_i(\omega_i)F_i(p_i(\omega_i')|\omega_i) \geq s_i(\omega_i')F_i(p_i(\omega_i')|\omega_i) - s_i(\omega_i)F_i(p_i(\omega_i')|\omega_i), \end{cases} \end{aligned}$$

which becomes

$$\Leftrightarrow \begin{cases} \dfrac{s_i(\omega_i') - s_i(\omega_i)}{\omega_i' - \omega_i}F_i(p_i(\omega_i')|\omega_i') \geq -\dfrac{F_i(p_i(\omega_i)|\omega_i') - F_i(p_i(\omega_i')|\omega_i')}{\omega_i - \omega_i'}s_i(\omega_i), \\ -\dfrac{F_i(p_i(\omega_i)|\omega_i) - F_i(p_i(\omega_i')|\omega_i)}{\omega_i - \omega_i'}s_i(\omega_i) \geq \dfrac{s_i(\omega_i') - s_i(\omega_i)}{\omega_i' - \omega_i}F_i(p_i(\omega_i')|\omega_i). \end{cases}$$

In the limit, we have

$$\begin{aligned} \left[\lim_{\omega_i' \to \omega_i} \frac{s_i(\omega_i') - s_i(\omega_i)}{\omega_i' - \omega_i}\right] F_i(p_i(\omega_i)|\omega_i) &\geq -s_i(\omega_i)f_i(p_i(\omega_i)|\omega_i))p_i'(\omega_i) \\ &\geq \left[\lim_{\omega_i' \to \omega_i} \frac{s_i(\omega_i') - s_i(\omega_i)}{\omega_i' - \omega_i}\right] F_i(p_i(\omega_i)|\omega_i). \end{aligned}$$

■

Proof of Theorem 1. *Part 1* The interior case. For any $\omega_i' > \omega_i$, $\dfrac{\partial u_i(\omega_i', \omega_i')}{\partial \omega_i'} = 0$, and by the reverse hazard dominance, $\phi_i(p_i(\omega_i'), \omega_i') < \phi_i(p_i(\omega_i'), \omega_i)$. Then, from Lemma 1,

$$\begin{aligned}
0 &= \frac{\partial u_i(\omega_i', \omega_i')}{\partial \omega_i'} \\
&= -s_i'(\omega_i') f_i(p_i(\omega_i')|\omega_i') \left[v(\omega_i') - \phi_i(p_i(\omega_i'), \omega_i')\right] \\
&> -s_i'(\omega_i') f_i(p_i(\omega_i')|\omega_i') \left[v(\omega_i') - \phi_i(p_i(\omega_i'), \omega_i)\right],
\end{aligned}$$

which implies $v(\omega_i') - \phi_i(p_i(\omega_i'), \omega_i) < 0$. Hence, we have

$$\frac{\partial u_i(\omega_i', \omega_i)}{\partial \omega_i'} = -s_i'(\omega_i') f_i(p_i(\omega_i')|\omega_i) \left[v(\omega_i') - \phi_i(p_i(\omega_i'), \omega_i)\right] < 0.$$

Similarly, one can show that for any $\omega_i' < \omega_i$, $\dfrac{\partial u_i(\omega_i', \omega_i)}{\partial \omega_i'} > 0$. From (17), we find $s_i(\omega_i)$ more explicitly such that

$$\frac{s_i'(\omega_i)}{s_i(\omega_i)} = \frac{p_i'(\omega_i)}{p_i(\omega_i) - v(\omega_i)}, \text{ or } [\ln s_i(\omega_i)]' = \frac{p_i'(\omega_i)}{p_i(\omega_i) - v(\omega_i)}. \tag{26}$$

Taking the integral of both sides results in

$$\int_{\omega_i}^{\overline{\omega}} [\ln s_i(x)]' dx = \int_{\omega_i}^{\overline{\omega}} \frac{p_i'(x)}{p_i(x) - v(x)} dx,$$

which can be rewritten as

$$\ln s_i(\omega_i) = \ln s_i(\overline{\omega}) - \int_{\omega_i}^{\overline{\omega}} \frac{p_i'(x)}{p_i(x) - v(x)} dx.$$

From

$$s_i(\omega_i) = s_i(\overline{\omega}) \exp\left\{\int_{\omega_i}^{\overline{\omega}} \frac{p_i'(x)}{v(x) - p_i(x)} dx\right\},$$

we can choose $(s_i(\overline{\omega}))_{i\in I}$ that satisfies

$$\sum_{i\in I} \left[\int_{\Omega} s_i(\overline{\omega}) \exp\left\{\int_{\omega_i}^{\overline{\omega}} \frac{p_i'(x)}{v(x) - p_i(x)} dx\right\} F_i(p_i(\omega_i)|\omega_i) dG_{\omega_i}(\omega_i)\right] = \overline{U} - B,$$

where $\overline{U} - B > 0$. Note that $s_i(\overline{\omega})$ *uniformly* changes the level of the auctioneer's ex ante payoff, while $s_i(\omega_i)$ for each $i \in I$ satisfies the incentive compatibility, and the result holds for any arbitrary B, smaller than $\overline{U}$.

Part 2 The non-interior case.

With the interior condition for the upper bound violated, i.e., $v(\overline{\omega}) \geq \phi_i(\overline{\theta}, \overline{\omega})$, we divide the case into two: the condition for the lower bound is satisfied, and it is not. The idea

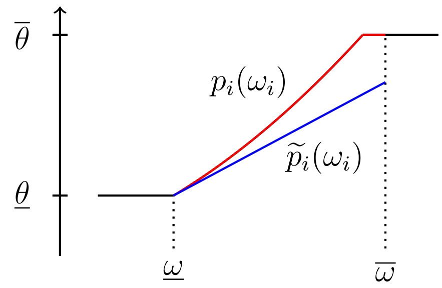

Figure 5: Non-interior case I

is simple and can be described with figures, case I in figure 5 and case II in figure 6. A transformation of $p_i(\omega_i)$ converts the previous corner points into interior ones to retain the first-order alignment. For the former, one transformation is sufficient, but for the latter, a shift to upward is necessary before the transformation, to attain the interior lower bound.

By defining $S_i \equiv [p_i^{-1}(\underline{\theta}), p_i^{-1}(\overline{\theta})]$, find that p_i is strictly increasing on S_i. Consider the case that the interior condition for the upper bound is not satisfied, i.e., $v(\overline{\omega}) \geq \phi_i(\overline{\theta}, \overline{\omega})$, and divide the proof into two parts: the condition for the lower bound is satisfied, and it is not.

Case 1 Suppose $v(\underline{\omega}) > \phi_i(\underline{\theta}, \underline{\omega})$, implying $p_i(\underline{\omega}) > \underline{\theta}$. Choose a strictly increasing function $\widetilde{p}_i(\omega_i)$ satisfying $\widetilde{p}_i(\underline{\omega}) = p_i(\underline{\omega})$, $\widetilde{p}_i(\omega_i) < p_i(\omega_i)$ for $\omega_i > \underline{\omega}$ and $\widetilde{p}_i(\overline{\omega}) < \overline{\theta}$. Then, for each $\omega_i \in [\underline{\omega}, \overline{\omega}]$, there exists $\omega_i' < \omega_i$ such that $\widetilde{p}_i(\omega_i) = p_i(\omega_i')$. Note that $\phi_i(p_i(\omega_i), \omega_i)$ is strictly increasing on S_i:

$$\frac{d\phi_i(p_i(\omega_i), \omega_i)}{d\omega_i} = \frac{\partial \phi_i}{\partial \theta_i} p_i'(\omega_i) + \frac{\partial \phi_i}{\partial \omega_i} = v'(\omega_i) > 0,$$

which implies that, by extending the definition of p_i to include those below $\underline{\omega}$, there exists $\widehat{\omega} < \underline{\omega}$ such that $\phi_i(p_i(\widehat{\omega}), \widehat{\omega}) = \underline{\theta}$. Choose any arbitrary $\omega_i \in [\underline{\omega}, \overline{\omega}]$. Then,

$$\phi_i(p_i(\widehat{\omega}), \widehat{\omega}) = \underline{\theta} < \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

On the other hand,

$$\phi_i(p_i(p_i^{-1}(\overline{\theta})), p_i^{-1}(\overline{\theta})) > \phi_i(p_i(\omega_i'), \omega_i') > \phi_i(p_i(\omega_i'), \omega_i) = \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

The first inequality follows from the monotonicity of $\phi_i(p_i(\omega_i), \omega_i)$, and the second follows from the property of ϕ_i and $\omega_i' < \omega_i$. The monotonicity, with the intermediate value theorem, entails the existence of an implicit function $\gamma_i(\omega_i) \in (\widehat{\omega}, \overline{\omega})$ such that

$$\phi_i(p_i(\gamma_i(\omega_i)), \gamma_i(\omega_i)) = \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

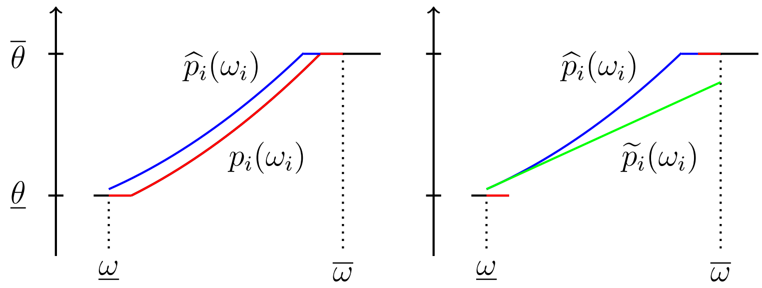

Figure 6: Non-interior case II

By choosing $s_i(\omega_i)$ such that

$$\widetilde{p}_i(\omega_i) - \frac{s_i(\omega_i)\widetilde{p}_i'(\omega_i)}{s_i'(\omega_i)} = v(\gamma_i(\omega_i)),$$

the derivative of $s_i(\omega_i)F_i(\widetilde{p}_i(\omega_i)|\omega_i)$ yields that

$$\begin{aligned}
&- s_i'(\omega_i)f_i(\widetilde{p}_i(\omega_i)|\omega_i)\left[v(\gamma_i(\omega_i)) - \phi_i(\widetilde{p}_i(\omega_i), \omega_i)\right] \\
=&- s_i'(\omega_i)f_i(\widetilde{p}_i(\omega_i)|\omega_i)\left[v(\gamma_i(\omega_i)) - \phi_i\big(p_i(\gamma_i(\omega_i)), \gamma_i(\omega_i)\big)\right].
\end{aligned}$$

Case 2 Suppose $v(\underline{\omega}) \leq \phi_i(\underline{\theta}, \underline{\omega})$. Choose $\delta > 0$ such that $v(\underline{\omega}) + \delta > \phi_i(\underline{\theta}, \underline{\omega})$. Find a threshold cost type $\widehat{p}_i(\omega_i)$ with the modification such that $v(\omega_i) + \delta = \phi_i(\widehat{p}_i(\omega_i), \omega_i)$ and $\widehat{p}_i(\underline{\omega}) > \underline{\theta}$. Choose a strictly increasing function $\widetilde{p}_i(\omega_i)$ satisfying $\widetilde{p}_i(\underline{\omega}) = \widehat{p}_i(\underline{\omega})$, $\widetilde{p}_i(\omega_i) < \widehat{p}_i(\omega_i)$ for $\omega_i > \underline{\omega}$ and $\widetilde{p}_i(\overline{\omega}) < \overline{\theta}$. Then, for each $\omega_i \in [\underline{\omega}, \overline{\omega}]$, there exists $\omega_i' < \omega_i$ such that $\widetilde{p}_i(\omega_i) = \widehat{p}_i(\omega_i')$. As with $p_i(\omega_i)$, we preserve the following property with $\widehat{p}_i(\omega_i)$:

$$\frac{d\phi_i(\widehat{p}_i(\omega_i), \omega_i)}{d\omega_i} = \frac{\partial \phi_i}{\partial \theta_i}\widehat{p}_i'(\omega_i) + \frac{\partial \phi_i}{\partial \omega_i} = v'(\omega_i) > 0.$$

This implies that there exists $\widehat{\omega} < \underline{\omega}$ such that $\phi_i(\widehat{p}_i(\widehat{\omega}), \widehat{\omega}) = \underline{\theta}$. Choose any arbitrary $\omega_i \in [\underline{\omega}, \overline{\omega}]$. Then,

$$\phi_i(\widehat{p}_i(\widehat{\omega}), \widehat{\omega}) = \underline{\theta} < \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

On the other hand,

$$\phi_i(\widehat{p}_i(\widehat{p}_i^{-1}(\overline{\theta})), \widehat{p}_i^{-1}(\overline{\theta})) > \phi_i(\widehat{p}_i(\omega_i'), \omega_i') > \phi_i(\widehat{p}_i(\omega_i'), \omega_i) = \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

The monotonicity implies the existence of an implicit function $\gamma_i(\omega_i) \in (\widehat{\omega}, \overline{\omega})$ such that

$$\phi_i(\widehat{p}_i(\gamma_i(\omega_i)), \gamma_i(\omega_i)) = \phi_i(\widetilde{p}_i(\omega_i), \omega_i).$$

By choosing $s_i(\omega_i)$ such that

$$\widetilde{p}_i(\omega_i) - \frac{s_i(\omega_i)\widetilde{p}_i'(\omega_i)}{s_i'(\omega_i)} = v(\gamma_i(\omega_i)),$$

the derivative of $s_i(\omega_i)F_i(\widetilde{p}_i(\omega_i)|\omega_i)$ yields that

$$\begin{aligned}
&- s_i'(\omega_i)f_i(\widetilde{p}_i(\omega_i)|\omega_i)\left[v(\gamma_i(\omega_i)) - \phi_i(\widetilde{p}_i(\omega_i), \omega_i)\right] \\
=& - s_i'(\omega_i)f_i(\widetilde{p}_i(\omega_i)|\omega_i)\left[v(\gamma_i(\omega_i)) - \phi_i\big(\widehat{p}_i(\gamma_i(\omega_i)), \gamma_i(\omega_i)\big)\right].
\end{aligned}$$

■

Proof of Lemma 2. First, assume that F_i satisfies the reverse hazard rate dominance. Then, $f_i(\theta_i|\omega_i')F_i(\theta_i|\omega_i) - f_i(\theta_i|\omega_i)F_i(\theta_i|\omega_i') > 0$, where the strict inequality follows from the reverse hazard rate dominance. Now, assume that it satisfied the cumulative sum ratio dominance, which can be rewritten as

$$\begin{aligned}
&F_i(\theta_i'|\omega_i')F_i(\theta_i|\omega_i) - F_i(\theta_i|\omega_i')F_i(\theta_i|\omega_i) > F_i(\theta_i'|\omega_i)F_i(\theta_i|\omega_i') - F_i(\theta_i|\omega_i')F_i(\theta_i|\omega_i), \\
\Leftrightarrow\ & \frac{F_i(\theta_i'|\omega_i') - F_i(\theta_i|\omega_i')}{\theta_i' - \theta_i}F_i(\theta_i|\omega_i) > \frac{F_i(\theta_i'|\omega_i) - F_i(\theta_i|\omega_i)}{\theta_i' - \theta_i}F_i(\theta_i|\omega_i'), \\
\Leftrightarrow\ & f_i(\theta_i|\omega_i')F_i(\theta_i|\omega_i) > f_i(\theta_i|\omega_i)F_i(\theta_i|\omega_i'),
\end{aligned}$$

where the last equivalence follows by taking the limit $\theta_i' \to \theta_i$. ■

Proof of Proposition 2. The simultaneous equations can be rewritten as

$$A_i s_i = \epsilon_i \Leftrightarrow \begin{bmatrix} F_{iH}(p_{iH}) & -F_{iH}(p_{iL}) \\ -F_{iL}(p_{iH}) & F_{iL}(p_{iL}) \end{bmatrix} \begin{bmatrix} s_{iH} \\ s_{iL} \end{bmatrix} = \begin{bmatrix} \epsilon_{iH} \\ \epsilon_{iL} \end{bmatrix}$$

Use Farkas' Lemma, by taking the transpose of the matrix above, that is, if there is no $y \in \mathbb{R}^2$, where $y^T \equiv [y_1, y_2]$ such that $A_i^T y \geq 0$ and $\epsilon_i^T y < 0$. Suppose, on the contrary, there exists such y. Since $\epsilon_i^T >> 0$, it is clear that $y_1 < 0$ or $y_2 < 0$. But, then, to satisfy $A_i^T y \geq 0$, $y_1 < 0$ and $y_2 < 0$. This results in

$$\frac{F_{iH}(p_{iH})}{F_{iL}(p_{iH})} \leq \frac{y_1}{y_2} \leq \frac{F_{iH}(p_{iL})}{F_{iL}(p_{iL})}.$$

This contradicts the local cumulative sum ratio dominance. ■

Proof of Theorem 2. Consider an arbitrary $1 < k < J$ with two bilateral Bayesian incentive compatibility conditions, one with $k-1$ and one with $k+1$.

$$\vdots$$

$$\text{BBIC for } k-1\,\&\,k \begin{cases} s_{ik-1}F_{ik-1}(p_{ik-1}) = s_{ik}F_{ik-1}(p_{ik}) + \epsilon_{ik-1}, \\ s_{ik}F_{ik}(p_{ik}) = s_{ik-1}F_{ik}(p_{ik-1}). \end{cases}$$

$$\text{BBIC for } k\,\&\,k+1 \begin{cases} s_{ik}F_{ik}(p_{ik}) = s_{ik+1}F_{ik}(p_{ik+1}) + \epsilon_{ik}, \\ s_{ik+1}F_{ik+1}(p_{ik+1}) = s_{ik}F_{ik+1}(p_{ik}). \end{cases}$$

$$\vdots$$

Denote Γ_{ik}:

$$\Gamma_{ik} \equiv \frac{1}{F_{ik+1}(p_{ik})\left[\dfrac{F_{ik}(p_{ik})}{F_{ik+1}(p_{ik})} - \dfrac{F_{ik}(p_{ik+1})}{F_{ik+1}(p_{ik+1})}\right]}.$$

Note that $\Gamma_{ik} > 0$ if and only if the local cumulative sum ratio dominance is satisfied. Then, the solution from k and $k+1$ is $s_{ik} = \Gamma_{ik}\epsilon_{ik}$ and $s_{ik+1} = \dfrac{F_{ik+1}(p_{ik})}{F_{ik+1}(p_{ik+1})}\Gamma_{ik}\epsilon_{ik}$. Hence, for any k such as $1 < k < J$, s_{ik} is derived from two different bilateral incentive compatibility conditions such that

$$s_{ik} = \frac{F_{ik}(p_{ik-1})}{F_{ik}(p_{ik})}\Gamma_{ik-1}\epsilon_{ik-1}; \text{and } s_{ik} = \Gamma_{ik}\epsilon_{ik},$$

so ϵ_{ik} and ϵ_{ik-1} must be chosen to satisfy

$$\epsilon_{ik} = \frac{F_{ik}(p_{ik-1})}{F_{ik}(p_{ik})}\frac{\Gamma_{ik-1}}{\Gamma_{ik}}\epsilon_{ik-1}.$$

Then, from

$$\Gamma_{ik}\epsilon_{ik} = \frac{F_{ik}(p_{ik-1})}{F_{ik}(p_{ik})}\Gamma_{ik-1}\epsilon_{ik-1}; \text{or } s_{ik} = \frac{F_{ik}(p_{ik-1})}{F_{ik}(p_{ik})}s_{ik-1},$$

a closed-form solution is derived:

$$s_{ik} = \prod_{j=2}^{k}\frac{F_{ij}(p_{ij-1})}{F_{ij}(p_{ij})}s_1. \tag{27}$$

Each s_{ik} for $k = 2, ..., J$ satisfies only two bilateral incentive compatibility conditions, i.e., $s_{ik}F_{ik}(p_{ik}) > s_{ik+1}F_{ik+1}(p_{ik+1})$ and $s_{ik}F_{ik}(p_{ik}) = s_{ik-1}F_{ik-1}(p_{ik-1})$. The second main result, however, establishes that it satisfies *all the other* incentive compatibility conditions as well, such that

$$s_{ik}F_{ik}(p_{ik}) \geq s_{ij}F_{ik}(p_{ij}) \text{ for all } j \neq k.$$

Last, $s \equiv (s_1, s_2, ..., s_K)$ satisfies incentive compatibility weakly, but the uniqueness can be obtained by modifying s such as, for $\delta > 1$ sufficiently close to 1,

$$s_{ik} = \delta^k \prod_{j=2}^{k}\frac{F_{ij}(p_{ij-1})}{F_{ij}(p_{ij})}s_1. \tag{28}$$

Consider first (27). Note that for all $j \geq k$,

$$\begin{aligned} s_{ij}F_{ik}(p_{ij}) - s_{ij+1}F_{ik}(p_{ij+1}) &= s_{ij}\left[F_{ik}(p_{ij}) - \frac{F_{ij+1}(p_{ij})}{F_{ij+1}(p_{ij+1})}F_{ik}(p_{ij+1})\right] \\ &= s_{ij}F_{ij+1}(p_{ij})\left[\frac{F_{ik}(p_{ij})}{F_{ij+1}(p_{ij})} - \frac{F_{ik}(p_{ij+1})}{F_{ij+1}(p_{ij+1})}\right] > 0, \end{aligned}$$

where the strict inequality follows from the local cumulative sum ratio dominance. Hence,

$$s_{ik}F_{ik}(p_{ik}) > s_{ik+1}F_{ik}(p_{ik+1}) > \cdots > s_{iK}F_{ik}(p_{iK}).$$

Similarly, for all $j \leq k$,

$$\begin{aligned} s_{ij}F_{ik}(p_{ij}) - s_{ij-1}F_{ik}(p_{ij-1}) &= s_{ij-1}\left[\frac{F_{ij}(p_{ij-1})}{F_{ij}(p_{ij})}F_{ik}(p_{ij}) - F_{ik}(p_{ij-1})\right] \\ &= s_{ij-1}\frac{F_{ik}(p_{ij})F_{ik}(p_{ij-1})}{F_{ij}(p_{ij})}\left[\frac{F_{ij}(p_{ij-1})}{F_{ik}(p_{ij-1})} - \frac{F_{ij}(p_{ij})}{F_{ik}(p_{ij})}\right]. \end{aligned}$$

The above is equal to zero only for $j = k$, and is greater than zero for all $j < k$. Hence,

$$s_{ik}F_{ik}(p_{ik}) = s_{ik-1}F_{ik}(p_{ik-1}) > \cdots > s_{i1}F_{ik}(p_{i1}).$$

To handle the equality problem, we introduce $\delta > 1$ such as (28). Then, for all $j \geq k$,

$$s_{ij}F_{ik}(p_{ij}) - s_{ij+1}F_{ik}(p_{ij+1}) = \delta^{j}s_{ij}F_{ij+1}(p_{ij})\left[\frac{F_{ik}(p_{ij})}{F_{ij+1}(p_{ij})} - \delta\frac{F_{ik}(p_{ij+1})}{F_{ij+1}(p_{ij+1})}\right] > 0,$$

for a δ sufficiently close to 1. Given $\delta > 1$, for all $j \leq k$,

$$s_{ij}F_{ik}(p_{ij}) - s_{ij-1}F_{ik}(p_{ij-1}) = \delta^{j-1}s_{ij-1}\frac{F_{ik}(p_{ij})F_{ik}(p_{ij-1})}{F_{ij}(p_{ij})}\left[\delta\frac{F_{ij}(p_{ij-1})}{F_{ik}(p_{ij-1})} - \frac{F_{ij}(p_{ij})}{F_{ik}(p_{ij})}\right] > 0.$$

■

Proof of Lemma 3. If $\theta_i > r_i$, it is clear that not participating is weakly dominant. Let $\theta_i \leq r_i$ for all $i \in I$ and $j \in \arg\max_{\{k \neq i\}} \pi_k(b_k, \omega'_k)$.

Part 1 $b_i < \theta_i$. Since π_i is a strictly decreasing function of b_i, $\pi_i(b_i, \omega'_i) > \pi_i(\theta_i, \omega'_i)$.

Case 1 Suppose $\pi_i(b_i, \omega'_i) > \pi_i(\theta_i, \omega'_i) \geq \pi_j(b_j, \omega'_j)$. Then, bidding b_i and θ_i yields either the same positive payoff to bidder i for $\pi_i(\theta_i, \omega'_i) > \pi_j(b_j, \omega'_j)$ from $\pi_i(x_i, \omega'_i) = \pi_j(b_j, \omega'_j) < \pi_i(\theta_i, \omega'_i)$ and $x_i - \theta_i > 0$, or zero payoff for $\pi_i(\theta_i, \omega'_i) = \pi_j(b_j, \omega'_j)$ from $\pi_i(x_i, \omega'_i) = \pi_j(b_j, \omega'_j) = \pi_i(\theta_i, \omega'_i)$.

Case 2 Suppose $\pi_i(b_i, \omega'_i) \geq \pi_j(b_j, \omega'_j) > \pi_i(\theta_i, \omega'_i)$. Then, bidding b_i yields a negative payoff from $\pi_i(x_i, \omega'_i) = \pi_j(b_j, \omega'_j) > \pi_i(\theta_i, \omega'_i)$ and $x_i - \theta_i < 0$. But bidding θ_i yields zero payoff.

Case 3 Suppose $\pi_j(b_j, \omega'_j) > \pi_i(b_i, \omega'_i) > \pi_i(\theta_i, \omega'_i)$. Then, both bidding b_i and bidding θ_i yield zero payoff.

Part 2 $b_i > \theta_i$. Since π_i is a strictly decreasing function of b_i, $\pi_i(b_i, \omega'_i) < \pi_i(\theta_i, \omega'_i)$.

Case 1 Suppose $\pi_i(\theta_i, \omega'_i) > \pi_i(b_i, \omega'_i) > \pi_j(b_j, \omega'_j)$. Then, both bidding b_i and bidding θ_i yield the same positive payoff $x_i - \theta_i > 0$ to bidder i from $\pi_i(x_i, \omega'_i) = \pi_j(b_j, \omega'_j) < \pi_i(\theta_i, \omega'_i)$.

Case 2 Suppose $\pi_i(\theta_i, \omega_i') > \pi_j(b_j, \omega_j') \geq \pi_i(b_i, \omega_i')$. Then, bidding b_i yields a payoff smaller than $x_i - \theta_i > 0$ since bidder i either loses or wins with a tie. But bidding θ_i yields $x_i - \theta_i > 0$ from $\pi_i(x_i, \omega_i') = \pi_j(b_j, \omega_j') < \pi_i(\theta_i, \omega_i')$.

Case 3 Suppose $\pi_j(b_j, \omega_j') \geq \pi_i(\theta_i, \omega_i') > \pi_i(b_i, \omega_i')$. Then, both bidding b_i and bidding θ_i yield zero payoff from $\pi_i(x_i, \omega_i') = \pi_j(b_j, \omega_j') = \pi_i(\theta_i, \omega_i')$.

■

Proof of Theorem 3. Using the constructions of $\widehat{S}(r, b)$ and $\widehat{s}_i(r_i)$,

$$\begin{aligned}\sum_{i \in I} \widehat{s}_i(r_i) F_i(r_i|\omega_i) &= \sum_{i \in I} s_i(p^{-1}(r_i)) F_i(r_i|\omega_i) \\ &= \sum_{i \in I} s_i(\omega_i') F_i(p_i(\omega_i')|\omega_i) \\ &= \sum_{i \in I} u_i(\omega_i', \omega_i).\end{aligned}$$

Then the result follows from Theorem 1. ■

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