Discussion Paper Series
No. 2002
April 2020

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Abstract

A principal *sells an influence* to firms that may undertake a project, through collusion with a seller of some item. For the optimal sale of influence, the principal chooses an influence level for a losing firm *above* the default level, while treating a winning firm even more favorably. Furthermore, the difference in influence between two firms does not change monotonically, and under some parameters, even the set of firm types participating in the sale is larger. The optimal mechanism highlights the *cashing-out process* of public resources by the principal, which comes at the cost of public welfare.

Keywords and Phrases: Sale of influence, collusion, optimal mechanism
JEL Classification Numbers: C73, D82

*I thank Masaki Aoyagi for helpful comments on an earlier version. Of course, all remaining errors are mine.

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1 Introduction

The means by which money is taken in exchange for a favor or influence have tended to advance in tandem with society. *Sale of influence* is indeed widely recognized as a potential threat to governance, but the authorities typically experience difficulty bringing a conviction against an influence peddler in court because often it is a third party that receives the benefit.[1]

This paper suggests a model of how influence can be sold seemingly legitimately. In the model, a principal has discretion to choose a level of influence on each firm (or a regulation rate), and a seller seeks to sell a license or *any* single-unit item that by its nature does not involve influence at all. The firms decide whether to undertake a project of which final realization hinges on the principal's influence. The principal can sell an influence through collusion with the seller. In particular, he or she can choose different influence levels depending on a firm's outcome in the sale of the item: winning, losing, or not-participating. In this way, the principal can *incorporate the value of influence into the item sale.*

We first show that, contrary to the conventional reward/penalty treatment, the principal chooses an influence level that is *higher* than the default for a losing firm as well as for a winning firm. The optimal mechanism can extract the payoff of a threshold firm type to participate, with a reserve price or an entry fee in an auction, from each participating firm, and such extraction effect, called the loss effect, dominates the typical penalty effect. Hence, to maximize the revenue from selling an influence, once a firm undertakes a project, the collusion raises the *overall* probability of success of the project above the default level. The consequence impairs social welfare. This greater overall influence highlights the *cashing-out process* of public resources by government officials, which comes at the cost of both public resources and welfare. Next, it is shown that the difference between an influence level for a winning firm and that for a losing firm does not monotonically change with changes in the threshold type value; it first increases, but after a single peak, diminishes. This shows that less discriminatory action on the part of the principal does not necessarily indicate *less* or no collusion. Last, despite a reserve price or an entry fee to optimally exclude low types, like a standard mechanism (Myerson (1981)), the mechanism's threshold can be even *lower* than the default level under some parameters. Thus, the collusion can not only increase the overall probability of success but also expand the set of types undertaking a project, which is detrimental to the public in general.[2]

In spite of overwhelming evidence (see Johnson, Kaufmann and Zoido-Lobaton (1998)) of senior bureaucrats exercising significant discretion in interpreting and enforcing laws and statutes, there is little discussion about its *overall* consequences. That is, instead of regulating a favored firm leniently, a bureaucrat can regulate other firms stringently. The key questions are whether the *distortion* in the distribution of regulation or influence implies a reduction in overall social welfare, and if so, what the mechanism is behind it. This paper aims to answer them.

The influence or regulation is an externality that firms can face. In this model, however, the principal internalizes the externality into the mechanism to maximize the payoff of the collusion. This exploitation process of public resources provides a different perspective on a related auction

[1]See "Trading in Influence" by Transparency International, and "Bribes that Benefit Third Party" by OECD (A Glossary of International Standards in Criminal Law).

[2]On the other hand, if each firm's project is legitimate, the results have implications for "efficient corruption."

design (Kirkegaard (2012) and Jehiel and Lamy (2015) among others). This paper considers an interim entry approach as in the literature on entry with costs (Samuelson (1985), Stegeman (1996) and Lu (2009)), and the difference is that unlike a fixed cost, the principal in this model manipulates each firm's consequence from entry.[3]

The next section introduces a model, and section 3 a benchmark. Section 4 contains the main results, and the last section concludes. All of the proofs are collected in an appendix.

2 Model

There are a principal, a seller and $N \geq 2$ firms, where $I \equiv \{1, ..., N\}$. Each firm $i \in I$ can undertake a project to obtain a value $v_i \in [\underline{v}, \overline{v}]$ with $\overline{v} > \underline{v} \geq 0$, but its final realization depends on the principal's action for the firm. The principal can choose a level of influence, a probability $p_i \in [0, 1]$ that firm i's project is realized. An alternative formulation for an influence level is to choose a regulation rate $1 - p_i$ so that firm i's project is *not* realized with probability $1 - p_i$. The seller seeks to sell a license or an item that has a common value to firms. The license/item or its common value is by nature *not related* to each firm's value v_i from the project.

With an influence level p_i, firm i's expected payoff from undertaking a project is $p_i v_i$, and without the principal's influence, it obtains dv_i, where $d \in [\underline{d}, \overline{d}]$ with $1 > \overline{d} > \underline{d} > 0$ denotes a default level.[4] Undertaking a project incurs a cost $c > 0$. Then, firm i's payoff $\pi(v_i, p_i)$ from its value v_i and an influence level p_i is given as

$$\pi(v_i, p_i) = p_i v_i - c. \tag{1}$$

The principal incurs a cost if he chooses firm i's success probability p_i different from d. The cost is captured by a loss function such that

$$-l(d - p_i), \tag{2}$$

which satisfies $l(0) = 0$, and it is differentiable, strictly convex having symmetry $l(a) = l(-a)$ around 0, that is, for all $a \neq 0$.[5] The common value for a license or an item is normalized as zero (no value at all). Firm i's value for a project v_i is its private information. The auction setting is symmetric: it is common knowledge that firms' valuations are identically and independently drawn from a differentiable cumulative distribution F.

The game's timeline after Nature chooses a type v_i for each firm is shown in Figure 1. We assume that that each firm i undertakes a project if indifferent and obtains 0 – a normalized outside option – if it does not undertake a project.

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[3]In the ex ante entry case, the optimal mechanism extracts all the surplus of each firm with no information rents for high types, so without any contrasting aspects from the bid effect and the loss effect shown in the main analysis, it can be readily demonstrated, following from McAfee and McMillan (1987), that the principal chooses an influence level for a losing firm higher than a default level as well as for a winning firm.

[4]A default level reflects how well the governmental system functions, and thus dv_i is what firm i expects to obtain in the pre-existing system without any intervention by the principal. Similarly, a default regulation level reflects how frequently the system checks unlawful activities or wrongdoings or how many loopholes the system embodies.

[5]One example is a quadratic loss function $l(x) = ax^2$ for $a > 0$.

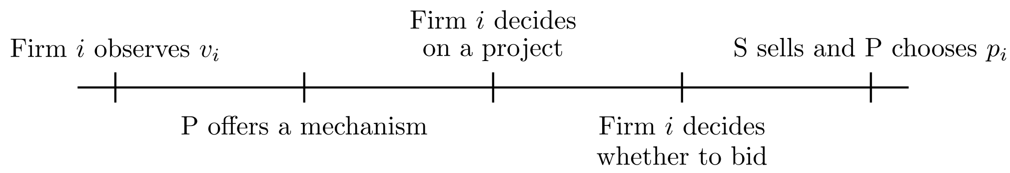

Figure 1: Timeline

3 Benchmark

If the principal and the seller are *separate* players, from the model in Section 2, there is, evidently, a unique equilibrium such that the principal chooses d for all firms, and both the principal and the seller obtain zero payoff. Firm i's payoff is dv_i, so only those firms with values $v_i \geq k^0 = \frac{c}{d}$ undertake a project, which is summarized below.

Proposition 1 *In the benchmark, the principal chooses $p_j = d$ for all j, and a firm with a value $v_i \geq k^0$ undertakes a project, where the threshold is given as $k^0 = \frac{c}{d}$.*

4 Influence sale

Now, the principal and the seller collude to maximize the sum of their payoffs. The collusion may yield the principal an incentive to choose an influence level different from the default level d depending on a firm's observable behavior, in particular, whether firm i participates in the sale of a license/item or not, and whether firm i wins it or not.[6] That is, the principal can *sell* an influence through a mechanism. We explore such possibility in this section. We denote by p_w an influence level for a winning firm, p_l a level for a losing firm, and p_n a level for a non-participating firm. Then, the principal's mechanism consists of a collection $p = (p_w, p_l, p_n) \in [0,1]^3$ and an auction mechanism Γ with functions q, t_i for all $i \in I$ such that

$$q : [\underline{v}, \overline{v}]^N \to \Delta \text{ and } t_i : [\underline{v}, \overline{v}] \to \mathbb{R}, \tag{3}$$

where $\Delta \equiv \{(q_1, ..., q_N) : q_i \in \mathbb{R}_+ \text{ and } \sum_{i \in I} q_i \leq 1\}$. In a direct mechanism, each firm i is asked to report its value v_i, and the seller commits to transferring a license/item to the buyer with probability $q_i(v)$ with $v = (v_1, ..., v_N)$ and the principal choosing a collection of influence levels p.[7] Each firm i must pay $t_i(v_i)$ if it participates in the influence sale.

Suppose that firm i undertakes a project and participates in the sale. Then, given a report v_i, denote by $Q_i(v_i)$ firm i's *expected* winning probability, and by $T_i(v_i)$ its expected payment. If the firm reports its type truthfully, its expected payoff from (1) is

$$\begin{aligned} u_i(v_i, p) &= \pi(v_i, p_w)Q_i(v_i) + \pi(v_i, p_l)(1 - Q_i(v_i)) - T_i(v_i) \\ &= \Delta p[v_i Q_i(v_i)] - T_i(v_i) + \pi(v_i, p_l), \end{aligned} \tag{4}$$

[6] Henceforth, the principal controls both his decision and the seller's so that the principal maximizes the sum of their payoffs.

[7] That is, as in most mechanism design problems, the principal *commits* to a collection of influence levels p. Such p can be alternatively formulated as a steady state equilibrium level in an infinitely repeated game between a long-run player, the principal, and a sequence of short-run players, different sets of firms, in each period.

where $\Delta p \equiv p_w - p_l$. On the other hand, if firm i undertakes a project but does not participate in the sale, the firm obtains $\pi(v_i, p_n)$. This model has three distinct features; it is otherwise a standard mechanism design problem. First, changes in p affect each firm i's expected value $\Delta p v_i$ from winning a license or an item. Hence, $\Delta p v_i$ can be interpreted as firm i's modified value in an auction setting, which can be positively related with the firm's bid in its implementation; for example, in a second price auction, it is indeed firm i's bid. Second, changes in p_n make each firm face a different outside option $\pi(v_i, p_n)$ from not participating in the sale. Last, the model has two separate participation time lines: investment and sale. In particular, firm i participates in the sale if $u_i(v_i, p) \geq \pi(v_i, p_n)$, and given the participation, firm i undertakes a project if $u_i(v_i, p) \geq 0$.

Yet, the main analysis can be simplified with two findings. It is immediate that for any $\Delta p \leq 0$, the maximum revenue is zero, identical to the benchmark. Furthermore, if firm i undertakes a project but does not participate in the sale, the collusion cannot collect revenue from it by selling an influence. However, if the principal chooses a sufficiently small p_n, any type of firm undertaking a project will participate in the sale, which in turn yields a set of participating types as an interval $[k, \overline{v}]$. We focus on such a collusive mechanism with $\Delta p > 0$ in what follows.

Lemma 1 *If the principal chooses a sufficiently small p_n such that for each $i \in I$ and every $v_i \in [\underline{v}, \overline{v}]$, then $u_i(v_i, p) \geq 0$ implies $u_i(v_i, p) \geq \pi(v_i, p_n)$. Moreover, for any mechanism satisfying incentive compatibility, a set of firm types participating in the sale is given as an interval $[k, \overline{v}]$ for $k \in [\underline{v}, \overline{v}]$.*

Hence, a collusive mechanism is said to be incentive compatible if for each $i \in I$ and every $v_i, v_i' \in [k, \overline{v}]$,

$$u_i(v_i, p) \geq \Delta p[v_i Q_i(v_i')] - T_i(v_i') + \pi(v_i, p_l), \tag{5}$$

and a collusive mechanism is individually rational if for each $i \in I$ and every $v_i \in [k, \overline{v}]$,

$$u_i(v_i, p) \geq 0. \tag{6}$$

For any revenue maximizing mechanism, the principal chooses a transfer rule such that $u_i(k, p) = 0$. The seller's revenue is

$$R(\Gamma, p) = \sum_{i \in I} \mathbb{E}[T_i(v_i)], \tag{7}$$

and the principal's payoff from the collusion combining (2) and (7) is

$$V(\Gamma, p) = R(\Gamma, p) - l(d - p_w) - (N-1)l(d - p_l). \tag{8}$$

We assume the standard regularity condition that the virtual valuation $\psi(v_i)$ is strictly increasing, where $\psi(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)}$, which is maintained throughout the analysis. Denote by $\widehat{v}$ the standard reserve price, without collusion, such that $\psi(\widehat{v}) = 0$. The optimal mechanism assigns an influence to a firm with the highest value, as in a typical case, but may exclude low values considering a firm's payoff when it loses as well as when it wins. In particular, the principal assigns it a positive probability only when $v_i > r$, where r is always greater than $\widehat{v}$.

Proposition 2 *For any p satisfying $\Delta p > 0$, the optimal mechanism is given by: for each $i \in I$ and every $(v_1, ..., v_N) \in [\underline{v}, \overline{v}]^N$,*

$$q_i(v) = \begin{cases} 1 & \textit{if } v_i > r \textit{ and } v_i > v_j \textit{ for all} j \neq i, \\ 0 & \textit{otherwise}, \end{cases} \tag{9}$$

where $r \in (\widehat{v}, \overline{v}]$ satisfies

$$r = \begin{cases} x \ s.t. \ [(p_w G(x) + p_l(1 - G(x))]\psi(x) = c & \textit{if } c < p_w \psi(\overline{v}), \\ \overline{v} & \textit{if } c \geq p_w \psi(\overline{v}). \end{cases}$$

The optimal revenue is $R^*(p) = N\Delta p\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big] + N(1 - F(r))\pi(r, p_l)$.

The optimal revenue $R^*(p)$ is based on r, where Δpr is the optimal reserve price in an auction setting for the optimal mechanism, which in turn depends on p. In particular, to maximize $\Delta p\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big] + (1 - F(r))\pi(r, p_l)$, an interior $r < \overline{v}$ satisfies

$$[(p_w G(r) + p_l(1 - G(r))]\psi(r) = c. \tag{10}$$

For the optimal mechanism, each firm's ex ante payment is $\Delta p\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big] + (1 - F(r))\pi(r, p_l)$, where the principal utilizes the public resource p to make the firm transfer its payoff to the principal. This exploitation process of public resource can be demonstrated effectively with its implementation. For instance, the principal can implement the optimal mechanism with a second price auction.[8] We denote X by an arbitrary firm i's value, since the auction model is symmetric, and $Y \equiv \max_{j \neq i}\{v_j\}$. It can be readily shown that each firm bidding its own value is weakly dominant, that is, $b_i = \pi(v_i, p_1) - \pi(v_i, p_0) = \Delta p v_i$, which can lead to the optimal allocation rule $q_i(v)$ in Proposition 2. Consider the following payment rule for $v_i \geq r$ such that for each $i \in I$ and every bid profile (b_i, b_{-i}),

$$t_i = \begin{cases} \pi(r, p_l) & \text{if } \max_{j \neq i}\{b_j\} > b_i, \\ \Delta pr + \pi(r, p_l) & \text{if } b_i \geq r > \max_{j \neq i}\{b_j\}, \\ \Delta p \max_{j \neq i}\{b_j\} + \pi(r, p_l) & \text{if } b_i > \max_{j \neq i}\{b_j\} \geq r. \end{cases}$$

This payment rule can be modified, more practically, as the standard second price auction payment combined with a reserve price Δpr and an entry fee $\pi(r, p_l)$ or the standard second price auction payment with only an entry fee $e = \Delta p[rG(r)] + \pi(r, p_l) = G(r)\pi(r, p_w) + (1 - G(r))\pi(r, p_l)$. Each of them yields a firm's payment if $x \geq r$ such that

$$\begin{aligned} m(x, r) &= \Pr(Y < r)\Delta pr + \Pr(r \leq Y < x)\mathbb{E}[\Delta pY \mid r \leq Y < x] + \pi(r, p_l) \\ &= \Delta p\left[rG(r) + \int_r^x yg(y)dy\right] + \pi(r, p_l), \end{aligned}$$

and its ex ante payment such that

$$\mathbb{E}[m(X, r)] = \Delta p\left[r(1 - F(r))G(r) + \int_r^{\overline{v}} y(1 - F(y)g(y)dy\right] + (1 - F(r))\pi(r, p_l).$$

This can be rewritten as $\Delta p\left[r(1 - F(r))G(r) + \int_r^{\overline{v}} y(1 - F(y)g(y)dy\right] = \Delta p\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big]$, so $\Delta p\left[r(1 - F(r))G(r) + \int_r^{\overline{v}} y(1 - F(y)g(y)dy\right] + (1 - F(r))\pi(r, p_l)$ is identical to the optimal revenue from each firm in Proposition 2. The last remark from the proposition is that the optimal

[8] See Krishna (2002) for the standard procedure.

mechanism yields type x the expected payoff $u(x,p) = \Delta p \int_r^x G(y)dy + \pi(x,p_l) - \pi(r,p_l)$, and despite zero payoff for the lowest type r, this type still participates in the auction due to the dire consequence from not participating $\pi(x,p_n)$.

The total effect of influence p on the optimal revenue $R^*(p)$ can be decomposed into three components. The direct effect of p on the expected value $\Delta p v_i$ is called the bid effect; the indirect effect through r, for $r \in (\underline{v}, \overline{v})$, on the set of values participating in the mechanism is the participation effect; and the change in a firm's payoff $\pi(v_i, p_l)$ when it loses is the loss effect. The reason for discriminatory choices from the direct effect is straightforward, confirming our intuition; the bid effect of influence for a winning firm is positive, but that of influence for a losing firm is negative. Yet, the participation effect for both firms is positive; for all $j \in \{w,l\}$, as p_i increases, the optimal r decreases, so $R^*(p)$ increases. The loss effect, only depending on p_l, is positive: $R^*(p)$ increases as p_l increases. We have a positive loss effect because the principal can extract the threshold value payoff when it loses, $\pi(r,p_l)$, from *all* participating firms. In other words, the principal can extract a payment from the participants, for example, with a reserve price or an entry fee, to the extent that it "compensates" them. This highlights the cashing-out process of public resource by the principal.

The optimal choice of r makes the participation effect disappear ultimately, by the envelope theorem, which leads to the following comparative statics. For $r \in (\underline{v}, \overline{v})$, changes in the optimal revenue $R^*(p)$ with respect to changes in p_w and p_l are, respectively, given as

$$\begin{aligned} \frac{\partial R^*(p)}{\partial p_w} &= N\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big], \\ \frac{\partial R^*(p)}{\partial p_l} &= -N\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big] + N(1-F(r))r. \end{aligned} \tag{11}$$

Unlike an influence level for a winning firm, that for a losing firm consists of two *contrasting* effects, the bid effect and the loss effect. By incorporating the optimal revenue $R^*(p)$ into the sum of payoffs in (8), the principal chooses (p_w, p_l) to maximize $V^*(p)$ such that

$$\max_{(p_w,p_l)\in[0,1]^2} V^*(p) = R^*(p) - l(d-p_w) - (N-1)l(d-p_l). \tag{12}$$

The first main result shows that, despite the contrasting effects of influence for a losing firm, interestingly, its overall effect turns out to always be positive; that is, for the optimal influence for a losing firm, the loss effect dominates the bid effect, with the principal extracting $\pi(r,p_l)$ for any participating firm. This finding, by examining only the necessary condition of the maximization problem above, leads to the subsequent result that the principal chooses an influence level for a losing firm that is higher than the default level d as well for $r < \overline{v}$.

Proposition 3 *There exists a solution to the problem in (12). Suppose $r < \overline{v}$. For any solution,*

$$p_w > d \text{ and } p_l > d.$$

For the sufficiency of the maximization problem in (12), note that l is strictly convex, but $R^*(p)$ might not be concave because the optimal r is strictly decreasing in p_j for all $j \in \{w,l\}$, that is, $\frac{\partial r}{\partial p_j} < 0$ for all j. Yet, for a sufficiently strong convexity of l, we can guarantee the condition for

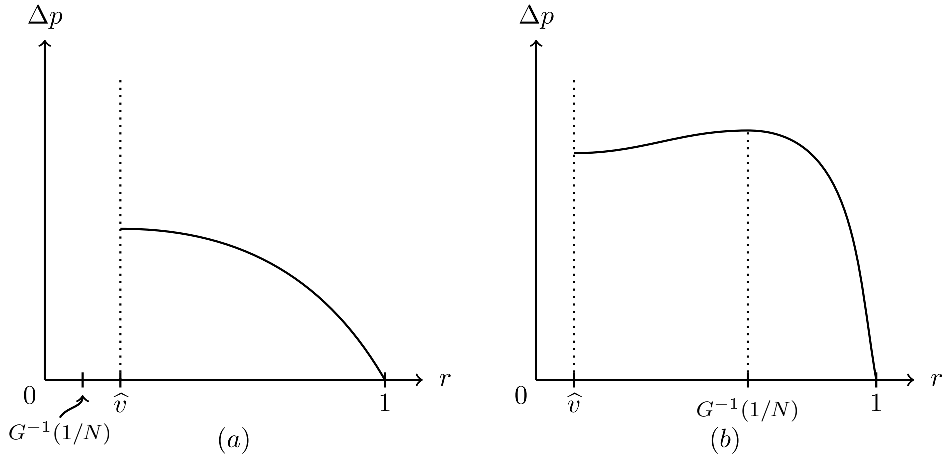

Figure 2: Δp for $N = 2$ (a) and Δp for $N = 10$ (b) when $U = [0, 1]$ with $l(x) = x^2$

a local max, which we maintain below.[9] We next turn to the question whether $\Delta p > 0$ for all interior solutions and how it changes as r changes. Since l is strictly convex and symmetric, for each $j \in \{w, l\}$, we have $-l'(d - p_j) = l'(p_j - d)$, and an inverse function of l' exists. Denote $h(x) \equiv l'^{-1}(x)$. Then, an interior p_w and p_l can be derived as

$$\begin{aligned} p_w &= d + h\Big(N\int_r^{\overline{v}} G(x)\psi(x)f(x)dx\Big), \\ p_l &= d + h\Big(-\frac{N}{N-1}\int_r^{\overline{v}} G(x)\psi(x)f(x)dx + \frac{N}{N-1}(1 - F(r))r\Big). \end{aligned} \tag{13}$$

By the mean value theorem, for some z, the difference is

$$\Delta p = h'(z)\frac{N}{2(N-1)}\Big(N\int_r^{\overline{v}} G(x)\psi(x)f(x)dx - (1 - F(r))r\Big), \tag{14}$$

where the strict convexity of l implies $h'(z) > 0$. The derivative of the term inside the brackets is

$$-NG(r)\psi(r)f(r) - [(1 - F(r))r]' = [NG(r) - 1][(1 - F(r))r]'. \tag{15}$$

If we have $G(r)$ instead of $NG(r)$ in (15), the sign is negative, which essentially results in $\frac{\partial R^*(p)}{\partial p_l} > 0$ in (11) and $p_l > d$ in Proposition 3. However, with $NG(r)$ for $N \geq 2$, if r is sufficiently close to 1, the sign is negative, whereas if r is sufficiently small, it is positive. For any distribution F with $[\underline{v}, \overline{v}]$, we show that for $r \in (\widehat{v}, \overline{v})$, as r increases, the gap Δp first starts expanding, but after a single peak, it diminishes.

Proposition 4 *For any $r \in (\widehat{v}, \overline{v})$ and an interior p, Δp satisfies the following properties.*

(i) Δp is strictly increasing in r if $r < G^{-1}(1/N)$ but strictly decreasing in r if $r > G^{-1}(1/N)$.

(ii) $\Delta p > 0$.

[9]In the Hessian matrix for the maximization problem in (12), for example, $\frac{\partial^2 V^*(p)}{\partial p_w^2} = -NG(r)\psi(r)f(r)\frac{\partial r}{\partial p_w} - l''(d - p_w)$, where the first term is positive because of negative $\frac{\partial r}{\partial p_w}$.

For the relatively low r, an additional exclusion of lower values must come with broadening of the difference between an influence level for a winning firm and that for a losing firm, but for a high r, it must come with narrowing of the optimal influence gap between them, which is illustrated in figure 2.

So far, we examine the case $r < \overline{v}$, but if there is no such a solution, the earlier outcomes will not arise. Hence, it is critical to find conditions, especially, parameter values (c, d), under which we obtain $r < \overline{v}$. To find a *feasible set* of (c, d) that generates $r < \overline{v}$, we substitute the solution (p_w, p_l) in (13) to the optimal reserve price formula for $r < \overline{v}$ in (10), which yields

$$c = \psi(r)d + D(r)\psi(r), \tag{16}$$

where we denote by $D(r)$ the convex combination of how p_w and p_l depart from the default level d such that

$$D(r) \equiv G(r)h\Big(N\int_r^{\overline{v}} G(x)\psi(x)f(x)dx\Big)+(1-G(r))h\Big(-\frac{N}{N-1}\int_r^{\overline{v}} G(x)\psi(x)f(x)dx+\frac{N}{N-1}(1-F(r))r\Big).$$

For each fixed $d \in [\underline{d}, \overline{d}]$, we find how $\psi(r)d + D(r)\psi(r)$ changes on $(\widehat{v}, \overline{v})$, and thereby, identify a feasible interval of c that generates $r \in (\widehat{v}, \overline{v})$ and furthermore a feasible set of (c, d) that generates $r \in (\widehat{v}, \overline{v})$.[10] The second result of Proposition 5 shows that although the principal chooses the optimal reserve price to exclude low values, it could still happen that the level of exclusion is *lower* than the benchmark case, that is, $r < k^0$, where k^0 is from Proposition 1 in the benchmark. In addition, there even exists a parameter value (c, d) such that the principal incentivizes some high firm values to undertake projects while under the benchmark, even the highest value does not.

Proposition 5 *For each $d \in (0, 1)$, there exists a feasible interval of c that generates $r \in (\widehat{v}, \overline{v})$. Furthermore, $\psi(r)d + D(r)\psi(r)$ satisfies properties as follows.*

(i) $\psi(r)d + D(r)\psi(r)$ is strictly increasing at $r = \widehat{v}$, and strictly decreasing at $r = \overline{v}$ for a sufficiently small $d > 0$.

(ii) There exists $d_1 > 0$ such that for any $d < d_1$, there exists c such that $r < k^0$.

(iii) There exists $d_2 \in (0, d_1)$ such that for any $d < d_2$, there exists c such that $r < \overline{v} < k^0$.

With $\psi(r)d + D(r)\psi(r)$, we can identify conditions under which the greater overall influence arises. For each d, there exists a corresponding interval of c such that $p_w > d$ and $p_l > d$. In particular, if d is lower than d_2, for intermediate values of c, the principal incentivizes some high firm values to undertake projects while under the benchmark, none of them undertakes.

Example Let $N = 2$ and a uniform distribution $U = [0, 1]$ with $l(x) = ax^2$ for $a > 0$. Then, $\psi(r)d + D(r)\psi(r)$ is given as

$$\psi(r)d + D(r)\psi(r) = (2k-1)d + \frac{1}{a}\Big[\Big(\int_k^{\overline{v}} x(2x-1)dx\Big)(2k-1) + (1-k)^2k\Big](2k-1),$$

and it can be shown that $d_1 = \frac{1}{a}$ an $d_2 = \frac{1}{2a}$. This example's equilibrium characterization is illustrated in figure 3.

[10] This could include a corner case, $p_w = 1$ or $p_l = 1$. For instance, it arises for d sufficiently close to 1 and for a sufficiently small c.

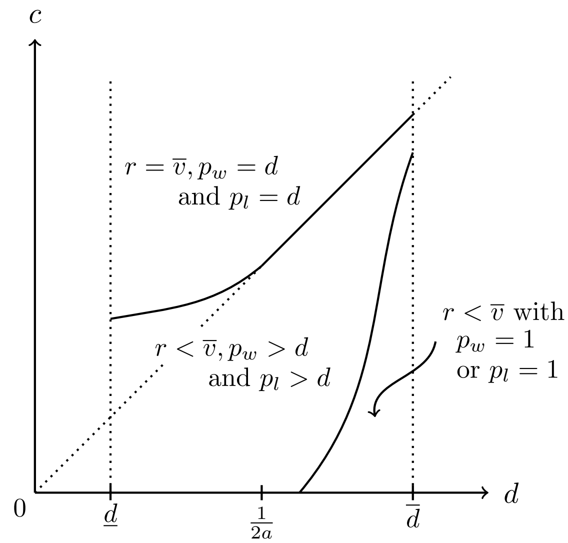

Figure 3: An equilibrium characterization

5 Concluding Remarks

This paper shows how the principal's discretion when selling an influence can impair social welfare. To maximize the revenue, he makes not only the overall probability of success higher but the set of types undertaking a project larger.

Appendix: Proofs

Proof of Lemma 1. Choose p_n such that $\pi(\overline{v}, p_n) \leq 0$. Then, if $u_i(v_i, p) \geq 0$, $u_i(v_i, p) \geq 0 \geq \pi(\overline{v}, p_n) \geq \pi(v_i, p_n)$ for all v_i, so $u_i(v_i, p) \geq 0$ implies $u_i(v_i, p) \geq \pi(v_i, p_n)$. For $\Delta p > 0$, if a direct mechanism is incentive compatible, then by the envelope theorem, $u_i(v_i, p)$ is increasing, so a set of types participating in the sale is given as an $[k, \overline{v}]$ for $k \in [\underline{v}, \overline{v}]$. ■

Proof of Proposition 2. Denote by v_{-i} vectors of values except for firm i's value and $f_{-i}(v_{-i})$ the probability density of v_{-i}. Then, $Q_i(v_i)$ and $T_i(v_i)$ are given as

$$Q_i(v_i) = \int_{[\underline{v},\overline{v}]^{N-1}} q_i(v_i, v_{-i}) f_{-i}(v_{-i}) dv_{-i} \text{ and } T_i(v_i) = \int_{[\underline{v},\overline{v}]^{N-1}} t_i(v_i, v_{-i}) f_{-i}(v_{-i}) dv_{-i}.$$

By the envelope theorem approach, we have $u_i(v_i, p) - u_i(k, p) = \Delta p \int_k^{v_i} Q_i(x)dx + \pi(v_i, p_l) - \pi(k, p_l)$, and from $u_i(k, p) = 0$, a type v_i's expected payment is $T_i(v_i) = \Delta p \left[v_i Q_i(v_i) - \int_k^{v_i} Q_i(x)dx\right] + \pi(k, p_l)$. Then, the expected revenue from firm i is $\mathbb{E}[T_i(v_i)] = \Delta p \Big[\int_k^{\overline{v}} Q_i(y)\psi(y)f(y)dy\Big] + (1 - F(k))\pi(k, p_l)$. By the monotone virtual valuation $\psi(v_i)$, the optimal q is given by (9), so $Q_i(y) =$

$G(y)$ for all i. To find the optimal r that maximizes $\mathbb{E}[T_i(v_i)]$, consider

$$\begin{aligned}\frac{\partial \mathbb{E}[T_i(v_i)]}{\partial k} &= -\Delta p G(k)\psi(k)f(k) + (1-F(k))p_l - f(k)\pi(k,p_l) \\ &= -f(k)\left[\Delta p G(k)\psi(k) - \frac{1-F(k)}{f(k)}p_l + p_l k - c\right] \\ &= -f(k)\left\{[(p_w G(k) + p_l(1-G(k))]\psi(k) - c\right\}.\end{aligned}$$

For any $\Delta p > 0$, if $k \leq \widehat{v}$, the partial derivative's sign is positive, which shows that $k > \widehat{v}$. Then, the optimal interior $r < \overline{v}$ and $r = \overline{v}$ in the proposition can be derived. ■

Proof of Proposition 3. There exists a solution because of the continuity of $V^*(p)$ and the compactness of $[0,1]^2$.

Part 1. Show that for any $r < \overline{v}$,

$$\frac{\partial R^*(p)}{\partial p_l} = -N\Big[\int_r^{\overline{v}} G(y)\psi(y)f(y)dy\Big] + N(1-F(r))r > 0.$$

The derivative of $\frac{\partial R^*(p)}{\partial p_l}$ with respect to r yields

$$NG(r)\psi(r)f(r) + N[(1-F(r))r]' = N[1-G(r)][(1-F(r))r]' < 0 \text{ for } r > \widehat{v}$$

where note $\psi(r)f(r) = -[(1-F(r))r]'$, and in addition, $\frac{\partial R^*(p)}{\partial p_l} = 0$ for $r = \overline{v}$. The negative sign above implies that $\frac{\partial R^*(p)}{\partial p_l}$ is positive for all $r < \overline{v}$.

Part 2. Show that $p_l > d$ and $p_l > d$ for $r < \overline{v}$. Suppose, on the contrary, that for either one, $p_i \leq d$. Then, from Part 1, $\frac{\partial R^*(p)}{\partial p_l} > 0$ as well as $\frac{\partial R^*(p)}{\partial p_w} > 0$, so $\frac{\partial V^*(p)}{\partial p_j} = \frac{\partial R^*(p)}{\partial p_j} + l'(d-p_j) > 0$, which yields a contradiction. ■

Proof of Proposition 4. *(i)* If $G^{-1}(1/N) \leq \widehat{v}$, Δp is strictly decreasing for all $r \in (\widehat{v}, \overline{v})$. If $G^{-1}(1/N) > \widehat{v}$, it is strictly increasing in r if $r < G^{-1}(1/N)$ but strictly decreasing in r if $r > G^{-1}(1/N)$.

(ii) Divide the proof into two cases. *Case 1.* If $G^{-1}(1/N) \leq \widehat{v}$, Δp is strictly decreasing for all $r \in (\widehat{v}, \overline{v})$ and $\Delta p = 0$ for $r = \overline{v}$. Hence, $\Delta p > 0$ for all $r \in (\widehat{v}, \overline{v})$. *Case 2.* Now, suppose $G^{-1}(1/N) > \widehat{v}$. The term inside the brackets in (14) can be rewritten as

$$N\int_r^{\overline{v}} G(x)\psi(x)f(x)dx - (1-F(r))r = N\int_r^{\overline{v}} G(x)\big(xf(x) - 1 + F(x)\big)dx - (1-F(r))r.$$

For any F and the corresponding optimal r from (10), there exists a sufficiently large m such that

$$\begin{aligned}&N\int_r^{\overline{v}} G(x)\big(xf(x) - 1 + F(x)\big)dx - (1-F(r))r \\ &= N\int_r^{\overline{v}} G(x)\big(xf(x) + F(x) - 1\big)dx + F(r)r - r \\ &> N\int_r^{\overline{v}} \left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{m(N-1)}\left[m\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{m-1}\frac{1}{\overline{v}-\underline{v}} + \left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^m - 1\right]dx + r\left(\frac{r-\underline{v}}{\overline{v}-\underline{v}}\right)^m - r.\end{aligned} \tag{17}$$

Note that, in particular, $m\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{m-1}$ is not monotonic; it increases first but eventually decreases in m. The integral term can be rewritten as

$$\begin{aligned}
&\int_r^{\overline{v}}\left((m+1)\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN}+\frac{m\underline{v}}{\overline{v}-\underline{v}}\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN-1}-\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{m(N-1)}\right)dx\\
&=\frac{(m+1)(\overline{v}-\underline{v})}{(mN+1)}+\frac{\underline{v}}{N}-\frac{(\overline{v}-\underline{v})}{m(N-1)+1}\\
&\quad-\left[\frac{(m+1)(\overline{v}-\underline{v})}{(mN+1)}\left(\frac{r-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN+1}+\frac{\underline{v}}{N}\left(\frac{r-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN}-\frac{(\overline{v}-\underline{v})}{m(N-1)+1}\left(\frac{r-\underline{v}}{\overline{v}-\underline{v}}\right)^{m(N-1)+1}\right].
\end{aligned}$$

Hence, in the limit, we have

$$\begin{aligned}
&\lim_{m\to\infty}N\int_r^{\overline{v}}\left((m+1)\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN}+\frac{m\underline{v}}{\overline{v}-\underline{v}}\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{mN-1}-\left(\frac{x-\underline{v}}{\overline{v}-\underline{v}}\right)^{m(N-1)}\right)dx+r\left(\frac{r-\underline{v}}{\overline{v}-\underline{v}}\right)^{m}-r\\
&=N\left[\frac{\overline{v}-\underline{v}}{N}+\frac{\underline{v}}{N}\right]-r=\overline{v}-r>0,
\end{aligned}$$

which implies that given each $r\in[\widehat{v},G^{-1}(1/N)]$ for a sufficiently large m, the sign of (17) is positive. ■

Proof of Proposition 5. *(i)* The derivative of $\psi(r)d+D(r)\psi(r)$ yields

$$\frac{d[\psi(r)d+D(r)\psi(r)]}{dr}=\psi'(r)d+D'(r)\psi(r)+D(r)\psi'(r).$$

If $r=\widehat{v}$, $\psi(\widehat{v})=0$, so $\frac{d[\psi(r)d+D(r)\psi(r)]}{dr}=\psi'(\widehat{v})d+D(\widehat{v})\psi'(\widehat{v})>0$. On the other hand, if $r=\overline{v}$, $D(\overline{v})=0$, and $D'(\overline{v})=h'(0)[-\psi(\overline{v})f(\overline{v})]<0$, so $\frac{d[\psi(r)d+D(r)\psi(r)]}{dr}=\psi'(\overline{v})d+h'(0)[-\psi(\overline{v})f(\overline{v})]\psi(\overline{v})$. Hence, if $d>0$ is sufficiently small, the derivative's sign is negative.

(ii) Show $r<k^0=\frac{c}{d}\Leftrightarrow dr<c$, which can be rewritten as $\psi(r)d+D(r)\psi(r)-dr>0$. The derivative at $r=\overline{v}$ is

$$\psi'(\overline{v})d+h'(0)[-\psi(\overline{v})f(\overline{v})]\psi(\overline{v})-d=h'(0)[-\psi(\overline{v})f(\overline{v})]\psi(\overline{v})+d(\psi'(\overline{v})-1),$$

which shows the result.

(iii) Show $\overline{v}<k^0\Leftrightarrow d\overline{v}<c$, which can be rewritten as $\psi(r)d+D(r)\psi(r)-d\overline{v}>0$. Note that if $r=\overline{v}$, $\psi(r)d+D(r)\psi(r)=d\overline{v}$, so $\psi(r)d+D(r)\psi(r)-d\overline{v}>0$ can be shown if $\psi(r)d+D(r)\psi(r)$ is strictly decreasing at $r=\overline{v}$ for a sufficiently small d, already proved in Part (i). ■

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