"Capital and Interest in Horizontal Innovation Models"

Man-Seop Park

Discussion Paper No. 07-28 (October 2007)
Capital and Interest in Horizontal Innovation Models

Abstract
A typical horizontal innovation model has three sequentially connected sectors. I argue that this structure of the model necessitates the compounding of interest on an input that goes through multiple production periods before the final good is produced. This aspect is missed (or deliberately assumed away) in typical horizontal innovation models and I contend that this practice generates internal inconsistency in relation to the long run nature of the models. I propose the correction which will restore internal consistency and discuss its implications. Though discussion is carried out in particular reference to Barro and Sala-i-Martin’s “lab-equipment” model, implications are general. (JEL O41)

Man-Seop Park
manseop@korea.ac.kr
Department of Economics, Korea University
5-1 Anam-dong, Seongbuk-gu, Seoul 136-701, Korea

* In polishing up the paper I benefited from Prof. Charles Jones, who drew my attention to his measure of “financial wealth” while commenting on a companion paper of the present one. Comments by Profs. Heinz Kurz, Neri Salvadori and Ian Steedman were conducive to improving earlier versions of the paper. Sincere thanks sans implication are due to them. This work was supported by the Korea Research Foundation Grant, funded by the Korean Government (MOEHRD) (KRF-2005-041-B00091).
The horizontal innovation modeling, initiated by Romer (1990), has become a staple of the recent approach to economic growth and development. Models proliferate, but they have a common structure. The model economy consists of three, sequentially connected, sectors: the first sector produces various “designs”; the second sector uses these “designs” to produce various intermediate goods; the third sector in turn uses intermediate goods to produce the final good.

Such an economy is bound to get implicated with the problem of value in two ways. First, as there are more than one kind of good in the economy (various designs, various intermediate goods, the final good), the problem of measurement arises regarding the quantities of these heterogeneous goods in terms of a common standard, so that they can be compared and, when necessary, added or deducted. The values of heterogeneous goods are their quantities made comparable. Second, the problem of interest should become more salient, for production here involves three different and sequentially connected sectors. If the final good is produced by means of an input (an intermediate good) which itself has been produced by means of an input (a design) in a previous stage of production, and if the latter input has also been produced by means of an input (the foregone final good) in a stage of production further back, this last input has gone through three stages before reaching the final good. The value of the final good output must reflect the three stages which that input has gone through, or, in other words, the length of time which has elapsed over those three stages. Because each stage of production is defined in reference to the same given length of time called “production period,” a multiplicity of production stages that an input goes through is equivalent to a multiplicity of production periods during which that input stays in production. The economic

---

1 Witness its appearance in recent major textbooks dealing with economic growth, such as Barro and Sala-i-Martin (2004), Heijdra and Van der Ploeg (2002) and Jones (2002). Gancia and Zilibotti (2005) in their survey of the literature on horizontal innovation, identify three variants of horizontal innovation models. The “baseline model” has Romer (1990) or Jones (2002, Ch. 5; 2005) as examples. Barro and Sala-i-Martin (2004, Ch. 6), along with Rivera-Batiz and Romer (1991), is an example of the “lab-equipment model,” whereas examples of the “labor-for-intermediates model” include Grossman and Helpman (1991, Ch. 3), Young (1993) and Heijdra and Van der Ploeg (2002, Ch. 14).

2 For the concept of the “period of production”—which is defined as the same length of time (more precisely, the greatest common factor of the lengths of time) required for production for all kinds of goods produced in the economy—see the early part of Section II.
method of reflecting this process of production is to \textit{compound} the unit-period rate of interest in accordance with for how many periods the input stays in production; after all, interest is the reward for waiting.

In the following pages I will argue that (currently available, representative) horizontal innovation models fail to consider (or deliberately assume away) the second problem of value,\textsuperscript{3} and that such failure (or deliberation) entails \textit{internal inconsistency} in relation to their \textit{long run equilibrium} characterization. The failure (or deliberation) simplifies a model greatly, but the simplicity of a model, especially if theoretical, should never be exchanged with its internal consistency (in particular, when a slight increase in the complexity of a model can restore its internal consistency).

Models in the literature differ in the specification of the economy; accordingly, the concrete form in which internal inconsistency creeps in varies across the models. In this paper, I will take up the model of Barro and Sala-i-Martin (2004, Ch. 6) (henceforth BSM) as the object to be dissected, but no substantially different conclusion will apply to other (currently available, representative) horizontal innovation models. The main reason of my choice is that the model is the best case in which the problem of interest is implicated in the fullest possible manner (in the sense that there is a produced input which goes through all the three production stages) and it is also an example that relies on the crudest way of “resolving” (not solving) that problem in horizontal innovation models; thus, its dissection will lay bare in the most conspicuous way how the problem of interest (and the problem of the measurement of “capital” in reference to which the rate of interest is to be calculated) reside generally in horizontal innovation models.\textsuperscript{4}

I will briefly summarize BSM in the following section. Then Section II will identify the internal inconsistency which has crept into the model. The identified internal inconsistency can be corrected outright once the logic regarding production and interest is accepted. The model is robust to the correction, in the sense that the corrected model, though more complicated, produces results similar to those of the uncorrected one (see Appendix A). However, Section III goes deeper and contends that the problem identified in BSM will \textit{mutatis mutandis} appear in all other horizontal innovation models. I will argue that the

\textsuperscript{3} The first problem—the problem of measurement—is properly taken into account: horizontal innovation models, typically and consistently, take the final good as the standard of measurement.

\textsuperscript{4} A companion paper to the present one deals with the Romer (1990) model, where internal inconsistency arises in a much more subtle way. The paper is available from the author on request.
literature invalidly generalizes the valid treatment in Solovian one-sector models of time (interest) in production and, impliedly, the measure of “capital” on which the long run equilibrium rate of interest is calculated. The literature invalidly reduces an essentially “multi-layered” economy to a “single-layered” one. (Appendix B illustrates and contrasts, with the aid of what is called the system of “price equations,” the original and the corrected BSM.) Section IV concludes by way of summary.

I. The Original Model

In this section I will simply repeat the crucial elements of BSM, with some minor modifications and additions; I reserve Section II for critical and constructive comments. In line with other horizontal innovation models, BSM has three sequentially connected sectors. The R&D sector (the \( D \) sector) produces new “designs” by using a non-physical input (the stock of old designs, which are public goods) and a physical input (the final good); the intermediate goods sector (the \( I \) sector) uses a non-physical input (designs, old and new) and a physical input (the final good) to produce intermediate goods; and the final good sector (the \( Y \) sector) uses physical inputs (intermediate goods) and labor to produce the final good. The final good is either used for consumption or foregone for investment in the \( D \) and \( I \) sectors.\(^5\)

Designs are the only durable (non-depreciating) input in the economy; all the physical inputs are nondurable (depreciate completely in one “round” of production).\(^6\)

Production in the \( Y \) sector is represented by the following Dixit-Stiglitz production function:

\[
Y = L^{1-\alpha} \sum_{i=1}^{N} x_i^\alpha,
\]

where \( Y \) = the final good output; \( L \) = labor; \( x_i \) = the \( i \)th type of intermediate goods; \( N \) = the number of the types of intermediate goods used in production; and \( 0 < \alpha < 1 \). Extra profit in

\(^5\) Of course, this sequential connection does not mean that production takes place in the form of separate full cycles, each starting from the input for designs and ending with the output of the final good; actually, for a given period of time, the three sectors operate side by side in the economy. The fundamental idea of the sequential connection is that each sector uses, as an input, an output that is produced in the other sector: the inputs (except labor) are produced goods. Note also that the sequential connection in BSM is in fact circular (the Figures in Section III show vividly the circularity of production process involved here). But not all horizontal innovation models have this circularity; for example, the “labor-for-intermediates” models mentioned earlier do not involve the ploughing back of the final good.

\(^6\) This is in contrast with, for example, Romer (1990) and Jones (2002, Ch. 5; 2005) where all produced inputs are durable.
this sector is

\[ \pi_Y = Y - wL - \sum_{i=1}^{N} p_i x_i , \]

where \( w \) = the wage rate, and \( p_i \) = the price of the \( i \)th-type intermediate goods (measured in terms of the final good). Since intermediate goods enter into production in a symmetrical way, profit maximization generates an identical demand function across the types of intermediate goods. Perfect competition prevails so that extra profit is zero in (long run) equilibrium; thus, in equilibrium,

\[ Y = wL + \sum_{i=1}^{N} p_i x_i . \]

The production of one unit of the \( i \)th intermediate good requires, besides the \( i \)th design, an amount \( \theta \) of the final good. The operating profit of the firm producing the \( i \)th-type intermediate goods is

\[ \pi_i = p_i x_i - \theta x_i . \]

The \( i \)th firm gets \( p_i x_i \) as the gross revenue by selling the \( i \)th intermediate goods to the final sector; the operating cost of producing the goods is \( \theta x_i \). The difference between the gross revenue and the operating cost is the “net revenue,” or the “monopolistic profit,” accruing to the firm which produces a differentiated type of intermediate goods on the basis of a differentiated design. \( \theta \) being uniform regardless of the types of intermediate goods, profit maximization by the firm yields an identical supply function across the different types of intermediate goods. The demand functions being also identical, one consequently has a “symmetric equilibrium” for the intermediate goods, where

\[ x_i = x, \quad p_i = p, \quad \forall i . \]

(Thus we drop the subscript \( i \) henceforth.) Long run equilibrium requires that the present value of the flow of operating profit accruing from a type of intermediate goods be equal to the cost of purchasing (or producing) the design used for that type of intermediate goods. With the assumption that labor supply is constant, this condition leads to the following “arbitrage equation” (Jones, 2002, Ch. 5):

\[ r P_D = px - \theta x , \]
where $r$ is the rate of interest and $P_D$ the price of a design (which is uniform across the designs).\footnote{It is sometimes convenient to rearrange (9) into what can be called the “price equation”: $px = rP_D + 0x$. A “price equation” expresses the price of a commodity as the sum of all the costs incurred in the production of that commodity. The price equation just referred to expresses the total price of a type of intermediate goods of the amount of $x$ as the sum of the interest cost on the corresponding design and the cost of the final good input. Equation (5) is the price equation for the final good (the price of a final good is the sum of wages and the cost of intermediate goods), and equation (8) below for a design (the price of a design is equal to its total production cost, that is the cost of the final good input).}

The $D$ sector produces a new variety of designs ($\hat{N}$) by means of the stock of old designs ($N_{-1}$) and the final good input of the amount of $\eta$ per design. Old designs are public goods, so that they do not incur any cost. Thus in this sector, only the final good input incurs current cost, and the extra profit is

$$\Pi_D = P_D \hat{N} - \eta \hat{N}. \quad (7)$$

Then profit maximization implies the following relationship:

$$P_D = \eta. \quad (8)$$

The final good is used for consumption or foregone for investment. The foregone final good is used in the $I$ sector ($F_I$) and the $D$ sector ($F_D$). To produce $N(= N_{-1} + \hat{N})$ kinds of intermediate goods at the current period, each type of the amount of $x$, the final good is used up by the amount of $0xN$:

$$F_I = 0xN. \quad (9)$$

Designs, being durable, remain available forever once produced; the stock of designs constitutes the “assets” of the economy. The final good which is foregone for investment in the $D$ sector each period is only to produce a new variety of designs.

$$F_D = \eta \hat{N}. \quad (10)$$

Thus, accounting regarding the use of the final good output in the economy leads to

$$Y = C + \eta \hat{N} + 0xN. \quad (11)$$

On the other hand, using the symmetry result and substituting (8) and (6) into (3), one gets
Comparing (11) and (12) results in the following relationship:

\[ wL + r\eta N = C + \eta \dot{N}. \]

This relationship serves as the budget constraint to the dynamic utility maximization problem for an infinitely-lived representative household which has the felicity function of the form

\[ U(t) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma}, \]

and the resulting Euler equation is

\[ \frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}. \]

where $\rho$ is the rate of time preference and $\sigma^{-1}$ the elasticity of intertemporal substitution.

III. The Internal Inconsistency

The place where the rate of interest plays the first substantial role in BSM is the "arbitrage equation" (6), where it serves as the discount rate in obtaining the present value of the flow of operating profit in the $I$ sector and, as a consequence, as the rate of interest accruing to a design rented to an $I$-sector firm. This leads to its appearance in the budget constraint (13) and the Euler equation (15) as the rate of interest on the "assets" of the economy. But is this rate really the rate of interest which is to be observed in the economy in long run equilibrium? The answer requires a clarification of the quantity of "capital" on which the rate of interest is defined in a theoretically proper way and, indeed, the concept of the rate of interest itself.

The rate of interest is, of course, the ratio between interest and a quantity that is usually called "capital." The problem of measurement (the first problem of value referred to at the beginning of the present paper) arises in calculating the rate of interest, because, to be expressed as a ratio, interest and "capital" must be comparable but they are in general aggregate measures of collections of heterogeneous goods; to be comparable, they must be comparable by nature.

---

8 I will use quotation marks for "capital" because the concept of “capital” expounded in immediate paragraphs covers not only “fixed capital” (or simply “capital” in the conventional usage of the term in the literature) but also “circulating capital” (physical inputs which depreciate completely in one round of production). The reason for this will be made clear in the course of argument; see Section III in particular.
expressed in terms of a common standard, namely, in value terms.\textsuperscript{9}

To express the quantity of a good in value terms is to express it in terms of its \textit{price} measured in a chosen \textit{numéraire}. Now, the price of a good in long run equilibrium, whether in perfect competition or in monopolistic competition, is the price which covers no less, and also no more, than all the costs incurred in the course of producing that good, \textit{including the normal level of interest} on the “capital” which has been used in producing that good. This is where the second problem of value—the problem of interest—enters the scene.

Before I proceed further, precautionary remarks are in order regarding the “period of production” in reference to which I define the rate of interest as accruing on various produced inputs (those inputs which are themselves produced in the economy). Indeed, the whole of my arguments in the present paper is based on the idea that the unit “period of production” is of the same length for all kinds of goods produced in the economy. In this I am following the traditional approach: the unit production period (usually called figuratively a “year”) is defined as the greatest common factor of the lengths of time required for producing the full units of the respective goods; if there is an “unfinished” good during the greatest common factor period, it can be treated as a different “good” from (though in an intermediate stage leading to) the “finished” good (see, for example, Neumann, 1945). That is, the definition of the “period of production” precedes the definition of a good (as distinguished from other goods) and also the definition of the “durability” of an input: if the period of production is a “day,” and if there is an input which lasts for “two days,” this input is fully “durable” as much as an input which lasts for a “year” and beyond. And the rate of interest to be applied to all kinds of inputs used in the economy must be the one corresponding to this same, pre-defined, period of production.

In contrast to this, a different idea—which seems taken up by many current practitioners and which could defend BSM successfully from my criticism—exists that if different kinds of inputs vary in durability, the length of time over which the rate of interest relevant to them is defined should also differ. According to this idea, the economic system is such that all the transactions—producing and selling a design, producing and selling intermediate goods, and producing the final good—are made on the first “day” of the “year,” with the consumption of the final good being carried out during the remaining days of the “year” until the new spurt of

\textsuperscript{9} In horizontal innovation models, the heterogeneity of goods is doubly the case: intermediate goods produced on the basis of differentiated designs are, by definition, different from each other (the very crux of the literature is indeed to show that the source of continuous growth is the increase in the \textit{variety} of intermediate goods used in the \textit{Y} sector), and intermediate goods are heterogeneous from the other goods of the economy (designs and the final good).
transactions takes place at the first “day” of the next “year.” The argument would be that because the design, being “durable,” existed in the economy throughout the “year” (and beyond), the rate of interest on the design was for a “year,” whilst the other (nondurable) inputs existed only for one “day,” so that the rate of interest relevant to these latter inputs was for that “day” only. This would imply that the rate of interest on the nondurable inputs should be much smaller than the one on the durable input, so that it could be safely ignored; that is, it could be safely assumed that \( r = 0 \) for a (fully) non-durable input whilst \( r > 0 \) for a durable input. However, the argument does not hold water at all. The period of time relevant to the determination of the prices of the goods—where the rate of interest has significance as part of production costs (see below)—is the period of production; here, the length of time for consumption, whether much shorter (an “instant”) or much longer (the remainder of the “year”) relative to the time for production (a “day”), is completely irrelevant. The setting just referred to merely means that the production period is a “day” for each sector, and that the relevant rate of interest which is to be applied to the various inputs—whether durable or nondurable—must be a “daily” rate. If the producer of the \( Y \) sector accounts, in the price of her output (the final good), only the “daily” rate of interest on the input (intermediate goods) for which she has paid the price that reflects the “annual” rate of interest on an input (a design), she—as a producer of a good—is behaving against economic rationality: the pursuit of higher returns. This is all the more the case, as I shall argue presently, because BSM (like all horizontal innovation models) are considering the state of long run equilibrium.

With these considerations, one is now ready to apply a surgeon’s scalpel to BSM, in particular to their understanding of profit (or cost) in the respective sectors—as is expressed by equations (2), (4) and (7). \(^{10}\) In the following I will assume, in conformity with the usual practice, that payment for all the produced inputs is made at the beginning of the production period. \(^{11}\) According to equation (2), the total cost of the \( Y \) sector is the sum of the wage cost of labor and, simply, the purchase value of intermediate goods. Equation (2) accounts only the depreciation cost of using the intermediate goods: no interest cost is incurred to the user of the intermediate goods input. This leads to the result, implied in (3), that the net revenue (the total revenue minus depreciation) of the \( Y \) sector is sufficient only to pay the wages of labor; the \( Y\)-

---

\(^{10}\) Most of the argument in the remaining part of this section, related to the concept of interest in production, are in fact elementary and might be tedious to some readers. I, however, proceed step by step in order to pinpoint what has gone “wrong” with BSM. The bored reader may jump straight to the paragraph containing equation (24) near the end of this section.

\(^{11}\) However, one can easily check that the ante factum payment does not make any difference to the results of the corrected BSM model.
sector firm obtains no net revenue. The $D$ sector is no different in this matter. As equation (7) expresses, the cost accounts only that quantity of the final good which is foregone for investment in this sector. Thus, as equation (8) implies, the $D$-sector firm obtains no net revenue. The situation of the $I$ sector is, in contrast, subtly different. A design is a fully durable input, so that it does not depreciate and the operating cost of using it is the interest to be paid on it; thus, part of the gross revenue of the sector is accounted by the interest accruing to the designs used in production. By contrast, the final good input attracts no interest at all; the cost of using it is merely the depreciation cost. As equation (6) implies, the net revenue of the $I$ sector consists of the interest accruing only to the designs.

Then, in the economy of BSM, the users of physical inputs, whether in the $D$, $I$ or $Y$ sector, must satisfy themselves with being compensated only for the cost that has been incurred to them in physical terms, but not for their opportunity cost. The opportunity cost on the physical inputs is zero. It is, of course, perfectly legitimate to assume a particular (but limited) case of zero rate of interest; this would simply means that waiting in production were of zero value. But—and this is the main argument of mine—this assumption generates inconsistency within the model. A series of observations are in order in this regard.

Equation (6) can be interpreted as follows. The $I$-sector firm pays the price of a design at once at the beginning of the period, and recoups that cost in the form of revenue of the amount equal to the “annual” interest on that cost, in perpetuity (the production period is defined as a “year,” this being however short—even “instantaneous”—but positive an elapse of time);\(^\text{12}\) that is, the cost for a design is a sunk cost. The $I$-sector firm is entitled to that revenue because it has to wait between the moment of paying for the design input and the (infinitely many) moments of producing and selling the output and thereby obtaining revenue. Here, waiting—the passing of time—is involved and this waiting is valued positively if and only if the rate of interest is positive. But, then, should exactly the same reasoning not apply to the other input of the $I$ sector, i.e., the final good input? Why should time (and the firm itself, for this matter) discriminate between a design input and a physical input while they, both produced inputs, are jointly involved in producing the same good during the same period of time? The only relevant difference is that the physical input is used up in a single period of production (thus generating revenue once and for all) while the design is used over and over

\(^{12}\) An alternative interpretation is that the provider of a design, who suffered the cost of producing it and now rents it to an $I$-sector firm in perpetuity, receives revenue at the end of every production period in perpetuity; the revenue of the design maker is the “annual” rent that the $I$-sector firm has to pay to him. Of course, this alternative interpretation does not at all affect my argument below.
again (thus generating revenue over and over again). But for this difference, a modification merely to that extent would be sufficient and necessary: the price of the physical input is paid at once and the cost of using it must be recouped by the revenue obtained in a single period; and the present value of this single occurrence of revenue must be equal to the price of the input. As there is a gap of one production period (and if the passage of time is valued positively), there is a corresponding gap between the current value and the present value of revenue, and that gap is represented by interest. The physical input, too, must attract interest if the design, used in the same round of production as the physical input, attracts interest.

This admitted, it is also noted that the rates at which interest accrues to the two inputs in the $I$ sector must be, in long run equilibrium, equal to each other. Otherwise, the provider of the input which attracts interest at a lower rate will be shifting his investment to the input which attracts interest at a higher rate; long run equilibrium is defined precisely as a state of affair in which there is no incentive for such shift (for this, as will be discussed further shortly, the monopolistic competition setting of the $I$ sector makes no difference). In long run equilibrium, the $I$-sector firm should value equally the waiting involved in using the design and that in using the physical input.

The reasoning behind the “arbitrage equation” (6) is:

$$P_D^* = \frac{\pi_i^*}{(1+r)} + \frac{\pi_i^*}{(1+r)^2} + \frac{\pi_i^*}{(1+r)^3} + \cdots,$$

where $\pi_i^*$ is the part of the total gross revenue of the $I$-sector firm that is accounted by a design (I have added a star to $\pi_i$ and $P_D$, in order to indicate that the starred ones are the correctly defined variables; similarly henceforth). Now, the physical input in the $I$ sector, which is non-durably used by the amount of $\theta x$ for each type of intermediate goods, accounts the revenue of the $I$-sector firm by a certain amount, denoted by $\pi_0^*$, which is realized just over one year at the end of the year. As $\theta x$ is the cost of purchasing the input at the beginning of the year, one will have, similarly to (16), using the same discount rate as for a design,

$$\theta x = \frac{\pi_0^*}{(1+r)}.$$

---

13 The reasoning is expressed in terms of discrete time (“year”) and a constant rate of interest over time; however, the same result will be obtained through a reasoning in terms of continuous time and a variable rate of interest over time, as Barro and Sala-i-Martin (2004, p. 291) do.
It is \( \pi_0^* \), in place of \( \theta x \), that must enter into the “arbitrage equation.” Accordingly, the correctly accounted gross revenue of an \( I \)-sector firm \( (p^* x) \) must consist of \( \pi_0^* \) and \( \pi_i^* \).

\[
(18) \quad \pi_i^* = p^* x - \pi_0^*,
\]
or, using (16) and (17),

\[
(19) \quad rP_D^* = p^* x - (1+r)\theta x .
\]

The same reasoning applies to the inputs in the other two sectors. Insofar as production involves some, however short, passage of time and also insofar as waiting arising from this passage of time is valued positively, positive interest must accrue to the cost (any cost) incurred at the beginning of the production period. There is, however, one important point to be made clear before proceeding further. As is well known, the firm in the \( I \) sector—though operating in the setting of monopolistic competition—obtains zero extra profit in the long run, just like the firms in the other sectors operating in the setting of perfect competition. The present value of the flow of monopolistic profit accruing to an \( I \)-sector firm is, in long run equilibrium, equal to the cost of purchasing the design it uses; this is precisely what is told by the “arbitrage equation” (6). The rate of interest prevailing in the \( I \) sector in long run equilibrium is the “normal” rate of interest (i.e., with zero extra profit), and this “normal” rate of interest is uniform across the sectors in accordance with the very definition of long run equilibrium.

In the \( D \) sector, the cost-incurring physical input (the final good) is applied by the amount of \( \eta \) per design. There is an elapsed of a “year” between the application of this input and the production of the output (a design). The gross revenue \( (\pi_\eta^*) \) that is generated by the selling of a design must, when discounted to the present value, be equal to the purchasing cost of it \( (\eta) \) incurred at the beginning of the production process. The physical input is nondurable, so that the generation of revenue is once and for all. Thus,

\[
(20) \quad \eta = \frac{\pi_\eta^*}{(1+r)},
\]

which implies

\[
(21) \quad P_D^* = \pi_\eta^* = (1+r)\eta .
\]

In the \( Y \) sector, the physical input is the stock of intermediate goods, purchased at the
price of \( p^* \) per unit at the beginning of the production period, and the output (the final good) is sold at the price of unity per unit (measured in terms of itself) at the end of the period. If the part of the gross revenue of the \( Y \) sector to be accounted by this physical input is denoted by \( \Pi_y \), the present value of its flow must be equal to its purchase price. The intermediate goods input is nondurable, so that the occurrence of revenue is for a single time, which leads to

\[
(22) \quad p^* xN = \frac{\Pi_y}{1+r}.
\]

The gross revenue of the \( Y \) sector is, in equilibrium, the sum of the revenue covering the wage of labor and the revenue covering the cost of intermediate goods (\( \Pi_y \)), this cost including the cost of interest on intermediate goods; and, for long run equilibrium, the cost of interest is incurred at the normal rate:

\[
(23) \quad Y^* = wL + \Pi_y = wL + (1+r)p^* xN.
\]

The correction is completed. Compare the arguments and formulations in this section with those of the original BSM. I argue that the key relationships in BSM—(3), (6) and (8)—should instead have been (23), (18) and (21), respectively. The corrected ones contain interest cost accruing to all the produced inputs (at a uniform “normal” rate, as the economy is in long run equilibrium), while the original ones have interest cost accruing only to the designs.

In fact, the same result could have been obtained in a more direct and usual way. The cost incurred by a produced input is accounted in terms of the rental on that input. There exists the general relationship among the rental rate on a produced input, the rate of interest thereupon, and the price of the input—the definition of the user cost of capital: \(^{14}\)

\[
(24) \quad \zeta = (\delta + r)z.
\]

where \( \zeta \) is the rental rate, \( \delta \) the depreciation rate, and \( z \) the price measured in terms of the numéraire. For a design, which is durable, \( \delta = 0 \) and \( z = P_D \); for the final good input in the \( I \) and \( D \) sectors, which is nondurable, \( \delta = 1 \) and \( z = 1 \); and, for an intermediate good which is also nondurable, \( \delta = 1 \) and \( z = p \). The rate of interest is uniformly \( r \), as we are concerned with long run equilibrium. The existence of a positive rate of interest implies that waiting is required in production and is valued as positive; once interest accrues to at least one of the produced inputs, all the other produced inputs will also attract interest in the long run, because

\(^{14}\) In the present case, capital gain is zero.
time does not discriminate among the various inputs of the economy. BSM has fallen into the trap of internal inconsistency exactly in this regard. In BSM, time applies discriminately to the various inputs of the economy: the design input in the I sector attracts interest; by contrast, no other produced inputs in the economy attract interest—not even the final good input which is used along with the design in the same sector—despite the fact that all the produced inputs in the respective stages of production are involved in production for the same length of time and the economy has settled at a state where the passage of the same length of time is to be valued as equal (that is, long run equilibrium).

The correction is straightforward, once one accepts the logic regarding the rate of interest and long run equilibrium. The solution, however, reveals some complications. Substituting (21) and (19) into (23), one gets

\[(25) \quad Y^* = wL + r(1+r)^2 \eta \hat{N} + (1+r)^3 \theta \hat{x} \hat{N} .\]

One will notice the difference from the original BSM in the powers of \(r\) (see Section III for the detailed discussion regarding this).

Meanwhile, the final good output is used, as before, for consumption \((C)\) or forgone for investment in the I sector \((F_I = \theta \hat{x} \hat{N})\) or in the D sector \((F_D = \eta \hat{N})\)—the relationship which was represented by equation (11). Juxtaposing (11) and (25) and rearranging the result, one gets the following budget constraint, corrected:

\[(26) \quad wL + \left[ r(1+r)^2 + r(2+r)\eta^{-1}\theta \hat{x} \right] \eta \hat{N} = C + \eta \hat{N} .\]

Contrast with BSM’s budget constraint (13) is salient. The coefficient to the “assets” of the economy in the budget constraint is not the rate of interest prevailing in the economy, but a variable \((\phi)\) composite of the rate of interest (compounded), technical parameters and the quantity of a type of intermediate goods:

\[(27) \quad \phi \equiv r(1+r)^2 + r(2+r)\eta^{-1}\theta \hat{x} .\]

It can be proved that the corrected BSM model has a unique solution for \(x\) and \(\hat{r}\) and, thus, \(\phi\) (see Appendix A). Then, the Euler equation for the representative household which maximizes utility with the same felicity function as before will be

\[(28) \quad \frac{\dot{C}}{C} = g^* = \frac{\phi - \rho}{\sigma} ,\]

with \(\phi\) replacing \(r\) in (15). Though the usual relationship between \(r\) on the one hand and \(g, \rho\)
and $\sigma$ on the other does not hold, the model is robust to the modification, in the sense that the corrected model produces results similar to those of the original BSM. The only difference is that, by having $r = \phi$, BSM overestimates the equilibrium rate of interest.

**IV. A “Single-layered” Economy vs. A “Multi-layered” Economy**

Then, is my correction much ado for nothing? I argue that, from the theoretical perspective, a more fundamental issue—which applies *mutatis mutandis* to all horizontal innovation models—is at stake here.

The economy of a horizontal innovation model has two characteristics: the positive (even if “instantaneous”) elapse of time for production in each of the three sectors and the sequential connection among them. This economy is a “multi-layered” one, the term “layer” carrying the connotations of both thickness (however thin) and sequence. The accounting relationship (25) fully reflects this “multi-layeredness” of the model economy. First, the passing of time in each sector is expressed by the occurrence of interest in each corresponding period of production (at a positive rate if and only if waiting is valued positively). Second, the powers of $r$ indicates that the produced input in the $D$ sector stays in production over three periods of production, and the inputs in the $I$ sector over two, before the final good is produced in the $Y$ sector. Figure 1 illustrates this process of production. (In the Figure, a thick solid arrow means that production actually takes place and that waiting involved is recognized, and a thin dotted arrow indicates that neither production nor waiting is involved: the final good, when foregone, is automatically and immediately “transformed” into “capital goods” for the $D$ and $I$ sectors. As the model is concerned with long run equilibrium, the rate of interest is uniform across the inputs and across the sectors.)

In contrast, Figure 2 corresponds to the process of production envisaged in BSM. In the $D$ and $Y$ sectors, waiting in production is not recognized (hence, dotted arrows—but thick ones because production actually takes place). In the $I$ sector, the recognition of waiting is inconsistent in that only the design input attracts interest whilst the final good input in the same sector does not (illustrated by a thick broken arrow). The inconsistency is double: within the $I$ sector and across the sectors.

This double inconsistency, however, does the model a special service: it reduces a “multi-layered” economy to a “single-layered” one, this latter best exemplified by the Solovian (or Clarkian) one-sector economy (Solow, 1956). Figure 3 depicts the typical Solovian “single-layered” economy. Here, there is only one “layer” of production—the $Y$
sector, where the input, which is simply the foregone final good, is applied to produce the final good. The Solow model reflects the “single-layeredness” of the economy in the consistent manner: the produced input (of the sole kind in the entire economy) attracts interest, and interest accrues for a single time (as is illustrated by one thick solid arrow) and hence at a simple rate (the durability of the input is represented by the depreciation rate, $\delta$). The reader will now notice that this is also precisely the effect that the double inconsistency of BSM brings about: as is seen in the accounting relation (12) and depicted in Figure 2, interest accrues only to one type of input, for a single time, at a simple rate.

My argument up to now indicates that the correct quantity of “capital” on which to calculate the long run equilibrium rate of interest in the $Y$ sector is

$$V^* = p^* xN = [rP_d^* + (1+r)x]N = [r(1+r)\eta + (1+r)x]N.$$  

This is the total value of the intermediate goods used in producing $Y$. This value is obtained, as it should be, by multiplying the physical quantities of the respective intermediate goods (uniformly $x$) by their respective prices (uniformly $p^*$) over the $N$ varieties. The total price of a type of intermediate goods ($p^* x$) accounts all the costs incurred in producing them: the cost of a design ($rP_d^*$) and the cost of the final good input ($x$), which both include the interest costs incurred on them. Further, the price of a design ($D^*$) also accounts all the costs incurred in producing the design: the user cost—depreciation cost ($\eta$) plus the interest cost ($r\eta$)—of the final good input. With this measure, the accounting relationship for the $Y$ sector is obtained through (23), which yields

$$Y^* = wL + (1+r)V^*.$$  

This is the all too familiar (and theoretically proper) form of the accounting relationship when the sector uses labor and intermediate goods which depreciate fully in the unit production period and which are paid for at the beginning of the production period. The rate of interest in the $Y$ sector is, as it should be, calculated on the value of the intermediate goods used in producing $Y$, for this value is precisely the cost incurred to the $Y$-sector firms when they purchase those intermediate goods. One can see from (29) and (30) that interest accrues to all the produced inputs in all the sectors at the same rate—the economy is in long run equilibrium: $r$ is the “normal” rate of interest prevailing uniformly across the economy in long run equilibrium.

For BSM, however, the measure of “capital” on which to calculate the rate of interest of the $Y$ sector (and also of the economy as a whole) is not the value of intermediate goods but,
as the accounting relationship (12) shows, the “assets” \((A)\) of the economy—the stock of designs valued in terms of the final good, which is in turn obtained as the accumulated consumption foregone for producing designs:

\[
A(t) = \eta N(t) = \int_0^t F_\rho(\tau) d\tau.
\]

BSM errs here. BSM calculates interest on the assets as \(rN\). As \(\eta N\) is the purchasing cost of the final good input incurred in producing designs, \(rN\) is the interest that accrues (or, is incurred) to the design makers. But (12) is the accounting relationship for the economy as a whole; in long run equilibrium, if \(D\) sector firms are subject to interest on their costs of production, then \(I\) and \(Y\)-sector firms must also be; \(rN\) being a part of the costs for the \(I\)- and \(Y\)-sector firms (which respectively use designs and intermediate goods, this latter having in turn used designs), the interest relevant to the \(I\) and \(Y\)-sector firms must involve interest on that interest (once and twice, respectively). BSM, in accounting the costs of production in the economy as a whole, includes only the interest that accrues to the design makers while excluding the interest that is to accrue to the \(I\) and \(Y\)-sector firms. BSM’s accounting is incongruent to the long run equilibrium nature of the model.

It is noted, however, that BSM’s measure accords fully with the usual practice in the horizontal innovation literature. Here, the quantity on which to calculate the rate of interest is the quantity, expressed in terms of the chosen numéraire, of that part of the goods in the economy which are accumulated as a stock (what is called “assets,” or “fixed capital” or simply “capital,” leaving in the air the physical inputs that depreciate fully in one period of production—what is usually called “circulating capital” in capital theory). For example, in both Romer (1990) and Jones (2005), for whom all the produced inputs in the economy are fully durable, the “assets” of the economy include intermediate goods (the produced input in the \(Y\) sector) in addition to designs (the produced input in the \(D\) sector).\(^{15}\) Jones takes the whole stock of assets (“financial wealth”) as the quantity on which to calculate the rate of interest. Romer takes the “accounting measure of capital” only, which is simply the quantity of the final good foregone for producing intermediate goods.\(^{16}\)

\(^{15}\) The \(D\) sector does not use any produced input; it uses only human capital input. Still, this measure excludes the other produced, durable, input used in the \(I\) sector (the final good input). The problem arising from this—and how Romer (1990) “resolves” this problem—is discussed in a companion paper of the present one mentioned earlier. See the next footnote.

\(^{16}\) However, the result is the same for Jones and Romer, because in Romer designs attract interest at the same rate as on (though separately from) the “accounting measure of capital,”
It is easy and, at the same time, utterly important to spot the common characteristic of these measures, whether BSM’s, Jones’ or Romer’s: they are independent of the rate of interest. It is equally important to note that this is the characteristic that is, seemingly, in full conformity with the measure of “capital” in a “single-layered” economy such as Solow’s (1956):

\[ K(t) = (1 - \delta) \int_0^{t-1} F_Y(\tau) d\tau + F_Y(t). \]

\( K \), the “capital” of the Solow economy on which the rate of interest is calculated, is simply the physical amount of accumulated foregone final good, thus measured independently of the rate of interest. However, there is a subtle difference between the horizontal innovation practice and the Solovian one. \( K \) of the Solovian economy is also the value of the accumulated stock of capital goods, measured in a theoretically consistent way: there is no real production process involved in the “transformation” of the final good into capital goods, so that no interest accrues in this process—which implies that capital goods are de facto homogeneous with the final good; the numéraire is the final good, so that the purchasing cost of capital goods is outright the quantity of foregone final good, this quantity being therefore the value of capital goods; and the rate of interest is in principle calculated on this value of capital goods.

By contrast, in horizontal innovation models, intermediate goods—on the value of which the rate of interest of the \( Y \) sector must be calculated (thus, corresponding to Solow’s \( K \))—are heterogeneous to the final good; one (Romer and Jones) or two (BSM) stages of “real transformation” (i.e., production) pass between the final good input and the intermediate goods output; together with it, hence, the involvement of interest with the value of intermediate goods. However, BSM’s measure (and, for that matter, Romer’s and Jones’, too), being independent of the rate of interest just like Solow’s \( K \), eliminates this aspect of production process, thus generating the internal inconsistency regarding the long run equilibrium nature of the model. The measure of “capital” used in horizontal innovation models is a theoretical device to render workable the reduction of a “multi-layered” economy and because for both Romer and Jones the value of designs is equal to the wage paid to human capital in the \( D \) sector (the sole kind of cost to the sector). Interestingly (in contrast with BSM), both Romer and Jones correctly conceive that all the produced inputs attract interest at the same rate. But their models eventually do exactly the same thing as BSM: reducing a “multi-layered” economy into a “single-layered” one; this time, the reduction is rendered possible by another, rather unexpected, kind of logical inconsistency in the models. A companion paper of the present one, mentioned earlier, shows that the culprit is the assumption that all the produced inputs are durable, and also that this assumption is logically at odds with the specification of the Dixit-Stiglitz production function in their models.
to a “single-layered” one. But it is a measure of “capital” which is not proper for calculating the rate of interest, because it does not properly reflect the cost incurred to the user of the “capital.” To be such a proper measure of “capital,” the measure must be obtained in reference to the prices of the goods constituting the “capital,” these prices fully reflecting the costs incurred in producing these capital goods, thus including the rate of interest, at the normal level in long run equilibrium—and all this has been well recognized in the established literature in capital theory.\(^{17}\)

**V. Conclusion**

A model abstracts. Abstraction involves a removal of what is considered as inessential for the insight a model wishes to convey. Thus the degree of “realism” of a model is not a strong criterion of its validity. But there is a lexicographically absolute criterion for the validity (and therefore the generality) of a model, especially if theoretical: *internal consistency*. If a correction of a model restores internal consistency while guaranteeing the robustness of the model, why not correct it? The present paper has argued that this is the case for the model of Barro and Sala-i-Martin (2004, Ch. 6) and, by extension, all other (currently available, representative) horizontal innovation models.

The Barro and Sala-i-Martin model falls into the trap of internal inconsistency in relation to the long run equilibrium characteristic of the model. The inconsistency is double: first, the passage of time (waiting) is valued positively in the intermediate goods sector whilst it is valued zero in the remaining two sectors of the economy; second, in the intermediate goods sector, only one of the two produced inputs (a design) is worth for waiting whilst the waiting regarding the other produced input (the final good input) is worthless. This is so despite the fact that, in the model, all the produced inputs in the economy are, in each round

---

\(^{17}\) See, for example, the works collected in Bliss, Cohen and Harcourt (2005). Indeed, the capital theory controversies in the 1950s and the 1960s took place precisely on this recognition when the Solovian one-sector model was generalized into a multi-sector one; see, for example, the famous symposium on “Paradoxes in Capital Theory” in the *Quarterly Journal of Economics* (Vol. 80, 1966); for a recent one, Cohen and Harcourt (2003). Appendix B considers the system of “price equations” for the corrected BSM in the long run equilibrium framework, in a similar way as was extensively studied by Dorfman, Samuelson and Solow (1958), Gale (1960), Sraffa (1960), and Kurz and Salvadori (1995) among others; even though the details are different from these works, the representation this way will make transparent the correct measurement of “capital” on which the long run, normal, rate of interest is calculated. (Also, since the system is one of simultaneous equations, it will help to understand a horizontal innovation economy as the one in which the three sectors operate side by side but are “connected sequentially.”)
of production, subject to the passage of time to an equal extent, and that the model deals with long run equilibrium, in which such equal passage of time is to be of equal value.

Internal inconsistency is not confined to the model under consideration. Different (representative) models have different model specifications, but they suffer from internal inconsistency of one or another kind which is, at bottom, similar to the one disclosed above. This is because the economy that horizontal innovation models wishes to deal with is by its very nature a “multi-layered” economy, whilst they effectively reduce it to a “single-layered” economy. The net output of this reduction is the negligence (unwitting or deliberate) of the problem of value that arises necessarily in a “multi-layered” economy. In this process of reduction, in the Barro and Sala-i-Martin model, time is forced to apply discriminatively. But Mother Nature whispers, sternly: tempus non discriminat.

APPENDIX A: PROOF OF THE EXISTENCE OF A UNIQUE EQUILIBRIUM OF THE CORRECTED MODEL

Profit maximization in each sector yields the following equations for the corrected model. First, the gross rental on an intermediate good used in the $Y$ sector is equal to the marginal product of that intermediate good:

$$(1 + r) p_i^* = \alpha L^{1-\alpha} x_i^{\alpha - 1}. \quad (33)$$

Second, the $I$ sector firms follow the following monopoly pricing rule:

$$p_i^* = (1 + r) \alpha^{-1} \theta. \quad (34)$$

Third, as was discussed in Section II, the “arbitrage equation” for the $I$ sector is

$$p_i^* x_i = rP_D^* + (1 + r) \theta x_i = r(1 + r) \eta + (1 + r) \theta x_i. \quad (35)$$

The three relationships (33), (34) and (35) constitute an independent system for three unknowns: $p^*$, $x_i$ and $r$. Arranging them for $r$, one will get

$$(1 - \alpha) \alpha^{-1} \theta \eta L(1 + r)^{\frac{-2}{\alpha - 2}} - \eta r = 0. \quad (36)$$

This equation in $r$ cannot in general be solved algebraically. However, expressing the left-hand side of (36) as $f(r; \alpha, \theta, \eta, L)$, one has

$$f(0; \alpha, \theta, \eta, L) > 0, \text{ and} \quad (37)$$
Thus, there must be a unique \( \hat{r} > 0 \) such that \( f(\hat{r}; \alpha, 0, \eta, L) = 0 \). On the basis of this, one can obtain, from (34), a unique \( \hat{p} > 0 \); and from (33), a unique \( \hat{x}_i = \hat{x} > 0, \forall i \). Then, \( \phi \) is obtained by substituting \( \hat{r} \) and \( \hat{x} \) into (27).

**APPENDIX B: THE SYSTEM OF “PRICE EQUATIONS”**

The three \((D, I \text{ and } Y)\) sectors operate side by side in a given period of time, each of which produces its own distinct kind of goods: designs, intermediate goods and the final good. These goods, while being produced in the economy, are also used as inputs in production in the other sector(s): designs in the \( I \) sector, intermediate goods in the \( Y \) sector, and the final good in the \( D \) and the \( Y \) sectors. The economy is one of “production of commodities by means of commodities” (Sraffa, 1960). The “price equation” of a good represents the price of that good as the sum of all the costs incurred in producing it, including interest accruing to the produced inputs used in the production. All the produced inputs attract interest. The economy is in long run equilibrium, so that interest accrues at the uniform (normal) rate across the produced inputs and across the sectors. The price equations of the economy, considering the durability of the respective produced inputs, are

\[
(39) \quad (1+r)\eta \hat{N} = P_d^* \hat{N} \quad \text{(the } D \text{ sector)};
\]

\[
(40) \quad rP_d^* N + (1+r)\theta x N = p^* x N \quad \text{(the } I \text{ sector)};
\]

\[
(41) \quad wL + (1+r)p^* x N = Y \quad \text{(the } Y \text{ sector)}.
\]

Expressed in a matrix form, with the rows for the sectors and the columns for the goods, the system is

\[
(42) \quad (1+r) \begin{bmatrix}
0 & 0 & \eta \hat{N} \\
0 & 0 & 0 \theta x N \\
0 & x N & 0
\end{bmatrix} + \begin{bmatrix}
P_d^* \\
p^*
\end{bmatrix} + \begin{bmatrix}
0 \\
N x N \\
0
\end{bmatrix} = \begin{bmatrix}
\hat{N} & 0 & 0 \\
N & x N & 0 \\
0 & 0 & Y
\end{bmatrix} \begin{bmatrix}
P_d^* \\
p^*
\end{bmatrix},
\]

or in a compact form (in obvious notations),

\[
(42') \quad (1+r)A p^* + wL = B p^*.
\]

Note in (42) and (42’) that, in the \( I \) sector, designs are treated as “depreciating” completely in one round of production but at the same time the same designs are “produced” intact together...
with intermediate goods (this is the way of representing a non-depreciating produced input, in
the framework of “joint production”; see, for example, Neumann, 1945; Sraffa, 1960). In
contrast, the system of price equations of the original BSM can be expressed as

\[
\mathbf{Ap} + r\mathbf{Cp} + \mathbf{wI} = \mathbf{Bp}, \quad \text{with } \mathbf{C}_0 = \begin{bmatrix} 0 & 0 & 0 \\ N & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

For either the original or the corrected BSM, the vector of (physical) net outputs of
the economy is, with \( \mathbf{e} \) as the summation vector,

\[
\mathbf{e}(\mathbf{B} - \mathbf{A}) = \begin{bmatrix} \dot{N}_r \\ 0 \\ Y - \eta\dot{N} - 0\times N \end{bmatrix},
\]

consisting of two kinds of heterogeneous goods in positive quantity: new designs and the final
good to be consumed. In physical aspects, there is no difference at all between the economy
of the corrected BSM and that of the original BSM; in particular, the system (43), as much as
(42), describes the feature of the “production of commodities by means of commodities.” But
they differ in value aspects.

The contrast between (43) and (42) lies in the conception of which inputs are to be
counted as the ones to which interest accrues (thus, what is entitled to be called “capital”).
Equations (42) are based on the idea that the rate of interest, in each sector, is calculated on
the value of all the produced inputs used in the sector, whether durable or nondurable; that
value for each sector is, with the final good as the numéraire,

\[
\mathbf{Ap}^* = \begin{bmatrix} \eta\dot{N} \\ P_D^* N + 0\times N \\ p^* x N \end{bmatrix}.
\]

The second and the third row of \( \mathbf{Ap}^* \) (the “capital” of the \( I \) and the \( Y \) sectors) will
eventually involve the rate of interest. In contrast, (43) reflects the original BSM’s idea that
only the durable input attracts interest; therefore, the value of the inputs in each sector on
which the rate of interest is calculated is

\[
\mathbf{Cp} = \begin{bmatrix} 0 \\ P_D N \\ 0 \end{bmatrix}.
\]

“Capital” exists only in the \( I \) sector; further, it counts only the durable input (designs),
excluding the other produced input used in the same sector (the final good input)—the double
inconsistency discussed in the text. From the first row of (43), one easily gets \( P_d = \eta \); thus, the measure of “capital” in this case is independent of the rate of interest, despite the sequential connectedness of the economy. The system (42) describes a “multi-layered” economy, whilst the system (43) describes a “single-layered” one—notwithstanding the fact that, in physical terms, both purport to describe the same economy where commodities are produced by means of commodities.

Now, one can rearrange (42’) into

\[
(47) \quad e(B - A)p^* = reAp^* + wel.
\]

This is the familiar relationship in which the aggregate net output (in value) is equal to the sum of the aggregate net interest and the aggregate wage, using the same set of inputs \( (A) \) both in calculating the net output and the aggregate interest. A further rearrangement of (47) leads to the budget constraint for the representative household (26) for the corrected BSM. Also, sequential substitutions of (39), (40) and (41) yield the accounting relationship for the \( Y \) sector in the corrected BSM, (30) with (29), showing the involvement of interest in the measure of “capital.”

For the original BSM, one has a rearrangement of (43):

\[
(48) \quad e(B - A)p = reCp + wel.
\]

One notices that the set of inputs used in calculating the net output \( (A) \) is different from the set of inputs in calculating the aggregate interest \( (C) \)—a tag of inconsistency. On the basis of this relationship are derived the accounting relationship for the \( Y \) sector (12) and the budget constraint (13).

References


FIGURE 1. PROCESS OF PRODUCTION (CORRECTED BSM)

\[ \begin{align*}
D & \quad I \quad Y \\
\eta \dot{N} & \quad (1+r)\eta \dot{N} & \quad r[(1+r)\eta \dot{N}] & \quad (1+r)[r(1+r)\eta \dot{N}] \\
0\dot{N} & \quad (1+r)0\dot{N} & \quad (1+r)[(1+r)0\dot{N}] \\
F_D & \quad F_I
\end{align*} \]

FIGURE 2. PROCESS OF PRODUCTION (BSM)

\[ \begin{align*}
D & \quad I \quad Y \\
\eta \dot{N} & \quad \eta \dot{N} & \quad r\eta \dot{N} & \quad r\eta \dot{N} \\
0\dot{N} & \quad 0\dot{N} & \quad 0\dot{N} \\
F_D & \quad F_I
\end{align*} \]

FIGURE 3. PROCESS OF PRODUCTION (SOLOW)

\[ \begin{align*}
Y & \quad C \\
F_Y & \quad (\delta+r)F_Y \\
F_Y
\end{align*} \]