WHAT IS EXPLOITATION?[1]

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ABSTRACT

This paper analyses John Roemer's theory of exploitation and classes in a dynamic context. An intertemporal generalisation of Roemer's (1982, 1988) economies is set up. Roemer's main results are extended to the intertemporal context and the theoretical similarities and differences between Roemer's concept of exploitation and the more traditional notion of welfare inequality are shown.

However, it is argued that absent time preference, differential ownership of scarce productive assets and classes are an inherent equilibrium feature of a capitalist economy, while exploitation tends to disappear in the long run. This suggests that *per se* asset inequalities are a normatively secondary wrong and that the emphasis should be placed on their being a causally primary wrong. Moreover, the paper highlights the importance of capital scarcity – and the conditions for its persistence - for any theory of exploitation. Finally, it is argued that these results raise some doubts about the possibility of providing robust micro-foundations to Marxian concepts by means of Walrasian general equilibrium models.

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1. INTRODUCTION

Egalitarians – more specifically, socialists – have long condemned capitalist economies on normative grounds by appealing to various concepts of justice. Often, the exploitative relations inherent in the capitalist mode of production have been denounced in addition to, and as distinct from the economic and social inequalities of a capitalist economy. However, the very definition of the concept of exploitation and the difference with inequality have been at the centre of philosophical and economic debate. This paper does not defend exploitation as a relevant normative (or, indeed positive) concept. Instead, assuming its relevance, it focuses on the differences between inequality and exploitation as distinct normative concepts and on the methodological issue of how exploitation can be modelled, taking as the point of departure John Roemer's seminal contributions on Marx's economic theory (e.g., Roemer, 1982A, 1982B, 1986, 1988).

Roemer's main methodological contribution concerns the possibility (indeed, the necessity) of providing microfoundations to Marx' economics. The concepts of class and exploitation – and the full class and exploitation structures of a society – are modelled as the product of constrained individual optimisation. From a substantive viewpoint, Roemer suggests that all relevant moral information is conveyed by the analysis of *Differential Ownership of Productive Assets* (DOPA) and the resulting welfare inequalities. Roemer develops an alternative game theoretical definition of exploitation based on DOPA which is meant to be a generalisation of Marx's theory capturing its essential normative content.

Due to the scope and relevance of the issues analysed, Roemer's theory has generated a vast literature. Several critiques have been expounded on his methodology and on his conclusions, mainly based on issues of interpretation of Marx's theory (e.g., Reiman, 1987; Foley, 1989; Howard and King, 1992; Lebowitz, 1994), but surprisingly little attention has been devoted to his models.[2] In this paper, *a priori* problems of interpretation are left aside, while both methodological and substantive issues are discussed by means of a dynamic generalisation of Roemer's subsistence economies.

From a methodological viewpoint, a dynamic model is particularly suitable to analyse the possibility of providing neoclassical (more precisely, Walrasian) microfoundations to Marxian economics. In fact, a model that aims to provide microfoundations to Marx's concepts of exploitation and class must be able to account for their persistence. "The economic problem for Marx, in examining capitalism, was to explain the persistent accumulation of wealth by one class and the persistent impoverishment of another, in a system characterized by voluntary trade" (Roemer, 1982A, p.6).

Roemer's models are essentially static in that there are no intertemporal trade-offs; they can be interpreted either as a succession of one-period economies (ibid., p.45) or as an infinitely lived generation, but in either case *intertemporal* credit markets are absent *and* savings are impossible. Thus, they do not seem suitable for analysing the persistence of exploitation and classes in a capitalist economy. Even though the absence of intertemporal credit markets is consistent with the subsistence hypothesis, the impossibility

[2] Devine and Dymski's (1991) article represents a partial exception.

of savings seems fairly restrictive. Moreover, savings and the intertemporal allocation of labour are particularly relevant, both because of the positive and normative importance of inter-class mobility, and because the introduction of a savings decision enlarges the set of choices available to agents.

From a substantive viewpoint, a dynamic model allows one to assess the causal and moral relevance of DOPA, focusing on its role in generating exploitation and classes as persistent features of a competitive economy with savings and a variable distribution of productive assets.

Given the importance of dynamics, the focus on Roemer's subsistence economies with labour-minimising agents might seem contradictory. However, arguably, the results obtained in Roemer's static economies depend on differential ownership of *scarce* productive assets (Skillman, 1995, 2001). Hence, it is not surprising *per se* that exploitation may disappear when accumulation is allowed (Devine and Dymski, 1991).[3] Focusing on subsistence, rather than accumulating economies (with revenue-maximising agents), allows one to abstract from the issue of capital scarcity.

Moreover, Roemer's main theoretical conclusions do not depend on accumulation. On the contrary, one of his most relevant results is precisely that "exploitation emerges logically prior to accumulation" (Roemer, 1982B, p.264). The analysis of subsistence economies allows one to examine the role of DOPA in a context where capital scarcity persists and it is theoretically crucial in order to evaluate Roemer's fundamental claim that "differential distribution of property and competitive markets are sufficient institutions to

[3] See Roemer's (1992) reply to Devine and Dymski (1991).

generate an exploitation phenomenon, under the simplest possible assumptions" (Roemer, 1982A, p.43).

2. THE INTERTEMPORAL MODEL

The economy consists of a sequence of nonoverlapping generations, each with $\nu = 1, \ldots, N$, identical producers, living for T periods, and indexed by the date of birth kT. For the sake of simplicity, in every period t, a single good is produced and consumed.[4] In every t, each agent ν requires a strictly positive amount b for her subsistence and can operate a fixed coefficient technology (A, L), where $A \in (0, 1)$ is the amount of input necessary to produce one unit of output and $L > 0$ is the direct labour coefficient.

In every t, (p_t, w_t) is the price vector, where w_t is the nominal wage; x_t^ν is ν's production as a self-employed producer; y_t^ν is the activity level that ν hires others to operate; z_t^ν is ν's labour supply; ω_t^ν is ν's productive endowment, where ω_{kT}^ν is the endowment inherited by ν, when born in kT. Finally, ν's wealth at t is $W_t^\nu \equiv p_t \omega_t^\nu$, while s_t^ν denotes ν's net savings.

As concerns credit markets and savings, it is assumed that productive assets must be bought with current wealth, while consumption and savings must be financed out of current revenue. As in Roemer (1982A, 1988), this assumption rules out intertemporal credit markets and intertemporal trade *between agents*, consistently with the subsistence hypothesis. However, due to the possibility of saving, it allows for intertemporal trade-offs in the allocation of labour *during an agent's life*, consistently with a dynamic

[4] See Veneziani (2005A) for an analysis of the n-good economy.

setting in which agents' lives are divided into more than one period and it represents a dynamic extension of Roemer's models.

Let $x^{\nu} = \{x_t^{\nu}\}_{t=kT,\ldots,(k+1)T-1}$ be ν's lifetime plan of activity levels operated as a self-employed producer, and let a similar notation hold for y^{ν}, z^{ν}, s^{ν}, and ω^{ν}. Let $(p, w) = \{p_t, w_t\}_{t=kT,\ldots,(k+1)T-1}$ be the intertemporal path of the price vector during the lifetime of a generation.[5] Let $\xi^{\nu} = (x^{\nu}, y^{\nu}, z^{\nu}, s^{\nu})$ denote a generic intertemporal plan for ν. Let $\Lambda_t^{\nu} \equiv Lx_t^{\nu} + z_t^{\nu}$: Λ_t^{ν} denotes total labour expenditure by ν in t. Let $0 < \beta \leq 1$ be the time preference factor. Given ω_{kT}^{ν}, each ν solves the programme MP, whose value is $V(\omega_{kT}^{\nu})$.[6]

$$\text{MP: } V(\omega_{kT}^{\nu}) = \min_{\xi^{\nu}} \sum_{t=kT}^{(k+1)T-1} \beta^t \Lambda_t^{\nu},$$

subject to: $p_t(1 - A)x_t^{\nu} + [p_t(1 - A) - w_tL]y_t^{\nu} + w_tz_t^{\nu} \geq p_tb + p_ts_t^{\nu}$, (1)

$p_tA(x_t^{\nu} + y_t^{\nu}) \leq p_t\omega_t^{\nu}$, (2)

$Lx_t^{\nu} + z_t^{\nu} \leq 1$, (3)

$\omega_{t+1}^{\nu} = \omega_t^{\nu} + s_t^{\nu}$, (4)

$\omega_{(k+1)T}^{\nu} \geq \omega_{kT}^{\nu}$, (5)

$x_t^{\nu}, y_t^{\nu}, \omega_t^{\nu} \geq 0$, and $z_t^{\nu} \geq 0$.

Thus, agent ν chooses ξ^{ν} to minimise lifetime labour, both when self employed and when working for somebody else, subject to the constraints that in every t: (1) revenues are sufficient to reach subsistence and for savings plans; (2) wealth is sufficient for production plans; (3) labour performed

[5] The index k is not included in x^{ν}, y^{ν}, z^{ν}, s^{ν}, and (p, w) in order to simplify the notation.

[6] As shown below, there are price vectors such that the value of MP is indeed attained.

cannot exceed the working day, normalised to one; (4) the path of productive endowments is determined by net savings. Finally, agents are required not to deplete their resources at the end of their lives.

Let $O^{\nu}(p, w) \equiv \{\xi^{\nu}$ solves MP$\}$ be the set of individually optimal ξ^{ν}. Let $\Omega_{kT} \equiv (\omega^1_{kT}, \omega^2_{kT}, \ldots, \omega^N_{kT})$; let $E(A, L, b, \Omega_{kT})$, or as a shorthand $E(\Omega_{kT})$, be the economy described by technology (A, L), subsistence b, and distribution of endowments Ω_{kT}. Let $x_t = \sum_{\nu=1}^{N} x_t^{\nu}$; and likewise for y_t, z_t, s_t, ω_t.

DEFINITION 1. A *reproducible solution* (RS) for $E(\Omega_{kT})$ is an intertemporal profile (p, w) of the price vector and an associated set of actions such that

(i) $\xi^{\nu} = (x^{\nu}, y^{\nu}, z^{\nu}, s^{\nu}) \in O^{\nu}(p, w)$, for all ν;

(ii) $(x_t + y_t) \geq A(x_t + y_t) + Nb + s_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(iii) $A(x_t + y_t) \leq \omega_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(iv) $Ly_t = z_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(v) $\omega_{(k+1)T} \geq \omega_{kT}$.

Let "for all t" stand for "for all t, $t = kT, \ldots, (k + 1)T\text{-}1$". Condition (*i*) requires individual optimisation; (*ii*) and (*iii*) require that at all t, there are enough resources for consumption and saving plans, and for production plans, respectively; (*iv*) requires the labour market to clear at all t; (*v*) is the intertemporal *reproducibility condition*, which significantly relaxes the one-period reproducibility condition (Roemer, 1982A, Definition 2.1.(*ii*), p.64).

DEFINITION 2. An *interior reproducible solution* (IRS) for $E(\Omega_{kT})$ is a RS such that $s_t^{\nu} = 0$, for all ν, t.

Let $\Lambda^{\nu} = \sum_{t=kT}^{(k+1)T-1} \Lambda_t^{\nu}$. As in Roemer (1982a, 1988), agents who are able to reproduce themselves without working are assumed to use just the amount of wealth strictly necessary to reach subsistence and to satisfy the reproducibility constraint (v). In a subsistence economy, wealthy agents have no reason to accumulate or to consume more than b; hence, by stating that they do not "waste" their capital, Assumption 1 endows them "with embryonic capitalist behavior" (Roemer, 1982a, p.65).

ASSUMPTION 1: Let (p, w) be a RS for $E(\Omega_{kT})$. If there is a $\xi^{\nu} \in O^{\nu}(p, w)$ such that $\Lambda^{\nu} = 0$, then agent ν chooses y^{ν}, s^{ν} to minimise capital outlay.

In the remainder of this section some preliminary results of the static model are extended to the dynamic setting. First, at a RS the net revenues constraint (1) binds, for all agents, in every period t.

LEMMA 1. *Let (p, w) be a RS for $E(\Omega_{kT})$. Then* $p_t(1 - A)x_t^{\nu} + [p_t(1 - A) - w_tL]y_t^{\nu} + w_t z_t^{\nu} = p_t b + p_t s_t^{\nu}$, *all* t, ν.

Proof. Suppose $p_t(1 - A)x_t^{\nu} + [p_t(1 - A) - w_tL]y_t^{\nu} + w_t z_t^{\nu} > p_t b + p_t s_t^{\nu}$, some t, ν.

If $\Lambda^{\nu} = 0$, capital outlay can be reduced without destroying feasibility, contradicting A.1. Suppose $\Lambda^{\nu} > 0$. *Case 1*: $\Lambda_t^{\nu} > 0$. It is feasible to decrease either x_t^{ν} or z_t^{ν}, contradicting optimality. *Case 2*: $\Lambda_t^{\nu} = 0$. Let $\tau = \min \{j| \Lambda_j^{\nu} > 0, (k + 1)T - 1 \geq j > t\}$; it is possible to increase s_t^{ν} with $x_j^{\nu} = z_j^{\nu} = 0$, all j, $\tau - 1 \geq j \geq t$, making constraint (1) slack in τ and *Case 1* obtains. The proof of *Case 2* with $\Lambda_{\tau}^{\nu} > 0$, $t > \tau \geq kT$, is similar. ■

Next, wealth constraints (2) bind at all t, for all ν who work at a RS.

Lemma 2. *Let (p, w) be a RS for $E(\Omega_{kT})$ such that $p_t > p_tA + w_tL$, all t. If $\Lambda^\nu >$ 0 for all $\xi^\nu \in O^\nu(p, w)$, then $p_tA(x_t^\nu + y_t^\nu) = p_t\omega_t^\nu$, all t.*

Proof. Suppose not. Then it is possible to increase y_t^ν making the net revenue constraint slack in t. By Lemma 1, given that $\Lambda^\nu > 0$, $\xi^\nu \notin O^\nu(p, w)$. ■

The profit rate at time t is $\pi_t = [p_t(1 - A) - w_tL]/p_tA$. Lemma 3 proves that at an RS π_t is nonnegative and the price vector is strictly positive, all t.

Lemma 3. *Let (p, w) be a RS for $E(\Omega_{kT})$ such that $Nb + s_t > 0$, all t. Then, for all t, (i) $\pi_t \geq 0$, and (ii) $w_t > 0$ and $p_t > 0$.*

Proof. Part (i). As in (Roemer, 1982a, Lemma 2.2, p.66), for every period t.

Part (ii). At every t, if $w_t = 0$ then $z_t^\nu = 0$, all ν, while by Lemma 1, at the solution to (MP), $y_t^\nu > 0$, for all ν with $p_t\omega_t^\nu > 0$, so that $Ly_t > z_t = 0$. Hence $w_t > 0$ and $p_t \geq w_tL(1 - A)^{-1} > 0$. ■

Let $\lambda = L(1 - A)^{-1}$. Proposition 1 derives aggregate production and aggregate labour, both in every t and during the lifetime of generation k.

Proposition 1. *Let (p, w) be a RS for $E(\Omega_{kT})$. Then:*

(i) $x_t + y_t = (1 - A)^{-1} (Nb + s_t)$, *all t;*

(ii) $\Lambda_t = Lx_t + z_t = N\lambda b + \lambda s_t$, *all t;*

(iii) $\sum_{t=kT}^{(k+1)T-1} (x_t + y_t) = (1 - A)^{-1} NTb$;

(iv) $\Lambda = \sum_{\nu=1}^{N} \Lambda^\nu = NT\lambda b.$

Proof. Part (i)-(ii). See Roemer (1982a, Theorem 2.1, p.67). *Part (iii)-(iv).* Given A.1, by optimality, $\sum_{t=kT}^{(k+1)T-1} s_t^\nu = \mathbf{0}$, all ν. ■

Two properties of a RS allow one to simplify the notation considerably. First, since at the solution to MP, $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$, all ν, if (p, w) is a RS for $E(\Omega_{kT})$, it is a RS for $E(\Omega_{(k+1)T})$ and (p, w) can be interpreted as a stationary solution. Thus, we shall consider $k = 0$ without loss of generality. Second, let $\mathbf{1} = (1, \ldots, 1)'$: since at a RS $w_t > 0$, all t, labour can be chosen as the numeraire, setting $w_t = 1$, all t, and we shall consider reproducible solutions of the form $(p, \mathbf{1})$ without loss of generality.

3. MARXIAN EXPLOITATION AND CLASSES

In order to analyse exploitation in the intertemporal context, first of all, it is necessary to extend the concept of *Socially Necessary Labour Time*.

DEFINITION 3. *Socially Necessary Labour Time in t* is the amount of labour time needed by ν to reproduce herself in t: $SNLT_t \equiv \lambda b$. *Aggregate Socially Necessary Labour Time in t* is the amount of time needed by society to reproduce itself in t: $ASNLT_t \equiv N\lambda b$. Similarly, considering whole lives, *Socially Necessary Labour Time* and *Aggregate Socially Necessary Labour Time* are defined, respectively, as $SNLT \equiv T\lambda b$ and $ASNLT \equiv TN\lambda b$.

Let $\Delta^{\nu} = \sum_{t=0}^{T-1}(\Lambda^{\nu}_t - \lambda b)$. Unlike in the static model, there are two different criteria to define the exploitation status, focusing on the amount of labour performed either in each period of her life, or during her whole life.[7]

[7] For a discussion of *single periods* and *whole lives* definitions of normative concepts in the context of egalitarian theories (see, McKerlie, 1989; Temkin, 1993; Veneziani, 2005c).

DEFINITION 4. Agent ν is *exploited within period t*, or WP_t *exploited*, if $\Lambda_t^\nu > \lambda b$; a WP_t *exploiter* if $\Lambda_t^\nu < \lambda b$; and WP_t *exploitation-neutral* if $\Lambda_t^\nu = \lambda b$. Similarly, ν is *exploited during her whole life*, or *WL exploited*, if $\Delta^\nu > 0$; a *WL exploiter* if $\Delta^\nu < 0$; and *WL exploitation-neutral* if $\Delta^\nu = 0$.

The *WP* and *WL* definitions incorporate different normative concerns. An analysis based on the *WL* definition reflects the intuition that, from an *individual's* viewpoint, to be exploited in every period is certainly worse than being exploited only in some periods. However, the *WL* criterion leads to the rather counterintuitive conclusion that there would be no Marxist objection to "*changing places capitalism*," i.e. to a capitalist economy in which exploitation, - no matter how significant and widespread, - existed in every period, but the agents' status changed over time so as to equalise the amount of exploitation suffered by every individual.

Instead, the *WP* definition seems more suitable to capture Marx's idea that the existence of exploitation in the economy is morally relevant *per se*, and even a society with significant upward and downward social mobility is not necessarily just. Indeed, from a Marxian perspective, "we might want to consider exploitation as a property of the economy as a whole, not just of individuals" (Elster, 1985, p.176), and as a qualitative as well as a quantitative condition, so that *society* should not want anyone to be in a relationship of exploiter or exploited with respect to anyone else. Hence, although both criteria convey normatively relevant information, the main conclusions of this paper focus on the *WP* definition,[8] which seems also more suitable to analyse the dynamics of exploitation.

Proposition 2 derives labour expended by each ν in every t.[9]

PROPOSITION 2. *Let* $(p, \mathbf{1})$ *be a RS for* $E(\Omega_0)$. *Then* $\Lambda_t^\nu = \max\{0, [p_t b + p_t s_t^\nu - \pi_t p_t \omega_t^\nu]\}$, *all* t, ν.

Proof. If $\Lambda^\nu > 0$ for all $\xi^\nu \in O^\nu(p, \mathbf{1})$, by Lemma 1, constraint (1) holds as an equality: $\pi_t p_t A(x_t^\nu + y_t^\nu) + (Lx_t^\nu + z_t^\nu) = p_t b + p_t s_t^\nu$. By constraint (2), $\Lambda_t^\nu = p_t b + p_t s_t^\nu - \pi_t p_t \omega_t^\nu \geq 0$, all t. If there is a $\xi^\nu \in O^\nu(p, \mathbf{1})$ such that $\Lambda^\nu = 0$. Then $\Lambda_t^\nu = 0$ and, by Lemma 1, $p_t b + p_t s_t^\nu - \pi_t p_t \omega_t^\nu \leq 0$, all t. $\blacksquare$

If agents save, it may be difficult to extend the asset-based theory of exploitation to the dynamic context: given the optimality of $\sum_{t=0}^{T-1} s_t^\nu = 0$, all ν, and the linearity of MP, an agent can be a WP_t exploiter while being WP_{t+j} exploited, $j \neq 0$, depending on the paths of savings and wealth (and only indirectly on ω_0^ν). However, such changes in *WP* exploitation status do not necessarily convey morally relevant information: the fact that in a non-interior RS a relatively wealthy agent might optimally work more than λb in t, in order to accumulate more assets and minimise labour in $t + j$, does not seem to raise serious moral concerns. Actually, by Proposition 1.(*ii*), if $s_t \neq 0$

[8] This applies to Roemer's concept of exploitation as "a *property* of individuals or of whole economies, not primarily as a relation between individuals" (Elster, 1985, p.173). In the latter case, the theoretical importance of *WP* exploitation would be even clearer.

[9] By Proposition 2, it follows that $p_t b \leq 1$, all t, is a sufficient condition for $Lx_t^\nu + z_t^\nu \leq 1$, for all ν, t, to hold at an IRS. It is also necessary if there are agents with $\omega_0^\nu = 0$.

then $\Lambda_t \neq ASNLT_t$ and there is no conceptual equivalence between *WP* exploitative and inegalitarian solutions: only at an IRS, if an agent works less than λb, there must be another agent working more than λb.[10]

Proposition 3 proves a necessary condition for the existence of an IRS.

PROPOSITION 3. *Let (p,* ***1****) be an IRS for* $E(\Omega_0)$*. Then* $p_t = \beta(1 + \pi_{t+1})p_{t+1}$*, all t.*

Proof. By Proposition 1, consider ν with $\Lambda^\nu > 0$ for all $\xi^\nu \in O^\nu(p, \mathbf{1})$, and take j such that $\Lambda_{j+1}{}^\nu > 0$. By Proposition 2, $\Lambda_j{}^\nu = p_j b - \pi_j p_j \omega_0{}^\nu \geq 0$ and $\Lambda_j{}^\nu + \beta\Lambda_{j+1}{}^\nu = p_j b + \beta p_{j+1} b - \pi_j p_j \omega_0{}^\nu - \beta\pi_{j+1} p_{j+1}\omega_0{}^\nu$. A necessary condition for $\omega_t{}^\nu = \omega_0{}^\nu$, all t, to be optimal is that there is no ξ'^ν such that $\omega_{j+1}{}'^\nu \neq \omega_0{}^\nu$, $0 \leq j \leq T - 2$, $\omega_t{}'^\nu = \omega_0{}^\nu$, all $t \neq j + 1$, and $\Lambda_j{}'^\nu + \beta\Lambda_{j+1}{}'^\nu < \Lambda_j{}^\nu + \beta\Lambda_{j+1}{}^\nu$. Consider a one-period perturbation $(s_j{}'^\nu, s_{j+1}{}'^\nu)$ of the putatively optimal ω^ν such that $\omega_{j+1}{}'^\nu = \omega_0{}^\nu + s_j{}'^\nu$, $\omega_{j+2}{}'^\nu = \omega_0{}^\nu = \omega_{j+1}{}'^\nu + s_{j+1}{}'^\nu$, and thus $s_j{}'^\nu = - s_{j+1}{}'^\nu$. Suppose $p_j < \beta(1 + \pi_{j+1})p_{j+1}$. By Proposition 2:

$$\Lambda_j{}'^\nu + \beta\Lambda_{j+1}{}'^\nu = \Lambda_j{}^\nu + \beta\Lambda_{j+1}{}^\nu + [p_j - \beta(1 + \pi_{j+1})p_{j+1}]s_j{}'^\nu.$$

Hence $s_j{}'^\nu > 0$ yields $\Lambda_j{}'^\nu + \beta\Lambda_{j+1}{}'^\nu < \Lambda_j{}^\nu + \beta\Lambda_{j+1}{}^\nu$. And likewise if $p_j > \beta(1 + \pi_{j+1})p_{j+1}$. ■

In other words, if $p_t < \beta(1 + \pi_{t+1})p_{t+1}$, by setting $s_t{}^\nu = - s_{t+1}{}^\nu > 0$, $\Lambda_t{}^\nu$ increases by $p_t s_t{}^\nu$, but $\Lambda_{t+1}{}^\nu$ decreases by a larger amount given by $\pi_{t+1}p_{t+1}s_t{}^\nu$, due to the additional hirings, and by $p_{t+1}s_{t+1}{}^\nu$, due to the decumulation of the

[10] This argument does not apply to the *WL* definition of exploitation: the existence of a general monotonic relationship between initial wealth and *WL* exploitation at a RS where agents save is an interesting issue for further research.

additional resources. The opposite holds if $p_t > \beta(1 + \pi_{t+1})p_{t+1}$ and in general, if $p_t \neq \beta(1 + \pi_{t+1})p_{t+1}$ it is not optimal to set $s_t^\nu = \mathbf{0}$.

Let $W_t^* \equiv (p_t b - \lambda b)/\pi_t$: by Proposition 2, at an IRS, W_t^* is the wealth at t associated with a working time of $\Lambda_t^\nu = \lambda b$ for its possessor. The next result proves that at an IRS, the *WL* and *WP* definitions of exploitation are equivalent and extends Roemer's theory of exploitation based on the agents' initial wealth W_0^ν to the dynamic context. This suggests that the static model can be interpreted as a special case of the intertemporal model under the *assumption* that $s_t^\nu = 0$, all ν.

PROPOSITION 4. *Let $(p, \mathbf{1})$ be an IRS for $E(\Omega_0)$ with $\pi_0 > 0$. Then $\Delta^\nu > 0$ and $\Lambda_t^\nu > \lambda b$, all t, if and only if $W_0^\nu < W_0^*$; $\Delta^\nu = 0$ and $\Lambda_t^\nu = \lambda b$, all t, if and only if $W_0^\nu = W_0^*$; and $\Delta^\nu < 0$ and $\Lambda_t^\nu < \lambda b$, all t, if and only if $W_0^\nu > W_0^*$.*

Proof. 1. At all t, $W_t^\nu = W_t^*$ is equivalent to $\pi_t W_t^\nu = [p_t(1 - A) - L](1 - A)^{-1}b$, or $p_t\omega_0^\nu = p_t A(1 - A)^{-1}b$. Thus, if $W_t^\nu = W_t^*$ then $W_{t+1}^\nu = W_{t+1}^*$, at all t. Similarly, $W_t^\nu > W_t^\mu$ implies $W_{t+1}^\nu > W_{t+1}^\mu$, for any ν, μ, and all t.

2. By Proposition 2 and the strict monotonicity of $p_t[b - \pi_t\omega_0^\nu]$ in W_t^ν, at all t: $\Lambda_t^\nu > \lambda b$ if and only if $W_t^\nu < W_t^*$, $\Lambda_t^\nu = \lambda b$ if and only if $W_t^\nu = W_t^*$, and $\Lambda_t^\nu < \lambda b$ if and only if $W_t^\nu > W_t^*$. Hence, by part 1 $\Lambda_0^\nu > \lambda b$ implies $\Lambda_t^\nu > \lambda b$, all $t > 0$, and thus $\Delta^\nu > 0$. Conversely, if $\Delta^\nu > 0$, it must be $\Lambda_t^\nu > \lambda b$, for at least some $t \geq 0$. However, as just shown, *WP* exploitation status cannot change over time, and thus $\Lambda_t^\nu > \lambda b$, all $t \geq 0$. The other two cases are proved similarly. ■

Let $X^\nu \equiv \sum_{t=0}^{T-1} x_t^\nu$, $Y^\nu \equiv \sum_{t=0}^{T-1} y_t^\nu$, $Z^\nu \equiv \sum_{t=0}^{T-1} z_t^\nu$; let $\Gamma^\nu \equiv \{(X^\nu, Y^\nu, Z^\nu) | \xi^\nu \in O^\nu(p, w)\}$ and $\Gamma_t^\nu \equiv \{(x_t^\nu, y_t^\nu, z_t^\nu) | \xi^\nu \in O^\nu(p, w)\}$; let (a_1, a_2, a_3) be a vector where $a_i = \{+, 0\}$, all i, and "+" denotes a non-zero value. There are two possible dynamic extensions of Roemer's definition of class.

DEFINITION 5. Let (p, w) be a RS for $E(\Omega_0)$. ν is said to be a member of *WP class* (a_1, a_2, a_3) in t, if there is a $\xi^\nu \in O^\nu(p, w)$ such that $(x_t^\nu, y_t^\nu, z_t^\nu)$ has the form (a_1, a_2, a_3) in t. Similarly, ν is said to be a member of *WL class* (a_1, a_2, a_3), if there is a $\xi^\nu \in O^\nu(p, w)$ such that (X^ν, Y^ν, Z^ν) has the form (a_1, a_2, a_3).

Although there are seven possible classes (a_1, a_2, a_3) for each definition, the theoretical relevance of $(+, +, +)$ and $(0, +, +)$ is unclear from a Marxian viewpoint. Instead, a more specific definition of the remaining five classes can be provided. According to the *WL* definition:

$C^1 = \{ \nu | \Gamma^\nu$ contains a solution $(0, +, 0)\}$,

$C^2 = \{ \nu | \Gamma^\nu$ contains a solution $(+, +, 0)$, but not one of form $(+, 0, 0)\}$,

$C^3 = \{ \nu | \Gamma^\nu$ contains a solution $(+, 0, 0)\}$,

$C^4 = \{ \nu | \Gamma^\nu$ contains a solution $(+, 0, +)$, but not one of form $(+, 0, 0)\}$,

$C^5 = \{ \nu | \Gamma^\nu$ contains a solution $(0, 0, +)\}$.

WP classes C_t^1 to C_t^5 are specified similarly, by replacing Γ^ν with Γ_t^ν. First, Lemmas 4 and 5 extend a result of the static model.

LEMMA 4. *Let $(p, \mathbf{1})$ be a RS for $E(\Omega_0)$. Let $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ be such that ν is a WP member of* $(+, +, +)$ *or* $(0, +, +)$ *in t: if $Ly_t^\nu > z_t^\nu$ then $\nu \in$* $(+, +, 0)$*, in t; if $Ly_t^\nu = z_t^\nu$ then $\nu \in$* $(+, 0, 0)$*, in t; and if $Ly_t^\nu < z_t^\nu$ then $\nu \in$* $(+, 0, +)$*, in t.*

Proof. Let ν's solution be ξ^ν with $(x_t^\nu, y_t^\nu, z_t^\nu, s_t^\nu)$ in t. The rest of the proof is as in Roemer (1982A, Lemma 2.4, p.75), for every t, given s^ν. $\blacksquare$

In other words, every *WP* member of (+, +, +) or (0, +, +) in t is also a *WP* member of either (+, 0, +), or (+, 0, 0), or (+, +, 0), in t. Therefore C_t^1 to C_t^5 are sufficient to fully describe the *WP* class structure of the economy in t.

LEMMA 5. *Let $(p, \mathbf{1})$ be a RS for $E(\Omega_0)$. Then (i) if ν is a WL member of* (+, +, 0) *then ν is a WP member of either* (+, +, 0), *or* (+, 0, 0), *or* (0, +, 0), *all t; (ii) if ν is a WL member of* (+, 0, +) *then ν is a WP member of* (+, 0, +) *or* (+, 0, 0) *or* (0, 0, +), *all t; (iii) $\nu \in C^1$ if and only if $\nu \in C_t^1$, all t, $\nu \in C^3$ if and only if $\nu \in C_t^3$, all t, and $\nu \in C^5$ if and only if $\nu \in C_t^5$, all t.*

Proof. Straightforward, given Lemma 4, $x_t^\nu, y_t^\nu \geq 0$, and $z_t^\nu \geq 0$. $\blacksquare$

Lemmas 4 and 5 highlight some limitations of a definition of classes based on the net amount of labour performed by an agent. Consider, for instance, ν such that $\Lambda^\nu > 0$. By Lemma 1 and rearranging ν's net revenues constraint in t, it ca be shown, as in Section 3, that, given the linearity of MP and the optimality of $\sum_{t=0}^{T-1} s_t^\nu = 0$, in a non-interior RS, the sign of $z_t^\nu - Ly_t^\nu$ and thus, by Lemma 4, *WP* class status can change over time, depending on dynamic paths of the price vector, optimal savings, and wealth. However, again, such change in *WP* class status does not necessarily reflect genuine inter-class mobility and may simply be the product of intertemporal labour trade-offs with little normative content.

As for the *WL* criterion, Lemmas 4 and 5 imply that in general the five *WL* classes are not exhaustive: agents whose *WP* class status switches, e.g., from C_t^2 to C^4_{t+j}, $j > 0$, do not belong to any of C^1 to C^5 and form instead a *WL* class whose members have a solution (+, +, +) or (0, +, +) in Γ^ν. Yet, the interpretation of the latter classes (a potentially large portion of the society) in Marxian terms is unclear, raising doubts on the *WL* criterion and *a fortiori* on the definition of classes based on the net amount of labour performed.

Moreover, due to the crucial role of savings, there is no general relation between *WP* and *WL* classes and W_0^ν. Thus, for instance, $W_0^\nu = 0$ does not necessarily imply $x_t^\nu = y_t^\nu = 0$, all t, and ν does not necessarily belong to C_t^5 after $t = 0$.[11] If time is added, the element of lack of freedom (intended as a severely limited set of available options) that is important in Marx's definition of a proletarian is lost. In the static economy, this element is incorporated in the initial conditions; in the intertemporal setting, ω_0^ν is a much weaker constraint on ν's set of options and ω_t^ν is a choice variable.

Proposition 5 can now be derived, which generalises Roemer's theory of classes to the dynamic context: at an IRS, *WL* and *WP* classes coincide, both *WP* classes C_t^1-C_t^5 and *WL* classes C^1-C^5 are pairwise disjoint and exhaustive, and the correspondence between class and exploitation status holds for both the *WL* and the *WP* definitions.

PROPOSITION 5. *Let $(p, \mathbf{1})$ be an IRS for $E(\Omega_0)$ with $\pi_0 > 0$. Then (i) for all $1 \leq i < j \leq 5$, $C_t^i \cap C_t^j = \{\emptyset\}$ and if $\nu \in C_t^i$ and $\mu \in C_t^j$, then $\Lambda_t^\mu > \Lambda_t^\nu$, all t; (ii) For all j, if $\nu \in C_0^j$ then $\nu \in C_t^j$, all t, and $\nu \in C^j$; conversely, if $\nu \in C^j$ then ν*

[11] Actually, they may be unable to save if $p_t b = 1$, i.e. if the real wage is set at the subsistence level (this is a common interpretation of Marx's theory of real wage determination).

$\in C_t^j$ *all t; (iii) (Class-Exploitation Correspondence Principle) If* $\nu \in C_0^1 \cup C_0^2$*, and thus* $\nu \in C^1 \cup C^2$*, then* $\Delta^\nu < 0$ *and* $\Lambda_t^\nu < \lambda b$*, all t while if* $\nu \in C_0^4 \cup C_0^5$*, and thus* $\nu \in C^4 \cup C^5$*, then* $\Delta^\nu > 0$ *and* $\Lambda_t^\nu > \lambda b$*, all t.*

Proof. Part (i). As in Roemer (1982A, Theorem 2.5, p.74). In particular, in every t: if $Ly_t^\nu > z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, then $\nu \in C_t^2$; if there is a $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ such that $Ly_t^\nu = z_t^\nu$, then $\nu \in C_t^3$; if $Ly_t^\nu < z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, then $\nu \in C_t^4$. Exactly one of these holds for every $\nu \notin C_t^1 \cup C_t^5$.

Part (ii). First, at an IRS, if $\nu \in C_t^5$ then $\nu \in C_{t+1}^5$, all t, and therefore $\nu \in C^5$. Conversely, $\nu \in C^5$ implies $\nu \in C_t^5$, all t. Next, if $p_0\omega_0^\nu > (p_0 b)/\pi_0$ but $p_t\omega_0^\nu < (p_t b)/\pi_t$, some t, clearly $s_0^\nu = 0$ cannot be optimal. Hence, if $\nu \in C_0^1$ then $\nu \in C_t^1$, all t, and $\nu \in C^1$. Conversely, $\nu \in C^1$ implies $\nu \in C_t^1$, all t. Finally, consider $\nu \in C_t^j$, $j = 2, 3, 4$. By Lemma 1 and Proposition 3, in any two adjacent periods:

$z_t^\nu - Ly_t^\nu = p_t[b - (1 - A)(x_t^\nu + y_t^\nu)]$,

$z_{t+1}^\nu - Ly_{t+1}^\nu = p_t\,[b - (1 - A)(x_{t+1}^\nu + y_{t+1}^\nu)]/\beta(1 + \pi_{t+1})$.

If $Ly_t^\nu > z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ then $Ly_{t+1}^\nu > z_{t+1}^\nu$ all $(x_{t+1}^\nu, y_{t+1}^\nu, z_{t+1}^\nu) \in \Gamma_{t+1}^\nu$. Similar arguments hold if $Ly_t^\nu < z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, or if there is a $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ with $Ly_t^\nu = z_t^\nu$. Hence, by part (*i*), $\nu \in C_t^j$ implies $\nu \in C_{t+1}^j$, all t, and thus $\nu \in C^j$. Conversely, suppose that $\nu \in C^j$: since $\nu \in C_t^j$ implies $\nu \in C_{t+1}^j$, all t, and by Lemma 5, part (*ii*) follows.

Part (*iii*). By Proposition 4, $\Lambda_t^\nu = \lambda b$, all t, and $\Delta^\nu = 0$ if and only if $W_0^\nu = W_0^*$, while if $W_0^\nu = W_0^*$, by setting $y_0^\nu = (1 - A)^{-1}b$, it is easy to verify that Γ_0^ν contains a solution with $Ly_0^\nu = z_0^\nu$ so that by part (*i*), $\nu \in C_0^3$.

Next, since by part (*i*) Λ_t^{ν} is monotonic in WP_t class status, if $\nu \in C_0^1 \cup C_0^2$ then $\Lambda_0^{\nu} < \lambda b$, while if $\nu \in C_0^4 \cup C_0^5$ then $\Lambda_0^{\nu} > \lambda b$. Then the result follows from part (*ii*) and Proposition 4. $\blacksquare$

4. EXPLOITATION, ASSET INEQUALITY, AND TIME

Given Propositions 1, 4, and 5, it is natural to focus on IRS's to analyse the links between exploitation, class, and wealth in the intertemporal context. The next results derive the conditions under which Roemer's (1982A, 1988) theory of exploitation can be extended to the intertemporal context, and at the same time highlight the conceptual links and differences between his definition of exploitation and neoclassical welfare inequalities.

THEOREM 1. *Let $\pi' = (1 - \beta)/\beta$ and let p' be the associated price vector. If $p_t = p'$, all t, and $p'b \leq 1$ then for all ν, $s_t^{\nu} = 0$, all t, is optimal and if T is finite, then $V(\omega_0^{\nu}) = \max\ \{0, (1 - \beta^T)[p'b\beta/(1 - \beta) - W_0^{\nu}]/\beta\}$, while if $T \to \infty$, then $V(\omega_0^{\nu}) = \max\ \{0, p'b/(1 - \beta) - W_0^{\nu}/\beta\}$.*

Proof. 1. Suppose $W_0^{\nu} \geq p'b\beta/(1 - \beta)$. The vector ξ^{ν} such that $s_t^{\nu} = 0$, all t, and $y_t = y'$ all t, with $\pi' Ay' = b$ is optimal and $\Lambda_t^{\nu} = 0$, all t.

2. Suppose $W_0^{\nu} < p'b\beta/(1 - \beta)$, so that $\Lambda^{\nu} > 0$ for all $\xi^{\nu} \in O^{\nu}(p, \mathbf{1})$. Write MP using dynamic optimisation theory. Let $W \subseteq \Re_+$ be the state space. Let Ψ: $W \to W$ be the feasibility correspondence. $\Psi(\omega_t^{\nu})$ is the set of feasible values for the state next period, ω_{t+1}^{ν}, if the current state is ω_t^{ν}: $\Psi(\omega_t^{\nu}) = \{\omega_{t+1}^{\nu} \in W$: $\omega_{t+1}^{\nu} \geq 0$ and $p_t\omega_{t+1}^{\nu} \leq 1 - p_tb + p_t\omega_t^{\nu} + \pi_tp_t\omega_t^{\nu}\}$. Let $\Pi(\omega_0^{\nu}) = \{\omega^{\nu}$: $\omega_{t+1}^{\nu} \in \Psi(\omega_t^{\nu})$, all t, $\omega_T^{\nu} \geq \omega_0^{\nu}$, and ω_0^{ν} given$\}$ be the set of feasible sequences ω^ν. Let $\Phi = \{(\omega_t^\nu, \omega_{t+1}^\nu) \in W \times W: \omega_{t+1}^\nu \in \Psi(\omega_t^\nu)\}$ be the graph of Ψ. The *one-period return function* F: $\Phi \to \Re_+$ at t is $F(\omega_t^\nu, \omega_{t+1}^\nu) = p_t b + p_t(\omega_{t+1}^\nu - \omega_t^\nu) - \pi_t p_t \omega_t^\nu$. Then,

MP $\quad V(\omega_0^\nu) = \min_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t [p_t b + p_t(\omega_{t+1}^\nu - \omega_t^\nu) - \pi_t p_t \omega_t^\nu]$.

If $p_t b - \pi_t p_t \omega_t^\nu \leq 1$, all t, then $\Psi(\omega_t^\nu) \neq \emptyset$, all $\omega_t^\nu \in W$. Then, since F is continuous and bounded, MP is well defined for all T.

2. If $p_t = p'$, all t, then $p_t b - \pi_t p_t \omega_t^\nu \leq 1$, all t, ν, and MP becomes:

$$V(\omega_0^\nu) = \min_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t p' b + \beta^{T-1} p' \omega_T^\nu - (1 + \pi') p' \omega_0^\nu.$$

Therefore, for all T, any feasible ω^ν such that $\omega_T^\nu = \omega_0^\nu$ (or $\lim_{T\to\infty} \omega_T^\nu = \omega_0^\nu$, if $T \to \infty$) is optimal and $V(\omega_0^\nu)$ immediately follows.

3. The last part of the statement is straightforward. ■

Given Theorem 1, the next result characterises welfare inequalities and exploitation at an IRS, if agents discount future labour.

THEOREM 2. *Let $1 > \beta$. Let $(p, \mathbf{1})$ be an IRS for $E(\Omega_0)$ with $\pi_t = (1 - \beta)/\beta$, all t. Then: (i) for all ν and μ, if $W_0^\mu < p'b\beta/(1 - \beta)$ then $V(\omega_0^\nu) < V(\omega_0^\mu)$ if and only if $W_0^\nu > W_0^\mu$; (ii) there is a constant k^ν such that $\Lambda_t^\nu - \lambda b = k^\nu$, all t, ν.*

Proof. *Part (i)*. Directly from Theorem 1, since $V(\omega_0^\nu) = 0$ if and only if $W_0^\nu \geq p'b/\pi'$; while if $V(\omega_0^\nu) > 0$ then $V(\omega_0^\nu) - V(\omega_0^\mu) = (1 - \beta^T)\,[W_0^\mu - W_0^\nu]/\beta$, if T is finite, while $V(\omega_0^\nu) - V(\omega_0^\mu) = [W_0^\mu - W_0^\nu]/\beta$, if $T \to \infty$.

Part (ii). Straightforward, given Proposition 2. ■

Theorems 1 - 2 complete the intertemporal generalisation of Roemer's theory: the dynamic economy with discounting displays the same pattern of *WP* and *WL* exploitation as the *T*-fold repetition of the static economy, and both *WP* and *WL* exploitation are persistent. Moreover, unlike in the static model, the introduction of time preference clarifies - at the *WL* level - the difference between Roemer's interpretation of Marxian exploitation as an objectivist measure of inequalities – "the exploitation–welfare criterion" (Roemer, 1982A, p.75) – and subjectivist neoclassical welfare inequalities, which instead depend on β. According to Theorems 1-2, the two views coincide at an IRS, but in principle they are conceptually distinct.

However, Theorems 1-2 crucially depend on the assumption that $\beta < 1$. If $\beta = 1$, the only constant price vector that satisfies Proposition 3 implies zero profits leading to an egalitarian, non-exploitative IRS. In general, it can be proved (Veneziani, 2005A) that in the RS that preserves asset inequalities and the class and exploitation structure of the capitalist economy, profits and *WP* exploitation decrease over time: *WP* exploiters work more, while the *WP* exploited work less, even if neither accumulates. The simple possibility of saving implies a decrease in the dispersion of agents' labour around λb, due to the decrease in profits. If $T \to \infty$, then at an IRS, profits and *WP* exploitation tend to disappear in the long run, even if DOSPA persists.[12]

This is arguably unsatisfactory. The moral relevance of time preference is disputable, even in non-Marxian approaches (e.g., Sidgwick, 1907;

[12] Moreover, there is a decrease in the dispersion of *WP* classes around the petty bourgeois and there are no big capitalists (Veneziani, 2005A, Theorem 4).

Ramsey, 1928; Rawls, 1971) and a theory of persistent inequalities that crucially depends on time preference seems objectionable. This is particularly relevant in this model since, by Theorems 1 - 2, both the persistence and the magnitude of exploitation and inequalities depend on time preference. Given the positive relation between the profit rate and inequalities-exploitation, the higher β, the lower the profit rate in the RS with constant prices, and thus the lower exploitation, *ceteris paribus*.

Even more importantly, although the above results highlight the conceptual links with the neoclassical analysis of welfare inequalities, Roemer's theory is intended to be an interpretation and generalisation of Marx's theory of exploitation. Arguably, time preference plays no essential role in the latter and thus an explanation of persistent exploitation based on exogenous time preference is far from Marx's.

Of course, it could be noted that *WL* exploitation does not disappear, even if $\beta = 1$, and if the condition in Proposition 3 holds, the relationship between initial wealth and *WL* exploitation status is preserved. Thus, from a mathematical viewpoint, the model may be interpreted as a generalisation of Roemer's theory of exploitation under the *WL* definition. Yet, this does not affect the main conclusions of the paper. First, given the theoretical relevance of the *WP* definition discussed above, Marxian exploitation should arguably be microfounded as a persistent *WP* phenomenon. Second, the tendential disappearance of *WP* exploitation is not only disturbing *per se* for a model that aims to provide microfoundations to Marx's theory; it also implies that *ceteris paribus*, *WL* exploitation, too, is lower in the dynamic model with agents living for *T* periods than in the *T*-fold iteration of the static model.

From a *methodological* viewpoint, these results suggest that the static models are a point of departure for the analysis of the microfoundations of Marxian economics. However, in order to provide robust and theoretically convincing microfoundations to exploitation theory, it may be necessary to go beyond "*standard* general equilibrium models" (Roemer, 1986, p.193, italics added). The model presented shows that the results of the static models depend crucially on the absence of intertemporal credit markets and either on the impossibility of savings, or on a strictly positive rate of time preference. Arguably, either pair of assumptions is far from Marx's theory and/or from the Walrasian benchmark.

From a *substantive* viewpoint, *WP* exploitation tends to disappear even if wealth inequalities - and, unlike in accumulation models (Devine and Dymsky, 1991), capital scarcity - remain an inherent equilibrium feature of the economy. At a RS where no agent accumulates and capital scarcity persists, DOPA is necessary to generate exploitation, but it is not sufficient for the latter to persist. This proves Cohen's claim that "the asset distribution is unjust because it enables or makes possible an unjust flow" (Cohen, 1995, p.207), but it does not necessitate such flow. Thus, the persistence of inequalities in the ownership of productive assets is not a sufficient statistic of the unfairness of the labour/capital relations (and more generally, of the society) from a Marxist perspective. Indeed, since DOSPA is not necessary and sufficient to generate persistent exploitation, asset inequalities seem "a normatively secondary (though causally primary) wrong" (ibid., p.199).

If correct, these conclusions raise two issues for further research. First, although the results hold at an RS with persistent capital scarcity, the analysis of the mechanisms that guarantee the persistent abundance of *labour* in a capitalist economy is a natural extension of Roemer's theory. Indeed, Skillman (1995, 2001) suggests that a dynamic model including growth in the labour force and/or labour-saving technical progress might provide microfoundations to persistent exploitation. The relation between economic inequality, growth, and relative factor scarcity is a long-debated issue in economics, and certainly a crucial one for socialists and egalitarians.

However, even if exploitation could be proved to be persistent under those assumptions, it is unclear whether the main conclusions of this paper would change. In particular, it would remain true that DOPA and competitive markets are not sufficient to yield persistent *WP* exploitation, which raises the issue of the definition of exploitation, the second line for further research.

5. WHAT IS EXPLOITATION?

In section 4, Roemer's definition of Marxian exploitation is shown to be distinct from, but conceptually related to neoclassical welfare inequalities. At the same time, even in Roemer's interpretation, exploitation is proved to be irreducible only to asset inequalities. But then, What is exploitation? In this section, Roemer's (1982A) game-theoretical approach to exploitation is considered, which focuses on property relations, a more general concept than simple asset inequalities. A coalition $J \subseteq N$ is exploited if and only if three conditions hold (ibid., pp.194-195): (1) there is an hypothetically feasible alternative in which J would be better off than in its present situation; (2) under this alternative, the complement to J, the coalition $N - J = J'$, would be worse than at present; (3) J' is in a relation of dominance to J.

Conditions (1)-(3) can capture various kinds of exploitation, including Marxian exploitation, by specifying different hypothetically feasible alternatives. Let $\{V^1, \ldots, V^N\}$ be the agents' payoffs at the existing allocation: in Roemer's game-theoretical framework, it is natural to consider $\{V^1, \ldots, V^N\}$ as *WL* values. For instance, at an RS for $E(\Omega_0)$, $V^1 = - V(\omega_0{}^1), \ldots, V^N = - V(\omega_0{}^N)$. Let $P(N)$ denote the power set of N and let K: $P(N) \to \mathfrak{R}_+$ be a *characteristic function* which assigns to every coalition J of agents in the economy an aggregate payoff $K(J)$ in the case it withdraws.

DEFINITION 6. Coalition J is exploited at allocation $\{V^1, \ldots, V^N\}$ with respect to alternative K if and only if J' is in a relation of dominance to J and

(i) $\sum_{\nu \in J} V^\nu < K(J)$,

(ii) $\sum_{\nu \in J'} V^\nu > K(J')$.

By Definition 6, the concept of exploitation is related to the *core* of an economy: the set of nonexploitative allocations coincides with the core of the game described by K (ibid., Theorem 7.1, p.198). The precise definition of exploitation depends on the specific function K chosen. A coalition is *feudally exploited* at a given allocation if it can improve by withdrawing from society with its own endowments and arranging production on its own. In $E(\Omega_0)$, the set of feudally nonexploitative allocations coincide with the *private ownership core* (POC). Formally, a coalition J is viable if it has enough assets to reproduce itself if it secedes from the parent economy.[13]

DEFINITION 7. Let N be the set of producers. Let $J \subseteq N$ be any subset of N. Coalition J is *viable* if $\sum_{\nu \in J} \omega_0^\nu \geq JA(1-A)^{-1}b$.

A reproducible allocation is a set of (not necessarily optimal) actions of all agents in $E(\Omega_0)$, that satisfy the feasibility and reproducibility constraints.

DEFINITION 8. A *reproducible allocation* (RA) for $E(\Omega_0)$ is a set of actions $\xi^\nu = (x^\nu, y^\nu, z^\nu, s^\nu)$, for all ν, such that

(i) $Lx_t^\nu + z_t^\nu \leq 1$, all ν, t;

(ii) $A(x_t + y_t) \leq \omega_t$, all t;

(iii) $(x_t + y_t) \geq A(x_t + y_t) + Nb + s_t$, all t;

(iv) $\omega_{t+1} = \omega_t + s_t$, all t;

(v) $\omega_T \geq \omega_0$.

A viable coalition J can block a RA $\{\xi^\nu\}_{\nu=1,\ldots,N}$ if there is another RA for the smaller economy that yields higher welfare to its members.

DEFINITION 9. A viable coalition J can *block* a RA $\{\xi^\nu\}_{\nu=1,\ldots,N}$ if there is a vector $\{\xi'^1, \ldots, \xi'^J\}$ such that

(i) $\sum_{t=0}^{T-1} \beta^t \Lambda'^\nu_t < \sum_{t=0}^{T-1} \beta^t \Lambda^\nu_t$, for all $\nu \in$ J;

(ii) $A \sum_{\nu \in J} x_t'^\nu \leq \sum_{\nu \in J} \omega_t'^\nu$, all t;

[13] See Roemer (1982A, pp.45-49). With a slight abuse of notation, the same symbols are used here to denote both the sets J and N and their cardinalities J and N.

(*iii*) $(1 - A)\sum_{\nu\in J} x_t^{\nu} = Jb + \sum_{\nu\in J} s_t^{\nu}$, all t;

(*iv*) $\sum_{\nu\in J} \omega_{t+1}^{\nu} = \sum_{\nu\in J} \omega_t^{\nu} + \sum_{\nu\in J} s_t^{\nu}$, all t;

(*v*) $\sum_{\nu\in J} \omega_T^{\nu} \geq \sum_{\nu\in J} \omega_0^{\nu}$.

The POC of $E(\Omega_0)$ is the set of RA's which no coalition can block. Theorem 3 proves the absence of feudal exploitation in $E(\Omega_0)$.

THEOREM 3: *Let $\beta \leq 1$. The IRS's of $E(\Omega_0)$ lie in its private ownership core and thus display no feudal exploitation.*

Proof. 1. If $\pi_t = 0$, all t, then the result is trivial. Hence, assume $\pi_0 > 0$.

2. Suppose that there is a coalition J that can block the IRS. By Definition 9.(*i*), no pure capitalist can be part of J; thus, by Lemmas 1 - 2 and Proposition 2, at an IRS $\pi_t p_t \omega_0^{\nu} = p_t b - \Lambda_t^{\nu}$, all t and all $\nu \in J$. Summing over $\nu \in J$ and t, $\sum_{t=0}^{T-1} \beta^t \pi_t p_t \sum_{\nu\in J} \omega_0^{\nu} = \sum_{t=0}^{T-1} \beta^t J p_t b$ - $\sum_{t=0}^{T-1} \beta^t \sum_{\nu\in J} \Lambda_t^{\nu}$. By Proposition 3, $\sum_{t=0}^{T-1} \beta^t \pi_t p_t \sum_{\nu\in J} \omega_0^{\nu} = [(1 + \pi_0)p_0 - \beta^{T-1} p_{T-1}] \sum_{\nu\in J} \omega_0^{\nu}$.

3. If J can block the IRS, multiplying Definition 9.(*iii*) by $\beta^t\lambda$ and summing over t, $\sum_{t=0}^{T-1} \beta^t \sum_{\nu\in J} \Lambda'^{\nu}_t = \sum_{t=0}^{T-1} \beta^t J\lambda b + \sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu\in J} s_t^{\nu}$. By Definition 9.(*i*) and part 2: $\sum_{t=0}^{T-1} \beta^t J(p_t - \lambda)b - \sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu\in J} s_t^{\nu}$ $> [(1 + \pi_0)p_0 - \beta^{T-1} p_{T-1}] \sum_{\nu\in J} \omega_0^{\nu}$.

4. If J can block the IRS, by Definition 9.(*ii*)-(*iii*), $A(1 - A)^{-1}(Jb + \sum_{\nu\in J} s_t^{\nu}) \leq \sum_{\nu\in J} \omega_t^{\nu}$, all t; multiplying both sides by $\beta^t \pi_t p_t$, $\beta^t(p_t$ -

$\lambda)Jb$ - $\beta^t\lambda\sum_{\nu\in J} s_t^{\nu} \leq \beta^t\pi_t p_t \sum_{\nu\in J} \omega_t^{\nu}$ - $\beta^t p_t \sum_{\nu\in J} s_t^{\nu}$, all t. Summing over t, by Definition 9.(*iv*), the latter expression becomes $\sum_{t=0}^{T-1} \beta^t (p_t$ - $\lambda)Jb$ - $\sum_{t=0}^{T-1} \beta^t\lambda\sum_{\nu\in J} s_t^{\nu} \leq \sum_{t=0}^{T-1} \beta^t[(1 + \pi_t)p_t\sum_{\nu\in J} \omega_t^{\nu}$ - $p_t\sum_{\nu\in J} \omega_{t+1}^{\nu}]$. Then, using $\beta(1 + \pi_{t+1})p_{t+1} = p_t$, all t, $\sum_{t=0}^{T-1} \beta^t(p_t - \lambda)Jb$ - $\sum_{t=0}^{T-1} \beta^t\lambda\sum_{\nu\in J} s_t^{\nu} \leq (1 + \pi_0)p_0\sum_{\nu\in J} \omega_0^{\nu} - \beta^{T-1}p_{T-1}\sum_{\nu\in J} \omega_T^{\nu}$.

4. The latter inequality and the inequality in part 3 can both hold only if $\sum_{\nu\in J} \omega_T^{\nu} < \sum_{\nu\in J} \omega_0^{\nu}$, which contradicts Definition 9.(v). ■

In Roemer's interpretation of historical materialism as predicting the progressive disappearance of various forms of exploitation, Theorem 3 proves that capitalist relations of production eliminate feudal exploitation in $E(\Omega_0)$.[14] However, a different specification of K is necessary to define *capitalist* exploitation. Let $\omega_0^J \equiv (J/N)\omega_0$; ω_0^J is coalition J's per-capita share of aggregate initial assets. Given the linear technology, all coalitions are viable if they withdraw with ω_0^E. Then, a coalition can communally block a RA if it can increase the welfare of its members by withdrawing with ω_0^E.

DEFINITION 10. A coalition J can *communally block* a RA ξ^{ν} if there is a vector $\{\xi'^1, \ldots, \xi'^J\}$ such that

(i) $\sum_{t=0}^{T-1}\beta^t \Lambda'^{\nu}_t < \sum_{t=0}^{T-1}\beta^t \Lambda^{\nu}_t$, for all $\nu \in J$;

(ii) $A\sum_{\nu\in J} x_t^{\nu} \leq \sum_{\nu\in J} \omega_t^{\nu}$, all t;

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[14] It also clarifies the neoclassical claim concerning the absence of exploitation in a competitive economy: there is no *feudal* exploitation (Roemer, 1982A, pp.205-8).

(iii) $(1 - A)\sum_{\nu\in J} x_t^{\nu} = Jb + \sum_{\nu\in J} s_t^{\nu}$, all t;

(iv) $\sum_{\nu\in J} \omega_{t+1}^{\nu} = \sum_{\nu\in J} \omega_t^{\nu} + \sum_{\nu\in J} s_t^{\nu}$, all t;

(v) $\sum_{\nu\in J} \omega_T^{\nu} \geq \omega_0^{E}$.

The *communal core* of $E(\Omega_0)$ is the set of RA's which no coalition can communally block; a coalition is *capitalistically exploited* if it can communally block the RA; and a RA is *capitalist nonexploitative* if it lies in the communal core of the economy. Theorem 4 proves that Marxian exploitation and capitalist exploitation coincide in $E(\Omega_0)$ at an IRS.

THEOREM 4: *Let $\beta \leq 1$. At an IRS, a coalition is WL Marxian exploited if and only if it is capitalistically exploited.*

Proof. If a coalition J is Marxian exploited, $\sum_{t=0}^{T-1} (\sum_{\nu\in J} \Lambda_t^{\nu} - J\lambda b) > 0$. But then by Proposition 4, at an IRS $\sum_{t=0}^{T-1} \beta^t(\sum_{\nu\in J} \Lambda_t^{\nu} - J\lambda b) > 0$, and J can communally block the allocation. The converse is proved similarly. ■

Theorem 4 suggests that Marxian exploitation can be seen as a special case of Roemer's property-relation definition of exploitation in a linear economy with labour minimising agents. The property-relation definition (which can be applied to a general set of economies; see Roemer, 1982A, chapter 7) can be seen as a generalisation of Marx's theory that captures its essential normative content. However, the results show some limitations of the property-relation definition both *per se* and as a generalisation of Marx's theory. In fact, "the legitimacy of Roemer's reformulation depends in large part on the validity of his claims concerning the role of DOPA in capitalist exploitation" (Skillman, 1995, p.311). Since DOPA is necessary but not sufficient to generate persistent Marxian exploitation, then arguably the game-theoretic definition may be seen as incorporating a different moral concern, rather than as a generalisation of Marx's definition. More generally, the question arises whether DOPA should be a basic moral concern, both in itself and in a theory of exploitation, or rather a different role of DOPA should be stressed as a causally primary, but normatively secondary wrong.

However, this raises again the issue of the definition of exploitation and its differences with the notion of (welfare or asset) inequality. Arguably, Roemer's own theory provides some interesting insights on a possible direction for further research in a roemerian spirit. In section 5, it is argued that Roemer's non-relational definition of Marxian exploitation is related to the notion of welfare inequalities, but it does not coincide with it. The same argument applies to the property-rights definition, which also differs from welfare inequalities due to its inherently relational nature.

In particular, it may be argued that the dominance condition (3), which is not formally defined by Roemer, is not just necessary "to rule out some bizarre examples" (Roemer, 1982A, p.195) and it might play a more prominent role in a theory of exploitation as a feature of relations between people. For instance, if a microfoundational approach is adopted, the above analysis suggests that the *property rights theory of the firm* (Grossman and Hart, 1986; Hart and Moore, 1990) may provide a promising analytical and theoretical framework to analyse Marxian exploitation, given its concern with power and the emphasis on the role of physical assets in explaining hierarchical relations and the existence of firms. This approach might provide a framework to model exploitation consistent with the idea that asset inequalities are causally primary, but normatively secondary, in the explanation of exploitative relations, given various sources of contractual incompleteness (e.g., Marx's labour/labour-power distinction).

7. CONCLUSIONS

In this paper an intertemporal model of a subsistence economy is set up to analyse exploitation and classes in a dynamic context, to evaluate the causal and moral relevance of *Differential Ownership of Productive Assets*, and to assess the possibility of providing neoclassical microfoundations to Marxian models. A dynamic extension of Roemer's theory of exploitation and classes is provided in an economy with positive time preference, where exploitation and classes are proved to be persistent and to be directly related to asset inequalities. However, the normative relevance of time preference is questioned and it is shown that, absent time preference, asset inequalities and classes persist, while exploitation decreases over time and eventually disappears. Hence, asset inequalities are argued to be normatively secondary, though causally primary in explaining exploitation and the normative relevance of asset inequalities *per se* is put into question. Moreover, some methodological doubts are raised on the possibility of providing robust microfoundations to Marx's theory by means of standard Walrasian models. Finally, various directions for further research are suggested.

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